Are Photons Massless or Massive?

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Abstract. Prevailing and conventional wisdom as drawn from Einstein's Special Theory *of* Relativity, holds that photons are massless particles and that, every particle that travels at the speed of light must – accordingly, be massless. Amongst other important but now resolved problems in physics, this assumption led to the Neutrino Mass Problem – namely,"Do neutrinos have mass?" Neutrinos appear very strongly to travel at the speed of light and according to the afore-stated, they must be massless. Massless neutrinos have a problem in that one is unable to explain the phenomenon of neutrino oscillations because this requires massive neutrinos. Experiments appear to strongly suggest that indeed, neutrinos most certainly are massive particles. While this solves the problem of neutrino oscillation, it directly leads to another problem, namely that of "How can a massive particle travel at the speed of light? Is not this speed a preserve and prerogative of only massless particles?" We argue herein that *in principle*, it is possible for massive particles to travel at the speed of light. In presenting the present letter, our hope is that this may aid or contribute significantly in solving the said problem of "How can massive particles travel at the speed of light?" Further in the horizon of this solution, we are left with a question hanging over the roof-top of our mind, namely the question of whether or not photons are massless of massive?

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1 Introduction

Despite the lack of solid experimental proof (see *e.g.* Hojman & Benjamin 2012, Burman 1972*a*,2,3, Goldhaber and Nieto 1971), it is generally agreed (perhaps believed) that photons have no mass. Though this notion of a zero-mass photon has been questioned over the years (see *e.g.* Nakamura *et al.* 2010, Tu *et al.* 2005, Weinburg 1972), this deeply entrenched fact has been deduced from two (seemingly) immutable facts of experience so well supported by experimental evidence. The first is Professor Albert Einstein (1905*b*)'s energy-momentum dispersion relation, namely:

$$E^2 = p^2c^2 + m_0^2c^4, \tag{1}$$

where E is the total energy of the particle, $p = |\boldsymbol{p}|$ is this particle's momentum, m_0 is this same particle's rest mass and $c = 2.99792458 \times 10^8\,\mathrm{ms}^{-1}$ is the speed of light in vacuum. The second fact is that the energy of the photon has been found from experience to be given by:

$$E = pc. \tag{2}$$

If (1) and (2) are both applicable to the photon with all the identical symbols holding the same meaning, then, it follows directly that $m_0 = 0$; that is, the rest mass of the photon must be zero. This is generalised and stated by saying a photon has no mass. It is thus accepted that if a particle has zero rest mass, it will travel at the speed of light. Conversely, if a particle travels at the speed of light, its rest mass must vanish identically. Herein, we place the two dispersion relations (1) and (2) into the dock for some *cross-examination*, where-after we come to the interesting conclusion that it must *in-principle* be possible to have massive photons (*i.e.* non-zero rest mass photons) obeying these two relations simultaneously and concurrently *i.e.*, massive particles that travel at the speed of light c.

The hidden assumption in all the reasoning leading to the fact that for photons $m_i = 0$, is that the energies (E) in the formulae $E^2 = p^2c^2 + m_0^2c^4$ and $E = pc$ are identical. On a more fundamental level, there is no *priori* nor *posteriori* justification for this clandestine assumption. If these two energies are different, that is, say the E in $E^2 = p^2c^2 + m_0^2c^4$ is the total gravitational energy $E_{\rm g}$ of the photon so that $E_{\rm g}^2 = p^2c^2 + m_0^2c^4$; and the E in $E = pc$ is say total kinetic energy $E_{\rm K}$ of the photon so that $E_{\rm K} = pc$, then, it is possible for $m_i \neq 0$. Combining these (*i.e.*, $E_{\rm g}^2 = p^2c^2 + m_0^2c^4$ and $E_{\rm K} = pc$) would lead to $E_{\rm g}^2 = E_{\rm K}^2 + m_0^2c^4$ where generally $m_0 \neq 0$.

The idea of a zero-mass particle usually presents a challenge to freshman students encountering this for the first time (Robles *&* Claro 2012) because, mass is generally thought to be the measure of the amount of matter in a substance. Based on this kind of thinking, zero-mass must mean no amount of matter present yet for the photon whose mass is zero, it has not only stuff in it, but lots of it. Can a massive photon solve this problem?

Using the arguments just presented in the ante-penultimate, together with *de* Brogile's wave-particle duality hypothesis, we argue herein that it must in-principle be possible to have non-zero rest mass photons. Actually, it is suggested that all photons may very well be massive all having the mass, in the same manner that electrons (and protons) have mass.

2 Inertial and Gravitational Mass

In building our thesis, we begin in this section by demonstrating a simple yet important point, namely that, the rest mass of a particle as defined in Einstein's Special Theory *of* Relativity (STR) can be identified with the inertial mass (as defined in Newtonian mechanics) of a particle. To that end – as is well known, there is at least two distinct and important kinds of mass that enter Newtonian mechanics. The first is the *inertial mass* (m_i) which enters in Newton's second law of motion. As it was first stated by the great Sir Isaac Newton, this law states that the resultant of all the forces ($\boldsymbol{F}_{\rm res}$) acting on a body is equal to the rate of change of motion of that body, *i.e.*:

$$\boldsymbol{F}_{\rm res} = \frac{d\boldsymbol{p}}{dt} \quad \text{where,} \quad \boldsymbol{p} = m_i\boldsymbol{v}. \tag{3}$$

By motion, Newton meant the momentum $\boldsymbol{p}$ of the body in question. Momentum ($\boldsymbol{p}$) is the product of inertial mass (m_i) and the velocity ($\boldsymbol{v}$) of the body in question. In most cases considered in natural systems, the inertial mass of the object is a constant of motion, so this law is often stated as:

$$\boldsymbol{F}_{\rm res} = m_i\boldsymbol{a} \quad \text{where,} \quad \boldsymbol{a} = \frac{d\boldsymbol{v}}{dt}. \tag{4}$$

The vector quantity $\boldsymbol{a}$ is the acceleration of the body in question.

The second kind of mass enters Newtonian mechanics in Newton's law of universal gravitation is the *gravitational mass* ($\mathcal{M}_{\rm g}$ and $m_{\rm g}$). Newton's law of universal gravitation states that the gravitational force drawing together two objects of gravitational mass $\mathcal{M}_{\rm g}$ and $m_{\rm g}$ that are separated by a distance r is:

$$\boldsymbol{F}_{\rm g} = -\frac{G\mathcal{M}_{\rm g}m_{\rm g}}{r^2}\hat{\boldsymbol{r}}, \tag{5}$$

where ($G > 0$) is Newton's constant of universal gravitation and $\hat{\boldsymbol{r}}$ is the unit vector along the line joining the centres of mass of these objects and the negative sign is there to denote the fact that the gravitational force is a force of attraction.

Pertaining the gravitational and inertial mass, we have the Weak Equivalence Principle (WEP) due to Galileo which states that test bodies fall with the same acceleration independent of their internal structure or composition: in other words, the gravitational mass appearing in (5) and inertial mass appearing (4) are the same *i.e.* $m_i \equiv m_{\rm g}$. Throughout this reading, in order to distinguish between gravitational and inertial mass, we shall use the subscripts "i" and "g" respectively *i.e.* m_i and $m_{\rm g}$.

As has been done in Nyambuya *&* Simango (2013), we are going to denote the ratio between the gravitational and inertial mass $m_{\rm g}/m_i$ as:

$$\frac{m_{\rm g}}{m_i} = 2\gamma \quad \Longrightarrow \quad \gamma = \frac{1}{2}\frac{m_{\rm g}}{m_i}. \tag{6}$$

Now, taking a step further toward our desired end, we know from Einstein's STR that the total energy of a particle E is such that:

$$E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} = \Gamma m_0 c^2 = mc^2, \tag{7}$$

where $v = |\boldsymbol{v}|$ is its speed and $\Gamma = 1/\sqrt{1 - v^2/c^2}$ and $m = \Gamma m_0$. In the non-relativistic limit where $v^2/c^2 \lll 1$, to first order approximation (7) is given by:

$$E = \frac{1}{2} m_0 v^2 + m_0 c^2. \tag{8}$$

The term $m_0 v^2/2$ is the usual classical kinetic energy $K = \frac{1}{2} m_i v^2$ of the particle where m_i the particle's inertial mass as defined in Newton second law, namely $\boldsymbol{F} = m_i \boldsymbol{a}$. It is generally agreed that in the non-relativistic limit, the equations of Einstein's STR must reduce to the well know Newtonian equations. Accepting this bare thesis – invariably, this means that the rest mass m_0 as it appears in (8) can or must be identified with the classical inertial mass of an object, *i.e.*:

$$m_0 \equiv m_i. \tag{9}$$

From the forgoing, it means we can write (8) as:

$$E = \frac{1}{2} m_i v^2 + m_i c^2. \tag{10}$$

Now, the energy E is equal to mc^2 *i.e.* $E = mc^2$. The question is what is this m in the formula $E = mc^2$; is it the gravitational or inertial mass? If as stated in the introduction of this section, we have only two kinds of masses, the gravitational or inertial mass, m can only be one of these two. If this mass is the inertial mass, it would mean that the kinetic energy of any particle must be zero for all times and all situations in life since: $m_i c^2 = \frac{1}{2} m_i v^2 + m_i c^2 \Longrightarrow \frac{1}{2} m_i v^2 = 0$. This is obviously nonsense and *must* be *rejected forthwith* without any further deliberations. This leaves us with no choice but to identify the m in $E = mc^2$ with the gravitational mass $m_{\rm g}$, *i.e.*:

$$E_{\rm g} = m_{\rm g} c^2. \tag{11}$$

In this case where $E_{\rm g} = m_{\rm g} c^2$, the kinetic energy is the nothing but the difference between the gravitational and inertial energy of a particle *i.e.* $E_K = (m_{\rm g} - m_i)c^2$. Therefore, written with all the masses well labelled *i.e.* in-terms of the gravitational and inertial mass, (10) must sure be given:

$$E_{\rm g} = \frac{1}{2} m_i v^2 + m_i c^2. \tag{12}$$

At this point, we have attained our desired objective *i.e.*, we have shown that in principle, one can define or identify the rest mass as the inertial mass. Further, we have defined the gravitational mass as-well. In the next section, we shall identify the particle and group velocity of a particle.

3 Wave-Particle Duality

In 1924, the *rightly-celebrated* French Prince, Louis Victor Pierre Raymond *de* Broglie (1892−1987) hypothesised in his *all-embellished* and unique[1] doctoral thesis, that there is an intimate and *all-enduring* duality between waves and particles. By so doing – *i.e.*, proposing or postulating the undulatory nature of matter, he opened a not only rich, but very wealthy scientific gold mine which to this day is under extensive exploration and there seems to be no end in-sight insofar as the quantum enigma brought forth by the wave-particle duality is concerned. At best – one can safely say that, *de* Broglie opened the *Scientific Pandora's Box* which led to the greatest scientific minds of the past century [such as Albert Einstein (1879 − 1955), Niels Bohr (1885 − 1962), Erwin Schrödinger (1887 − 1961), David Bohm (1917 − 1992), John Bell (1928 − 1990) *etc*] to debate and end-up up in stalemate. Their debates ended up in the deepest and non-objective trenches of psychology, philosophy and region where it is difficult to obtain an objective and straitjacket answer to the questions at hand.

Beginning with his photoelectric paper, Einstein (1905*a*) showed that light, which was previously assumed to be a wave, would be treated as a stream of very tiny "*hard-billiard-ball-like*" particles which he [Einstein] called photons. He did this in the full glare of the fact that the wave model of light by the brilliant Dutch physicist – Christian Huygens (1629 − 1695), was a successful model in its own right; it is well supported by a number of experiments. The English scientist – Thomas Young (1773 − 1829)'s famous doubleslit experiment is a classic, clearest and unequivocal demonstration of the wave nature of light. On the same footing, Einstein's particle model rests on well founded and solid experimental findings of the photoelectric effect. This experiment can only be understood in-terms of Einstein's corpuscular model of light.

The rare and acute genius of *de* Broglie lay in not only worrying about whether or not, light was a wave or a particle, but in accepting this as a bare fact of experience and exploiting further the fruits thereof. That is, on accepting this, *de* Broglie's agile and sharp genius made a further great leap by esoterically wondering if this duality only applied to light?! Why not to matter as-well – he asked? After all, *Nature* appears to favour symmetries. Matter is modelled on the particle model. If light behaves as both a particle (matter) and a wave, matter must too behave as a wave if She [*Nature*] is to preserve Her symmetrical nature! So, *de* Broglie set-forth his *all-daring* and *all-pervasive* grand hypothesis that matter must have a dual nature of both particle and wave in much the same manner as light does. As we know, the rest is nothing but a beautiful and ornate history of quantum mechanics and the quantum enigma.

An intricate and intimate relation between Quantum Mechanics (QM) and the STR exists and one of these relations is embodied in the relation (1) which is founded on the particle model of *Nature*. This relation is extended to explain matter as a wave *via* the well known quantum canonical quantization procedure were this equation is transformed into the Klein-Gordon equation, namely:

$$\boldsymbol{\nabla}^2\Psi - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = \left(\frac{m_0c^2}{\hbar}\right)^2\Psi, \tag{13}$$

where Ψ is the wavefunction describing the matter particle in question and $\hbar$ is Planck's normalised constant. The wavefunction $\Psi = Ne^{\pm\boldsymbol{k}\cdot\boldsymbol{r}+\omega t}$ reproduces the Einstein dispersion relation (1). For this wavefunction $\Psi = Ne^{\pm\boldsymbol{k}\cdot\boldsymbol{r}+\omega t}$, N is some normalisation constant, $\boldsymbol{k} = \boldsymbol{p}/\hbar$ is the wave-number or wave-vector, $\boldsymbol{r}$ is the position vector, $\omega = E/\hbar$ is the angular frequency and t the time. Traditionally,to describe a wave it suffices to only to have knowledge of these two quantities *i.e.* the angular frequency (equivalently this can be expressed simple as the frequency ν) and wave-number (equivalently this can be expressed as the wavelength λ).

Other than the wavelength and frequency, one thing that characterises a wave is the wave's *group velocity* which we shall denote as $\boldsymbol{v}_{\mathrm{g}}$. If the total energy E of the wave is known in-terms of the wave's momentum $\boldsymbol{p}$, then, the group velocity of the wave is given by the differential relation:

[1] This thesis is unique in that it is so far the only doctoral thesis in history to wholly be award the highest Honour and Prize *of* Science – *i.e.*, the Nobel Prize in Physics.

$$\boldsymbol{v}_{\mathrm{g}}=\frac{\partial\omega}{\partial\boldsymbol{k}}=\frac{\partial E}{\partial\boldsymbol{p}}. \tag{14}$$

Another is the *phase velocity*. The phase velocity of a wave is the rate at which the phase of the wave propagates in space. This is the velocity at which the phase of any one frequency component of the wave travels.

$$\boldsymbol{v}_{p}=\frac{\omega}{k}\hat{\boldsymbol{k}}=\frac{E}{p}\hat{\boldsymbol{k}}, \tag{15}$$

where $\hat{\boldsymbol{k}}$ is the unit vector along $\boldsymbol{k}$. The group velocity can be measured, this is the velocity with which the entire structure of a particle will be seen to travel. The phase velocity on the other hand is not a directly and easily measurable quantity.

Now, because the velocity of a particle that we measure in the macroscopic world is its group velocity, we can associate the group velocity with the particulate nature of matter. On the same footing, we can associate the phase velocity with the undulatory nature matter. In simple terms, we are saying that the group velocity is a particle-like property of matter, while the phase velocity is a wave-like property of matter. Without any justification, from the foregoing, we shall assume that matter is described by the condition ($\boldsymbol{v}_p=\boldsymbol{v}_{\mathrm{g}}$) and ($v_p=v_{\mathrm{g}}$).

If the Universe only contains two constitutes, matter and energy (radiation); and if the condition ($\boldsymbol{v}_p=\boldsymbol{v}_{\mathrm{g}}$) and ($v_p=v_{\mathrm{g}}$) describes matter, then, logically, the contrary condition must somehow describe radiation. Once again, without any justification, we shall assume that radiation is described by the condition ($\boldsymbol{v}_p\neq\boldsymbol{v}_{\mathrm{g}}$) and ($v_p\neq v_{\mathrm{g}}$). In the next section, we are going to use these definitions of calculate the γ-factor for both matter and radiation.

Before we close this section, we need to point out an important point about (1). In Einstein's STR, this relation is derived from two equations, namely $E=\Gamma m_0c^2$ which is the total energy of the particle and $\boldsymbol{p}=\Gamma m_0\boldsymbol{v}$ which is the relativistic momentum of the particle. By multiplying $\boldsymbol{p}=\Gamma m_0\boldsymbol{v}$ by c *i.e.* $\boldsymbol{p}c=\Gamma m_0c\boldsymbol{v}$ and squaring the resultant equation ($p^2c^2=\Gamma^2m_0^2c^2v^2$) and thereafter subtracting this from the square of $E=\Gamma m_0c^2$ *i.e.* $E^2=\Gamma^2m_0^2c^4$, one obtains (1). Now, taking matters from these two fundamental equations $E=\Gamma m_0c^2$ and $\boldsymbol{p}=\Gamma m_0\boldsymbol{v}$, one will be forgiven for thinking that a zero-mass particle must have zero energy and momentum. This is one of the difficulties normally encountered in trying to comprehend a zero-mass object (*e.g.* Robles *&* Claro 2012). This problem is overcome by treating QM as more fundamental than Einstein's STR. If one does this, then, by requiring that (13) be Lorentz invariant, they can derive all the results of Einstein's STR from a purely quantum mechanical standpoint as was done by the great French mathematician Henri Poincaré $(1882-1934)$ in 1904. The resulting theory solves the above stated problem.

4 Massive Photon

Now, we come to the main issue of the present letter. First, in the ante-penultimate paragraph of the introductory section, we stated that there is a hidden assumption in all the reasoning leading to the fact that for photons $m_i=0$. This clandestine assumption lies in that the energies (E) in the formulae $E^2=p^2c^2+m_0^2c^4$ and $E=pc$ are assumed to be identical *i.e.* they represent the same energy. Are they really the same energy? Strangely, as far as our survey of the available literature that we have had the fortune to set our eyes and mind, we have come not across anyone that has wondered whether or not these energies are one and the same energies.

Is not the energy "*pc*" the kinetic energy of the photon since this energy is wholly associated with the motion of the photon? For example, in the photoelectric effect that led the great Einstein to make the hypothesis that light comprises a stream of tiny *billiard-ball-like* particles called photons, this kinetic energy "*pc*" of the photon is transformed not into another form of energy, but into the kinetic energy of the electron that gets ejected from the metal surface. This strongly suggests that "*pc*" is actually the kinetic of the photon and nothing more. The energy E in $E^2=p^2c^2+m_0^2c^4$ is not only the kinetic energy as this energy includes the potent locked-up energy

m_0c^2. Perhaps we have not been all correct in assuming that the energy E in $E^2 = p^2c^2 + m_0^2c^4$ is the kinetic energy of the photon. Actually, on a most pristine and fundamental level or reasoning, there really is no *priori* nor *posteriori* justification for this hitherto clandestine assumption.

If these two energies are different, that is, say the energy E in $E^2 = p^2c^2 + m_0^2c^4$ is the total gravitational energy $E_{\rm g}$ as argued in §(2) and the E in $E = pc$ is say total kinetic energy $E_{\rm K}$ of the photon so that $E_{\rm K} = pc$, then, it is possible for $m_i \neq 0$ because if we combine these two formulae (*i.e.*, $E_{\rm g}^2 = p^2c^2 + m_0^2c^4$ and $E_{\rm K} = pc$), one is led to:

$$E_{\rm g}^2 = E_{\rm K}^2 + m_0^2c^4, \tag{16}$$

where generally $m_0 \neq 0$. This is not the end of the road for our quest. Before that, we have to demonstrate that the dispersion relation (16) does contain the solution of "a non-zero rest mass or non-zero mass wave-packet the travels at the speed of light". This solution is the solution describing massive photons.

Applying (14) to (16), one obtains:

$$\boldsymbol{v}_{\rm g} = \left(\frac{\Gamma \boldsymbol{p}}{E_{\rm g}}\right) c = \left(\frac{\Gamma}{2\gamma}\right) \boldsymbol{v}_p. \tag{17}$$

Now, our task is to apply the above relation to energy and matter waves. We shall set the following limits for v_p and $v_{\rm g}$, that is $(v_p < c)$ and $(v_{\rm g} \leq c)$.

4.1 γ-Factor for Matter

If $v_p = v_{\rm g}$, it follows that:

$$\gamma_M = \frac{1}{2}\left(1 - \frac{v_p^2}{c^2}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(1 - \frac{v_{\rm g}^2}{c^2}\right)^{-\frac{1}{2}}. \tag{18}$$

Since $(0 \leq v_p = v_{\rm g} < c)$, it follows that:

$$\gamma_M \geq \frac{1}{2}. \tag{19}$$

From this, it follows that for ordinary matter where the speeds are close to zero when compared to the speed of light *i.e.* $(v_p = v_{\rm g} \sim 0)$, we will have $\gamma \sim 1/2$.

$$m_{\rm g} = \frac{m_i}{\sqrt{1 - v_p^2/c^2}} = \frac{m_i}{\sqrt{1 - v_{\rm g}^2/c^2}}. \tag{20}$$

In Nyambuya & Simango (2013), we have presented a way to measure the γ-factor for matter. Further, we have also presented a way out of the problem that may arise from the violation of the WEP by showing that a conformal theory of gravitation will be needed to preserve Einstein's Equivalence Principle which stands as the foundational bases of his much celebrated General Theory *of* Relativity (GTR). We will not repeat ourself here but direct the reader to the said reading (*i.e.* Nyambuya & Simango 2013).

4.2 γ-Factor for Photons

Contemporary physics (which assumes a massless photon) as currently constituted holds that for light $v_g = v_p = c$. Applying this *i.e.* $v_g = v_p = c$ into the present ideas leads to nonsensical infinities. Sanity can be restored by assuming $v_p \neq v_g = c$, thus leading to:

$$\gamma_L = \frac{1}{2}\frac{v_p}{c}\left(1 - \frac{v_p^2}{c^2}\right)^{-\frac{1}{2}}. \tag{21}$$

The problem with (21) is that $\gamma_L \longmapsto \infty$ as $v_p \longmapsto c$. The singularity at the light speed barrier causes some problems. In order to resolved this problem of infinities that arises when the assumption $v_p = c$ is applied, one may very-well need to invoke a Doubly Special Relativity (DSR) theory that contains no singularity when the assumption $v_p = c$ is applied. One such theory is that presented in Nyambuya (2012). In this new theory which is yet to be evaluated by the scientific community, there is no singularity at the light speed barrier – even ordinary massive particles can attain the light speed barrier under extenuating energy conditions. For low quantum fluctuations as is the case in our ordinary state of exist, $\Gamma \simeq 1$ for $v_p = c$ according to Nyambuya (2012). We shall adopt this position. So doing, we lead to:

$$\gamma_L = \frac{1}{2}\frac{c}{v_g}, \tag{22}$$

where the subscript "L" has been inserted to indicate that this γ-factor is the γ-factor for light or electromagnetic waves in general.

Now, when ($\gamma_L \simeq 1$) as is the case with radio waves (see Nyambuya *&* Simango 2013), this means that these waves are such that ($v_g \sim 0.5c$). As for the limit for γ_L, we know that ($0 < v_g \leq c$), from this it follows that:

$$\gamma_L \geq 1/2. \tag{23}$$

What the above means is that deflection of light by the Sun should take at least the Newtonian deflection of $0.87''$. This means that deflections in the range $0.87'' - 1.75''$ are permitted, this deepens of the value of γ. Not only that, deflections exceeding Einstein's $1.75''$ prediction are also permitted. We have made the endeavour to address these and other issues arising from the present findings in the letter Nyambuya *&* Simango (2013).

Further, if γ_L is known, we can calculate the mass of the photon from *de* Brogile's relation $p\lambda = 2\pi\hbar$. In this relation, we take the momentum p to be given by $p = \Gamma m_i v_g \simeq m_i v_g$. Now, making v_p the subject of the formula in (22) and substituting this into the *de* Brogile's relation $p\lambda = 2\pi\hbar$ relation and making m_i the subject of the formula, we will have:

$$m_i = \frac{\hbar k}{v_g} = \frac{2\gamma\hbar k}{c}. \tag{24}$$

Now, considering for example that the radio waves used by Fomalont *et al.* (2009) to measure the deflection of these waves by the Sun yielded the value ($\gamma_L = 0.9998 \pm 0.0003$) and that the dominant wavelength that produced this result is the 43 GHz spectral line, from this information, it follows that:

$$m_A = 9.20 \times 10^{-32}\,\mathrm{eV/c^2} = 1.50 \times 10^{-50}\,\mathrm{kg}. \tag{25}$$

However small this mass maybe, it is not zero! Such a small mass is consistent with the upper bounds placed by various experiments that have been performed to measure the mass of the photon. These experiments place upper bounds of $\sim 10^{-54} - 10^{-40}$ kg *e.g.* (see e.g. Tu *et al.* 2005, p.106, 123). These upper bounds determined by experiment may mean that the mass of photons may vary from one photon to the other.

On a more interesting note, what (22) implies is that a gravitational field will not only alter the path of electromagnetic waves, but slow the speed of these waves as-well. This prediction can be computed by measuring the time of fly of electromagnetic waves as that travel in the gravitational field. If the verification of this prediction can be made by experiments, it would be the most definitive proof that the photon has mass because according to Einstein's STR and GTR, light travel null geodesics without altering its speed. We will not go into the details of this but merely point this out as we have already done.

5 Discussion and Conclusion

5.1 General Discussion

If what we have presented herein is proven to hold, then, the implications thereof have serious repercussions across the broad spectrum of physics as contemporary physics hitherto assumes that the mass of a photon is identically zero, especially the embellished Standard Model *of* Particle Physics (SMPP). We have argued that this assumption may not be correct as it is based on a hitherto hidden and clandestine assumption that is not necessary; this assumption can be gotten reed off. More than this, we have argued that, in principle, physics (*i.e.* Einstein's STR and the theory of waves) has no problem with a massive photon that travels at the speed of light. If the ideas herein are accepted or acceptable, then, this places physics on a sure pedestal to consider massive photons as plausible physical objects of the Universe.

For example, in Quantum Electrodynamics (QED) and Quantum Field Theory (QFT) which are one of the two critical foundational pillars of the SMPP, a massive photon is not consistent with gauge invariance or renormalizability. However, *via* Proca Electrodynamics, one can device a theory of massive photons (see *e.g.* Tu *et al.* 2005, Jackson 1998). Besides, one can also accomodate massive photons in Podolsky Electrodynamics as has been argued by Fonseca *&* Vargas-Paredes (2012). From the foregoing, it is clear that physics has never ruled out massive photons (further see *e.g.* Goldhaber and Nieto 2010, Nakamura *et al.* 2010, Tu *et al.* 2005). Perhaps the reason they have been neglected in main-stream science is the shear difficulty that may be brought about by trying to renormalize the resulting theory. Renormalisation of QFT with a massless photon is already a nightmare, what more with a massive photon? However, from a theoretical perspective, if the rest mass of the photon where non-zero, classical electromagnetism, QED and QFT would remain untroubled in spite of the undesired loss of gauge invariance (Tu *et al.* 2005). On the beautiful side of things, loss of gauge invariance is too high a prize to pay – few physicist would be prepared to pay this prize. However, despite the desideratum of the physicist, if observations are to point in that direction, the only choice we have is to submit and move on.

On the more realistic side of things, if one can obtain results which are in satisfactory agreement with experience using a massless photon, why bother with an additional unessential? *Simple let the sleeping dogs lay.* In any case, *Occam's "all-powerful and very sharp" Razor* forbids the unnecessary addition of non-essentials, so, there are very many good reasons to ignore massive photons. Perhaps physicist will consider them when they can longer avoid them at all. For now, there strongly appears to be no real need for them.

However, the notion that every particle that travels at the speed of light must – accordingly, be massless lead to an important but now solved problem in physics, this assumption has led to the Neutrino Mass Problem – namely,"Do neutrinos have mass?" According to Einstein's STR, if neutrinos are massless, they must travel at the speed of light and conversely, if neutrinos travel at the speed of light, they must me massless. According of a recent CERN press release[2] refuting the claim of *faster-than light* speed for neutrinos supposedly detected by the OPERA Collaboration in September of 2011, neutrinos strongly appear to travel at the speed of light.

Further, in order to explain neutrino oscillation, that this, the change of neutrinos from one state[3], neutrinos must have mass. Endowing neutrinos with mass helped solve one of the outstanding problems in Solar physics known as the Solar neutrino problem. The Standard Solar Model (SSM) which is a theory detailing how the Sun produces its energy predicted that the Sun must produce a specific amount of neutrinos. Prior to the 1988 Sudbury Neutrino Observatory (SNO) Experiment, all neutrino detectors that had ever attempted to measure the rate at which the Sun made neutrinos were getting values between 1/3 and 2/3 of what was expected from theory. This meant that there was a dearth in neutrinos produced by the Sun if the SSM was right — this deficiency came to be known as the of the *Solar Neutrino Problem* (SNP). Neutrino oscillations resolved the SNP: the electron neutrinos produced in the Sun partly change into other flavors

[2]http://press.web.cern.ch/Press/PressReleases/Releases2011/PR19.11E.html

[3]Neutrinos come in three states known as flavors and these are the electron neutrino, muon neutrino and the tau neutrino. Neutrino oscillation is when one state changes to the other.

which prior experiments could not detect. Other than this, measurements indicate that neutrino most certainly have mass. Additionally – as already stated above, neutrinos most certainly travel at the speed of light. We seem to have a apparent contradiction here!

Ceteris paribus: in the light of popular contemporary physics, how can a massive particle travel at the speed of light? Is not this speed a preserve and prerogative of only massless particles? If observations are to take their rightful place in Science, which is that they take precedence over all our theories, then, we have but no choice expect to accept that massive particles most certainly can travel at the speed of light as strongly appears to be the case with neutrinos. The only way is to amend our theories to conform with observations and experiments. To that end, we have demonstrated herein that *in principle*, it is possible for massive particles to travel at the speed of light. In our supposition that only massless particles must travel at the speed of light and conversely, that, if a particle does happen to travel at the speed of light, it must accordingly be massless, we have argued that this assumption may not be correct as it hinges on a hitherto hidden and clandestine assumption that is surely not necessary; this assumption can swiftly be gotten reed off. In-closing, allow us to say that in presenting the present letter, our hope is that this may aid or contribute significantly in solving the said problem of *"How can massive particles travel at the speed of light?"*

5.2 Conclusion

Assuming the correctness (or acceptability) of the ideas presented herein, we hereby make the following conclusions:

1. The current belief or position that for an object to travel at the speed of light it must be massless may not entirely be correct as this is based on a hitherto hidden and clandestine assumption that the energies (E) in the formulae $E^2 = p^2c^2 + m_0^2c^4$ and $E = pc$ are identical *i.e.* they represent the same form of energy. This hitherto *priori* and *posteriori* unjustified assumption is not really necessary. Dropping this assumption leads to the plausibility of massive photons and massive particles that travel at the speed of light.

2. Neutrinos may very well be good candidates to be described by the present ideas of massive particles travelling at the speed of light. These neutrinos must have a non-zero γ-factor, the meaning of which is that they must suffer gravitational deflection as happens with normal electromagnetic waves when they graze the limb of massive gravitating objects like the Sun.

3. Gravitational deflection of monochromatic electromagnetic waves (such as gamma and radio waves) by the Sun presents the best way to measure the mass of photons by measuring γ. The theory on how to interpret γ is presented in Nyambuya *&* Simango (2013).

4. It has been shown herein that it is possible for a photon to be massive and yet travel at exactly the speed $c = 2.99792458 \times 10^8\text{ms}^{-1}$ in such a manner that the photon's speed is independent of the mass of the photon. If this were the case, there would be no need to modify Einstein's STR to accommodate massive photons. Present theories strongly advocate for a scenario were a massive photon hypothesis leads to the speed of the photon to be dependent on the mass of the photon, leading to slight but significant modifications to Einstein's STR (see *e.g.* Tu *et al.* 2005).

5. As much as a gravitational field alters the path of electromagnetic waves by bending their otherwise straight path, this same gravitational field may significantly slow down light by causing it to move at speeds less than c. This prediction can be tested by measuring the time of flight of these electromagnetic waves as they pass through these gravitational fields.

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