The Concept of the Effective Mass in the General Relativity

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Abstract

We consider the concept of the effective mass in the General Relativity. We analyze if the concept of the effective mass is suitable to describe the relation between curved geometry and the particle dynamics. We propose the physical experiment which could decide if the curvature of space-time is equivalent to the effective mass of the body.

keywords: general theory of gravity, Einstein's field equation, the effective mass of the body

PACS: 04.20.-q, 04.20.Cv

1. Introduction

Typically, mass of the body (the inertial or gravitational) is a scalar. The concept of mass in the General Relativity (GR) is more complex than the concept of mass in the special relativity. In fact, general relativity does not offer a single definition for the term mass, but offers several different definitions which are applicable under different circumstances [1, 4]. In this paper we will analyze if the concept of the effective mass is suitable to describe the relation between curved geometry and particle dynamics (see sections 2 - 4). We will propose the physical experiment (see section 5) which could decide if the curvature of space-time is equivalent to the effective mass of the body.

2. The concept of effective mass

The concept of effective mass of the body plays important role in the contemporary physics. The effective mass is well-known in the solid-state physics. When an electron is moving inside a solid material, the force between other atoms will affect its movement and it will not be described by Newton's law. So we introduce the concept of the effective mass to describe the movement of the electron in Newton's law. The effective mass can be negative or different due to circumstances. Generally, in the absence of an electric or magnetic field, the concept of effective mass does not apply. In the solid-state physics a three-dimensional effective mass tensor is define:

$$\frac{1}{m^*_{ij}} = \frac{d^2E}{dp_i dp_j} \tag{1}$$

where: E is the energy, p is the momentum of the electron, components i and j are the Roman indices to denote spatial components (i, j = 1, 2, 3). For the classical kinetic energy $E = p^2/2m$ and in the isotropic medium the effective mass tensor $m^*_{ij} = m$, where m denotes the bare mass [5].

The concept of effective mass is a very attractive because the effective mass of the body in the equations of the motion includes full information about all fields (for example electromagnetic etc.) surrounding the body without their exact analysis. Effective mass can be isotropic or anisotropic, positive or negative. For the free body his effective mass is equal to the bare mass.

3. Does the effective mass of the body exist in the GR?

Carl Gustav Jacob Jacobi was a first who studied the relation between curved geometry and particle dynamics. After him, this relation was detailed researched by Liouville, Lipschitz, Thomson, Tait, and Hertz. With the origin of tensor calculus, it became clear that there existed a map between the trajectories of the certain mechanical systems in configuration space and the geodesics of a curved manifold [6]. Nevertheless, despite the intense use of geometrical techniques in the context of dynamics, it seems that the relation between mechanics and geometry was not clearly appreciated in the literature of the GR in the aspect of the concept of the effective mass.

Let's us consider a classical dynamical system with kinetic energy T

$$\mathrm{T} = \frac{1}{2}\mathrm{m}^*_{\mathrm{ij}}(\mathbf{r})\frac{\mathrm{dx}^{\mathrm{i}}}{\mathrm{dt}}\frac{\mathrm{dx}^{\mathrm{j}}}{\mathrm{dt}} \tag{2}$$

where: $m^*_{ij}(\mathbf{r})$ is the position-dependent effective mass of the body. It is well-known from the classical mechanics that there exists mathematical relation between the metric tensor g_{ij} and the position-dependent effective mass of the body $m^*_{ij}(\mathbf{r})$ [7, 8] and

$$\mathrm{ds}^2 = \mathrm{g}_{\mathrm{ij}}\mathrm{dx}^{\mathrm{i}}\mathrm{dx}^{\mathrm{j}} = \frac{\mathrm{m}_{\mathrm{ij}}(\mathbf{r})}{\mathrm{m}}\mathrm{dx}^{\mathrm{i}}\mathrm{dx}^{\mathrm{j}} \tag{3}$$

where the elements of the g_{ij} are identical with the elements of the $m^*_{ij}(\mathbf{r})$.

Now we will analyze the concept of the effective mass in the GR. We will look for if exists mathematical relation between the metric tensor $\mathrm{g}_{\mu\nu}$ and the effective mass of the body $\mathrm{m}^*_{\mu\nu}$ in the GR, where μ, $\nu = 0, 1, 2, 3$. Einstein's field equation has form:

$$\mathrm{R}_{\mu\nu} - \frac{1}{2}\mathrm{g}_{\mu\nu}\mathrm{R} = -\frac{8\pi\mathrm{G}}{\mathrm{c}^4}\mathrm{T}_{\mu\nu} \tag{4}$$

where: $R_{\mu\nu}$ is the Ricci curvature tensor, R is the Ricci scalar, $g_{\mu\nu}$ is the metric tensor, G is Newton's gravitational constant, c is the speed of light in the vacuum, and $T_{\mu\nu}$ is the stress-energy tensor. The GR explains gravitation as a consequence of the curvature of space-time, while in turn space-time curvature is a consequence of the presence of matter. Space-time curvature affects the movement of matter, which reciprocally determines the geometric properties and evolution of space-time [9, 10].

Considering the components of (0, 0) in the equation (4), in the approximation of the static and the weakness of the gravitational field, we can get the Poisson equation. We will use the alternative and covariant form of the equation (4):

$$\mathrm{R}^{\mu\nu} = \frac{8\pi\mathrm{G}}{\mathrm{c}^4}\left(\mathrm{T}^{\mu\nu} - \frac{1}{2}\mathrm{T}\mathrm{g}^{\mu\nu}\right) \tag{5}$$

where $T \equiv T_\mu^\mu$ .We are interested the expression in the form:

$$R_{00} = \frac{8\pi G}{c^4}\left(T_{00} - \frac{1}{2}Tg_{00}\right) \tag{6}$$

Our considerations we will realize in the weakness of the gravitational field, which allows us to decompose the metric tensor into the flat Minkowski metric plus a small perturbation,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{7}$$

where: $\left|h_{\mu\nu}\right| << 1$ is a small perturbation. We will restrict ourselves to coordinates in which $\eta_{\mu\nu}$ takes its canonical form, $\eta_{\mu\nu}$ = diag(-1, +1, +1, +1). In the particular case for the weakness of the static gravitational field, for the small velocity v, where v << *c* and for the perfect fluids $p/c^2 << \rho$, where p is the pressure, ρ is the mass density of the fluid element, the stress-energy tensor has simple form:

$$T_{\mu\nu} = \rho u_\mu u_\nu \tag{8}$$

what gives $T = \rho c^2$ and eq. (6) has form:

$$R_{00} = \frac{8\pi G}{c^4}\rho\left(u_0 u_0 - \frac{1}{2}c^2 g_{00}\right) \tag{9}$$

But $u_0 \approx c$ and $g_{00} \approx 1$ therefore

$$R_{00} = \frac{4\pi G}{c^4}\rho c^2 \tag{10}$$

From the second side of eq. (10) [9, 10]

$$R_{00} = \frac{1}{2}\delta^{ij}\partial_i\partial_j h_{00} \tag{11}$$

but $\delta^{ij}\partial_i\partial_j = \nabla^2$. In the GR we have to assume that

$$h_{00} = -\frac{2V}{c^2} = \frac{2GM}{c^2 r} \tag{12}$$

because Einstein sought dependences between the metric tensor and Newton's potential V in the non-relativistic limit.

Let's us consider the planet which is moving on the elliptic orbit in the weak gravitational field. The source of this field is the star with the mass M. We assume that, if the planet is in the perihelion then star will "*generates*" additional mass, which we will call *the effective mass* m^*. We assume also that the effective mass m^* is appear in the equation:

$$h_{00} = -\frac{2V}{c^2} = \frac{2GM}{c^2 r} = \frac{m^*}{m} \tag{13}$$

We will analyze now if the concept of the effective mass m^* will be suitable to describe the relation between curved geometry and particle dynamics.

4. Can we assume that the space-time curvature is equivalent to the effective mass?

This question we can write in the mathematical form:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \overset{?}{=} \frac{m^*_{\mu\nu}}{m} dx^\mu dx^\nu \tag{14}$$

According to the eq. (13) the metric tensor g_{00} will have the form

$$g_{00} = -1 + h_{00} = -1 + \frac{m^*}{m} \tag{15}$$

Now we will try to find the h_{ij} components. The eq. (4) is the nonlinear equation. We can use perturbation theory to compute the weak-field, non-relativistic perturbation to the metric (see eq. 7) and we get the wave equation

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu} \tag{16}$$

where: $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$ and the gauge condition $\partial_\mu \bar{h}^\mu_\lambda = 0$ [10]. In the vacuum the eq. (16) has form:

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}_{\mu\nu} = 0 \tag{17}$$

A few calculations [10] gives

$$h_{ij} = \frac{2GM}{c^2 r}\delta_{ij} = \frac{m^*}{m}\delta_{ij} \tag{18}$$

and we have the metric form for a star (or planet) in the weak-field limit (in the GR)

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right)c^2 dt^2 + \left(1 + \frac{2GM}{c^2 r}\right)\left(dx^2 + dy^2 + dz^2\right) \tag{19}$$

or (for the effective mass)

$$ds^2 = -\left(1 - \frac{m^*}{m}\right)c^2 dt^2 + \left(1 + \frac{m^*}{m}\right)\left(dx^2 + dy^2 + dz^2\right) \tag{20}$$

and finally we get **a very important result**

$$g_{\mu\nu} = \frac{m^*_{\mu\nu}}{m} \tag{21}$$

The concept that the curvature of space-time is equivalent to the effective mass of the body is a very attractive and opens a new physical interpretation for the classical and quantum physics. We postulate the physical experiment which could decide if the curvature of space-time is equivalent to the effective mass of the body.

5. Postulated physical experiment

The eq. $\frac{2GM}{c^2 r} = \frac{m^*}{m}$ makes calculation of Earth's effective mass in the perihelion and aphelion and the difference is equals

$$\frac{\delta m^*}{m} = \left(\frac{m^*}{m}\right)_{perih} - \left(\frac{m^*}{m}\right)_{aphel} = \frac{2GM}{c^2}\left(\frac{1}{r_{perih}} - \frac{1}{r_{aphel}}\right) \approx 6{,}6 \cdot 10^{-10} \tag{22}$$

It seems that if the phenomenon of the effective mass exists, then should be measured in the Solar System. The (effective) mass of the Earth in the perihelion and in the aphelion probably was never measured.

The concept of the effective mass of the body (if exists) should satisfy the classical tests of the GR – *the perihelion shift*, *the deflection of light by the Sun* and *the gravitational redshift*, but their physical interpretation should be different. These physical phenomena would not be generated by the curvature of space-time, but by the effective mass of the body.

6. Conclusion

In this paper we considered the concept of the effective mass of the body in the GR. If the phenomenon of the effective mass exists, then should be measured in the Solar System. The difference of measurements of Earth's mass in perihelion and aphelion is equal:

$$\frac{\delta m^*}{m} \approx 6{,}6 \cdot 10^{-10}$$

We pointed out (see assumption in eq. (13)) that in the static and weak gravitational field exists the relation between the space-time curvature and the particle dynamics. If the phenomenon of the effective mass will be experimentally confirmed, then the relation between the space-time curvature and the effective mass we will describe with help of the very simple formula:

$$g_{\mu\nu} = \frac{m^*_{\mu\nu}}{m}$$

This a very important equation which can open a new way to unify quantum mechanics (describing three of the four known fundamental interactions) with GR (describing the fourth, gravity).

We believe that the concept of the effective mass in the GR will help better understand the phenomena of the classical and quantum gravitation.

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