On Exponential Decay and the Riemann Hypothesis

JEFFREY N. COOK

ABSTRACT. A Riemann operator is constructed in which sequential elements are removed from a decaying set by means of prime factorization, leading to a form of exponential decay with zero degeneration, referred to as the root of exponential decay. A proportionate operator is then constructed in a similar manner in terms of the non-trivial zeros of the Riemann zeta function, extending proportionately, mapping expectedly always to zero, which imposes a ratio of the primes to said zeta roots. Thirdly, a statistical oscillation function is constructed algebraically into an expression of the Laplace transform that links the two operators and binds the roots of the functions in such a manner that the period of the oscillation is defined (and derived) by the eigenvalues of one and the elements of another. A proof then of the Riemann hypothesis is obtained with a set of algebraic paradoxes that unmanageably occur for the single incident of any non-trivial real part greater or less than a rational one half.

1. Introduction

Chronological (sequential) time n, an element of $\mathbb{N}, \mathbb{Z}$ (or clock time, "chronos" in Latin) in whole values (clocks, rings, etc.) is not always considered for physical phenomena [1]. In terms of exponential decay, the fractions of time a decaying quantity N is measured by is instead considered, called lifetimes or durations of time, most typically expressed in terms of a quantity half-life [2] in seemingly chaotic systems. The half-life is the time required for the decaying quantity to fall to $1/2$ its initial value

$$t_{\frac{1}{y}} = \frac{\ln(2)}{\lambda} = \tau \ln(\lambda) : y = 2,$$

where λ is the decay constant and τ the mean lifetime. Rational time $t_{1/y}$ can be measured in any fraction of the mean lifetime ($t_{\pi/3}$ for instance where it would be consider real time); $t_{1/2}$ is conveniently considered [2].

The decay constant λ is always a positive number, such that

$$\tau\lambda = 1.$$

Because the progression of natural time n itself (not the progression of something over time) is incrementally considered for all positive whole numbers, and not a ring,

$$\mathbb{N}, \mathbb{Z} \in n;$$

rather, τ and t are durations, instead being number fields in that

$$\mathbb{Q}, \mathbb{R}, \mathbb{C} \in \tau, t \in F.$$

The Riemann zeta function can be expressed in terms of the progression of natural time n, considering its infinite series

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s},$$

as involves argument powers of $n \in \mathbb{N} \neq 0$ [3]. Considering too Euler's derivation [4];

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \sum_{p} (1 - p^{-s})^{-1},$$

a method for mapping clocks $n \in \mathbb{N} \neq 0$ to fields $\tau \in \mathbb{R} = p/\ln(p) \to \infty$ becomes convenient for describing the moment an event occurs ("kairos" in latin), meaning the time defining an opportune moment, which may be though higher ordered over linear chronological time if vectors may be considered higher ordered points on planes over points on an axis.

In terms of exponential decay over distance, rather than decay over time, the propagation constant of an electromagnetic wave is the eigenvalue of the change undergone by amplitude A (identically interchangeable in study for quantity N over time t) of the wave as it propagates in a given direction. Typically, this can be voltage or current in a circuit or a field vector, such as electric field strength or flux density [5], in that the propagation constant itself measures change per distance rather than change per time.

Propagation constant γ (identically interchangeable—in study—for decay constant λ) for a given system is defined by the ratio of the amplitude at the source of the wave to the amplitude at some distance x, such that,

$$\frac{A_0}{A_x} = e^{\gamma x}.$$

The propagation constant being a complex quantity, we can write

$$\gamma = \sigma + i\omega,$$

where σ, the real part (more conventionally symbolized α), is the attenuation constant and ω, the imaginary part (more conventionally symbolized as β), is the phase constant—though not accurately "constant", varying in frequency. Both σ, ω can in any given circumstance be equal to zero, thereby γ may be treated as a real number when $\omega = 0$, but the propagation constant is always mapped on the complex plane [6].

ω represents phase by means of Euler's formula;

$$e^{i\theta} = cos\theta + isin\theta,$$

which is a sinusoid that varies in phase as θ varies, but having a constant amplitude, as

$$\left|e^{i\theta}\right| = \sqrt{\cos^2\theta + \sin^2\theta} = 1.$$

The two parts form a single complex number that can be handled in one mathematical operation, provided they are to the same base (most typically) e [5].

This study follows loosely from telecommunications terminology [7], in that the attenuation constant is the attenuation of an electromagnetic wave propagating through a medium per unit distance from the source and defined by the amplitude ratio;

$$\left|\frac{A_0}{A_x}\right| = e^{\sigma x}.$$

The general continuous form is written as

$$A\, e^{\gamma x}.$$

Because both exponential decay of N over kairos time t and amplitude decay of A over distance x, can be counted n-at-a-time, $n \in \mathbb{N}$ (but in some consideration provides richer meaning in terms of kairos time t and field x when represented as a change through space or time), we simply refer to kairos as "field time t" (dropping the Latin terminology). In this manner we compare its nature equally with field distance x, a point somewhere in space. Any countable measurement of distance n, is then referred to simply as natural distance or natural time. We consider that certain progressions of natural processes, such as exponential decay (even in terms of rational numbers, half-lives), may be defined but tend toward reduced meaning in terms of their countability without first consideration of n (i.e. the harmonic series requires first the countability of n, as well as other infinite series and the like).

Exponential decay (inside or outside the study of propagation and amplitudes) is typically represented as

$$\frac{N(t)}{N_0} = e^{-\lambda t}$$

or

$$\frac{A(x)}{A_0} = e^{-\gamma x}.$$

Thus, it is in terms of decay over time that we will proceed, so long as we understand that A refers to any quantity decaying exponentially over distance x (most conventionally amplitude) and N a quantity decaying exponentially over time t. We study this from the perspective of N in that the following methods apply to both forms of decay. There will come a point in this paper where we return to the study of a dimensionless A (though, we will not change in the middle of any derivation) after we suppress the dimensions by means of the Buckingham π theorem, as it will prove desirable (though not entirely required) to discuss A instead, as the meanings of our expressions become straight forward for the study of general dimensionless periodic functions analogous to waves.

Theorems.

Theorem 1. *Let* ν *be a parameterized constant that corresponds to any given argument s of the Riemann zeta function, so that*

$$\nu = e^{\frac{-s\ln(p_n)}{p_n}} : \begin{matrix} \lambda = s \\ t = \dfrac{\ln(p)}{p}. \end{matrix}$$

One gets the infinite sequence $\nu_s(p)$ *whose points in the Riemann zeta function for any argument* s *correspond to a single prime number;*

$$\zeta(s) - \prod_p (1 - \nu_s(p)^p)^{-1} = 0 \; \forall \, p, s,$$

so that an expression of exponential decay is defined in

$$\nu_s(p) = \frac{N(t)}{N_0},$$

and

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \prod_p \left(1 - \left(\frac{N(t)}{N_0}\right)^p\right)^{-1},$$

where the Riemann zeta function is equal to the product of the inverse of one minus the ratio of the number of discrete elements $N(t)$ *in a certain set per the initial quantity, the quantity at* $t = 0$.

Theorem 2. $N(t) \sim N_0$ *for Riemann decay.*

Theorem 3. *By means of the fundamental theorem of arithmetic,* ν *for all* $s < 0$ *becomes the numerator of the set of elements of the field of fractions* $Quot(R_\sigma)$ *of the integral domain* R_σ*, containing all numbers factored out of the set of by a lowest common multiple (*lcm*). Said* $Quot(R_\sigma)$ *for a given* s *then parameterizes the denominator* $\delta_s(p)$*, where* $\lim \nu_s(p)$ *demands* $\lim \delta_s(p) = 1/Quot(R)$*, the prime numbers then considered the atomic elements which, when combined together, make up composite number* $-s$.

Theorem 4. *Let partial sum function* $\{c_k\}$ *converge absolutely, whereby the definition of the series* $\sum_{n=0}^{\infty} a_n$ *converges to a limit* Z *if and only if the associated sequence of partial sums*$\{c_k\}$ *converges to* Z*, written as*

$$Z = c_k = \sum_{n=0}^{\infty} a_n \Leftrightarrow Z = \lim_{k \to \infty} c_k,$$

or when $\{c_k\}$ *is undefined due to* $n = 0$*, and if continuity exists between* $n, n+1$*, then*

$$Z = c_k = \sum_{n=1}^{\infty} a_n \Leftrightarrow Z = \begin{matrix} lim \\ k \to \infty \end{matrix} c_k,$$

such that

$$1 = \int_1^{\infty} c_k \cdot N_0 \, e^{-\lambda t} \, dt = c_k \cdot \frac{N_0}{\lambda} \, .$$

The root of exponential decay, its degeneration being zero, imposes

$$Z = c = 0.$$

Theorem 4.1. *The Riemann zeta function* s *is the eigenvalue of a system that forms the root of all natural exponential decay out of consideration of natural numbers and the primes in that domain where* s *becomes the eigenvalue of the opposite of the differentiation operator with* $N(t)$ *as the corresponding eigenfunction;*

$$-s \, N(t) - \lim \nu_s(p) = 0 \; \forall \; s > 0.$$

Theorem 4.2. *Riemann decay involves a Boltzmann distribution, where its partition function* $Z = c_s(p) = 0$*, such that* $q = Z/g_n$*.*

Theorem 5*. Given* $\nu_s(p) = A(x)/A_0$*, all the prime numbers are to the ratio of* $A(x)/A_0$ *as the reciprocal of* $A(x)/A_0$ *is to all the values of the imaginary part of all the non-trivial zeros of the Riemann zeta function;*

$$p \colon \frac{A(x)}{A_0} :: \frac{A_0}{A(x)} \colon \omega_\rho,$$

such that

$$\frac{k_\rho \, \mathrm{A}_0 \ln(\omega_\rho)}{p \, A(x) \ln(A_0 \, Ax^{-1})} \sim -A(x) = \frac{1}{s} \; \forall \; s > 0,$$

where k_ρ *is simply the value* $1.1580 \ldots$*, and such that*

$$\omega_\rho(p) = e^{\frac{\ln(p) \, A(x)}{k_\rho(p) \, A_0}},$$

in that the roots of the Riemann zeta function become defined in terms of an infinite sequence over the primes and Riemann decay.

Theorem 6*. The period of the triangle periodic function*

$$\hat{\phi}_\sigma(\omega) = \frac{1}{h} - \frac{1}{g} = \frac{\chi_\sigma(\omega)}{\eta(\sigma)},$$

is equal to the greatest common divisor of the reciprocal of the real part σ *of the argument* s *(the eigenvalue of the root of natural decay) and the reciprocal of the product of the real part* σ *of* s

and the real part ξ *of the corresponding elements* $z_s(p)$ *of the Hermitian matrix derived from* s*;*

$$P = \gcd\left(\frac{1}{\sigma}, \frac{1}{\sigma \lim \xi}\right): \begin{array}{c} \sigma = Re(s) \\ \xi = Re\big(z_s(p)\big) \end{array}.$$

Theorem 7. *Given that* $\hat{\phi}_\sigma(\omega)$ *defines the period through a greatest common divisor, a necessary condition is imposed on the Fourier series convergence of the Riemann triangle periodic function. The Dirichlet Conditions may be replaced with a single necessary and sufficient condition: The Fourier series converges because the period is equal to ("or corresponds to", in the case of multiplicative factor cases of the arguments of the function), the greatest common divisor of the reciprocal of the real part of the argument and the reciprocal of the product of the real part of the argument and the real part* ξ *of its corresponding* $z_s(p)$ *(i.e. it converges due to reducibility near infinity);*

$$\lim -g\hat{\phi}_\sigma(\omega) + g = dirac(-h) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{im(-h)}\, dm = 0 \;\forall\; \sigma > 0,$$

$$dirac_\sigma(\omega) = -g\hat{\phi}_\sigma(\omega) + g \;\forall\; \sigma > 0.$$

Theorem 8. *Given an equation in the form of*

$$(\lim g - h)^{-4} - (h)^2 = \sqrt{\Delta_h} \Leftrightarrow h = 0,$$

where Δ_h *is the discriminant of* h *(the function of the polynomial's coefficients that gives information about the nature of its roots) there is only one rational argument* h *that provides any meaningful solution to the equation.*

Theorem 9. *Let the cancellation property of all prime numbers* p *occur in the subtraction of the mean value of assigned probabilities from the ratio of a multiplicative factor of the field of fractions from its numerator, also the eigenfunction of the root of natural decay containing the elements of prime factors, further subtracted from a continuous real part of the Riemann zeta function;*

$$\sigma - \frac{e^{-\frac{s\, ln(p)}{p}}}{2\; Quot(R_\sigma)} - \mu.$$

If the cancellation property of the product of a multiplicative factor of the field of fractions and the root determinants of a Hermitian matrix expressed by $z_s(p)$*, also required for determining the period of the oscillations* μ *is the mean value of, occurs correspondingly to*

$$-16\; Quot(R_\sigma)\; |z_s(p)|^2 = 1,$$

then continuity exists in σ_ρ *(of the roots of the Riemann zeta function) as* $z_s(p)$ *too correlates to a factor of the ratio of the primes to the roots. Given*

$$\lim \frac{\ln(p)}{\ln(\omega_\rho(s))} = 1.22(1) \ldots,$$

where $\omega_p(s)$ are the values of the imaginary parts of the roots of the Riemann zeta function,

$$\lim\left(\left(\frac{1.22(1)\ldots}{-4\,|z_s(p)|^2}+4\right)^{-1}-1\right)=1\ \Leftrightarrow \sigma=\frac{1}{2},$$

such that

$$\zeta(s)=0 \Leftrightarrow \sigma_p=\frac{1}{2}\ \forall\, 1>\sigma>0,$$

where all the real parts of the non-trivial zeros of the Riemann zeta function equal one half.

2. Construction of Riemann Decay

Consider quantity N (a quantity that experiences exponential decay over distance t), and the number of discrete elements $N(t)$ in a certain set. The common definition is

$$N(t)=N_0\,e^{-\frac{t}{\tau}},$$

where e is Euler's number and A_0 is the initial quantity, quantity at field time $t=0$ and τ is the exponential time constant [8]. Exponential decay is a scalar multiple of the exponential distribution, which has a well-known expected value, so the solution of the differential equation that expresses the progression is

$$\ln(N)=-\lambda t+C,$$

where C is the constant of integration, which allows

$$N(t)=e^C\,e^{-\lambda t}=N_0\,e^{-\lambda t}.$$

Upon inspection of $t=0$, the final substitution,

$$N_0=e^C,$$

may be obtained, as N_0 is defined as being the quantity at field time $t=0$. λ, the decay constant, then becomes the eigenvalue of the opposite of the differentiation operator with $N(t)$ as the corresponding eigenfunction and given an assembly of elements, the number of which decreases ultimately to zero, the exponential time constant τ, becomes the expected value of the amount of time before an object is removed from the assembly [8].

The expected value is obtained from the standard normalizing conversion to a probability space, where c is the normalizing factor in

$$1=\int_0^\infty c\cdot N_0\,e^{-\lambda t}\,dt=c\cdot\frac{N_0}{\lambda}$$

to convert a probability space [9], which can be rearranged to

$$c = \frac{\lambda}{N_0}.$$

The following them allows computation of integration by parts [8];

$$\tau = \langle t \rangle = \int_0^\infty x \ \cdot c \cdot N_0 e^{-\lambda t} = \int_0^\infty \lambda t e^{-\lambda t}\, dt = \frac{1}{\lambda}.$$

Basis for Riemann Decay. The Montgomery-Odlyzko Law [4][10] states

> The distribution of the spacings between successive non-trivial zeros of the Riemann zeta function (suitably normalized) is statistically identical with the distribution of eigenvalue spacings in a GUE operator.

Based on this (or perhaps more internalized reasoning), Alain Connes has constructed just such an operator [11], but regrettably his construction provided no proof (in and of itself) of whether or not all the non-trivial zeros have a real part equal to one half. We therefore reconstruct from a different approach, diverging shy of Connes' fuller development and redirect to focus on one particular aspect of the operator, most of our result not pertaining directly to his work (though correlations may be made). Now, if we may determine that the normalizing factor c is also applicable to the Riemann zeta function with a probability density function of various parameters, then a parameterized normalizing constant c becomes directly applicable to the partition function for the Boltzmann distribution, known to play a central role in statistical mechanics [12].

Proposition 1. *Let* ν *be a parameterized constant that corresponds to any given argument s of the Riemann zeta function, so that*

$$\nu = e^{\frac{-s\ln(p_n)}{p_n}} : \begin{matrix} \lambda = s \\ t = \dfrac{\ln(p)}{p}. \end{matrix}$$

One gets the infinite sequence $\nu_s(p)$ *whose points in the Riemann zeta function for any argument* s *correspond to a single prime number;*

$$\zeta(s) - \prod_p (1 - \nu_s(p)^p)^{-1} = 0 \ \forall\, p, s,$$

so that an expression of exponential decay is defined in

$$\nu_s(p) = \frac{N(t)}{N_0},$$

and

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \prod_{p} \left(1 - \left(\frac{N(t)}{N_0}\right)^{p}\right)^{-1},$$

where the Riemann zeta function is equal to the product of the inverse of one minus the ratio of the number of discrete elements $N(t)$ *in a certain set per the initial quantity, the quantity at* $t = 0$*, where* p *represent the prime numbers.*

Typically throughout this paper we will not show all derivations, but since proof of this proposition is made simply through the derivation, and that this derivation will be used twice, we do so in this case. In order to prove the proposition, we first begin with the exponential decay form

$$\nu = e^{\frac{-s\ln(p)}{p}} : \begin{matrix} \lambda = s \\ t = \dfrac{\ln(p)}{p} \end{matrix},$$

allowing ν to become some parameterized constant for any given s, corresponding to any given p, and then rearrange in order to solve for minus s in order to apply it to Euler's derivation of the zeta function;

$$\zeta(s) = \prod_{p} (1 - p^{-s})^{-1}.$$

One gets

$$-s = \frac{p\ln(\nu)}{\ln(p)}.$$

Then apply the right hand side of the equation above in place of minus s in

$$\zeta(s) = \prod_{p} (1 - p^{-s})^{-1},$$

which gives

$$\zeta(s) = \prod_{p} \left(1 - p^{\frac{p\ln(\nu)}{\ln(p)}}\right)^{-1}.$$

Express the right side as

$$\zeta(s) = \left(1 - 2^{2\ln(\nu_1)\div\ln(2)}\right)^{-1}\left(1 - 3^{3\ln(\nu_2)\div\ln(3)}\right)^{-1}\left(1 - 5^{5\ln(\nu_3)\div\ln(5)}\right)^{-1} \dots$$

and give the inverse of both sides. One gets

$$\zeta(s)^{-1} = \left(1 - 2^{2\ln(\nu_1)\div\ln(2)}\right)\left(1 - 3^{3\ln(\nu_2)\div\ln(3)}\right)\left(1 - 5^{5\ln(\nu_3)\div\ln(5)}\right) \dots.$$

Then give the natural logarithm of both sides. One gets

$$\ln(\zeta(s)^{-1}) = \ln\left(\left(1 - 2^{2\ln(\nu_1)\div\ln(2)}\right)\left(1 - 3^{3\ln(\nu_2)\div\ln(3)}\right)\dots\right).$$

Because $\ln(x \times y) = \ln(x) + \ln(y)$, express the equation above as an infinite series of natural logarithms. This gives

$$\ln(\zeta(s)^{-1}) = \ln\left(1 - 2^{2\,ln(\nu_1)\div ln(2)}\right) + \ln\left(1 - 3^{3\,ln(\nu_2)\div ln(3)}\right) + \cdots.$$

Next, subtract all the natural logarithms to the right of the first from both sides, which gives

$$\ln(\zeta(s)^{-1}) - (\ \ln\left(1 - 3^{3\,ln(\nu_2)\div ln(3)}\right) + \cdots) = \ln\left(1 - 2^{2\,ln(\nu_1)\div ln(2)}\right),$$

and give the exponent of both sides. One gets

$$\exp(\ln(\zeta(s)^{-1}) - (\ \ln\left(1 - 3^{3\,ln(\nu_2)\div ln(3)}\right) + \cdots) = 1 - 2^{2\ln(\nu_1)\div\ln(2)}.$$

Then subtract one from both sides, and multiply both sides by minus one, getting

$$1 - \exp(\ln(\zeta(s)^{-1}) - (\ \ln\left(1 - 3^{3\,ln(\nu_2)\div ln(3)}\right) + \cdots) = 2^{2\ln(\nu_1)\div\ln(2)}.$$

Next give the natural logarithm of both sides. This gives

$$\begin{aligned}\ln(1 - \exp\left(\ln(\zeta(s)^{-1}) - \left(\ \ln\left(1 - 3^{3\,ln(\nu_2)\div ln(3)}\right) + \cdots\right)\right) \\ = \ln\left(2^{2\ln(\nu_1)\div\ln(2)}\right).\end{aligned}$$

Then, because $\ln(x^Y) = Y\ \times \ln(x)$, express the right hand side of the equation above as

$$\ln(1 - \exp(\ln(\zeta(s)^{-1}) - (\ \ln\left(1 - 3^{3\,ln(\nu_2)\div ln(3)}\right) + \cdots)) = \frac{2\ln(2)\ln(\nu_1)}{\ln(2)}.$$

The natural logarithm of two cancels out on the right side of the equation, which gives

$$\ln\left(1 - \exp\left(\ln(\zeta(s)^{-1}) - \left(\ \ln\left(1 - 3^{3\,ln(\nu_2)\div ln(3)}\right) + \cdots\right)\right)\right) = 2\ln(\nu_1).$$

Once more, because $\ln(m^N) = N\ \times \ln(m)$, express the right hand side of the equation above as

$$\ln\left(1 - \exp\left(\ln(\zeta(s)^{-1}) - \left(\ \ln\left(1 - 3^{3\,ln(\nu_2)\div ln(3)}\right) + \cdots\right)\right)\right) = \ln({\nu_1}^2),$$

and then give the exponent of both sides. One gets

$$1 - \exp\left(\ln(\zeta(s)^{-1}) - \left(\ \ln\left(1 - 3^{3\,ln(\nu_2)\div ln(3)}\right) + \cdots\right)\right) = {\nu_1}^2.$$

Multiply both sides by minus one and subtract both sides from one. Written as

$$\exp\left(\ln(\zeta(s)^{-1}) - \left(\ln\left(1 - 3^{3\,ln(\nu_2) \div ln(3)}\right) + \cdots \right)\right) = 1 - {\nu_1}^2,$$

give the natural logarithm of both sides. This gives

$$\ln(\zeta(s)^{-1}) - \left(\ln\left(1 - 3^{3\,ln(\nu_2) \div ln(3)}\right) + \cdots \right) = \ln(1 - {\nu_1}^2).$$

Subtract the natural logarithm of one minus ν_1 to the power of two from both sides. One gets

$$\ln(\zeta(s)^{-1}) - (\ln\left(1 - 3^{3\,ln(\nu_2) \div ln(3)}\right) + \cdots) - \ln(1 - {\nu_1}^2) = 0.$$

Next add the natural logarithm of one minus three to the power of three times the natural logarithm of ν_2 divided by the natural logarithm of three to both sides. This gives

$$\ln(\zeta(s)^{-1}) - \left(\ln\left(1 - 5^{5\,ln(\nu_3) \div ln(5)}\right) + \cdots \right) - \ln(1 - {\nu_1}^2) . \\ = \ln\left(1 - 3^{3\ln(\nu_2) \div \ln(3)}\right)$$

Then repeat the previous nine equations (except for the one directly above) for ν_2 as we did for ν_1. We get

$$\ln(\zeta(s)^{-1}) - (\ln\left(1 - 5^{5\,ln(\nu_3) \div ln(5)}\right) + \cdots) - \ln(1 - {\nu_1}^2) - \ln(1 - {\nu_2}^3) \\ = 0.$$

Then repeat in the same for all remaining prime numbers to infinity. One gets

$$\ln(\zeta(s)^{-1}) - \ln(1 - {\nu_1}^2) - \ln(1 - {\nu_2}^3) - \ln(1 - {\nu_3}^5) - \cdots = 0.$$

Next, add the natural logarithm of one minus ν_1 to the power of two to both sides. This gives

$$\ln(\zeta(s)^{-1}) - \ln(1 - {\nu_2}^3) - \ln(1 - {\nu_3}^5) - \ln(1 - {\nu_4}^7) - \cdots \ldots = \ln(1 - {\nu_1}^2).$$

Repeat for all the remaining natural logarithms of one minus $\nu(x)$ to the power of the successive prime number, which gives

$$\ln(\zeta(s)^{-1}) = \ln(1 - {\nu_1}^2) + \ln(1 - {\nu_2}^3) + \ln(1 - {\nu_3}^5) \ldots.$$

Once more, because $\ln(x \times y) = \ln(x) + \ln(y)$, express the above equation as

$$\ln(\zeta(s)^{-1}) = \ln\left((1 - {\nu_1}^2)(1 - {\nu_2}^3)(1 - {\nu_3}^5) \ldots\right),$$

and give the exponent of both sides. One gets

$$\zeta(s)^{-1} = (1 - {\nu_1}^2)(1 - {\nu_2}^3)(1 - {\nu_3}^5).$$

Next give the inverse of both sides, which is written as

$$\zeta(s) = (1 - {\nu_1}^2)^{-1}(1 - {\nu_2}^3)^{-1}(1 - {\nu_3}^5)^{-1}.$$

We then finally get the Riemann zeta function in terms of ν raised to a single prime number in

$$\zeta(s) = \prod_p (1 - \nu^p)^{-1}\,.$$

Considering the exponential decay form ν originated from, in terms of s being the decay constant (and thus the eigenvalue of the matrix) and the natural log of a prime number divided by the same prime number, we can write the Riemann zeta function in terms of exponential decay;

$$\zeta(s) = \prod_p \left(1 - \left(\frac{N(t)}{N_0}\right)^p\right)^{-1} \forall\, s > 0. \ \blacksquare$$

The Limit of $\boldsymbol{\nu_s(p)}$. We construct an eigenfunction sequence $\nu_s(p)$ over all prime numbers p.

Proposition 2. $N(t) \sim N_0$ *for Riemann decay.*

To prove the proposition, we simply define the limit of the sequence

$$\nu_s(p) = e^{\frac{-s\ln(p)}{p}} = e^{\frac{-s\ln(2)}{2}}, e^{\frac{-s\ln(3)}{3}} e^{\frac{-s\ln(5)}{5}}, \ldots,$$

where also any value of sequential eigenfunction $\nu_s(p)$ for any given s and any given prime number p results equally to the exponent of minus s times the natural logarithm of said given prime number divided by said given prime number, as

$$\nu_s(p_n) - e^{\frac{-s\ln(p_n)}{p_n}} = \zeta(s) - \prod_p (1 - \nu_s(p)^p)^{-1} = 0 \;\forall\, p, s.$$

p_n becomes the prime number that corresponds to natural time n from the prime counting function $\pi(n)$ $(n = 1{:}\, p = 2, n = 2{:}\, p = 3, n = 3{:}\, p = 5,$ etc.) [13]. Thus, out of consideration of fields

$$\lambda = s, t = \frac{\ln(p_n)}{p_n}.$$

We saw in the last section that

$$\frac{N_n}{N_0} = \frac{-s\ln(p_n)}{p_n} \;\forall\, s > 0,$$

resulting in

$$\frac{N(t)}{N_0} = \nu_s(p) \;\forall\, s > 0,$$

where N is a quantity valued with respect to n, such that N_n corresponds to prime number p_n. Also, s must always be a positive number if it can be considered a decay constant by definitions of exponential decay [8]. But also, mathematically in terms of our sequence, because $\nu_s(p)$ is continuous at point $s = 0$ as s approaches zero, resulting in a curve (having null curvature) at point $s = 0$, sufficiently defined with

$$\lim_{n \to \infty} \nu_s(p) = 1 \; \forall \, s <> 0.$$

The curve at point $s = 0$ with null curvature results in all values of $\nu_0(p) = 1,1,1,1,1, \ldots$, as the exponent of zero times the natural logarithm of any p_n divided by said p_n always will reduce to one. We show that $\nu_s(p) = N(t)/N_0$ has a limit for all decaying quantities, the limit of one. Since any $s > 0$ is an argument for $N(t)/N_0$ from the ratio over all the primes $\nu_s(p)$, the Riemann zeta function is equal to the product of the inverse of one minus the ratio of the number of discrete elements $N(t)$ in a certain set per the initial quantity N_0, the quantity at field time $t = 0$. The physical meaning of a calculation involving $t = 0$ in time $\ln(p)/p$ for

$$\frac{N(t)}{N_0} = \nu_s(p)$$

when there is no prime number equal to zero, such that $\ln(p)/p$ converges absolutely, and where $N_O, N(t)$ becomes asymptotically equivalent, is that it expresses the sum of a decaying quantity backward in time. This should not mean that it is time reversible, however, as a meaningful solution may not exist in solving for it forward in time.

The above should only be a necessary mathematical procedure in a case where the end time of the decay is undefined (the decaying quantity has zero degeneration). The above form of decay could not represent any form of matter, but it could theoretically represent time (or space) itself, as if time could have degeneration greater than zero, then all forms of exponential decay would become undefined, which they do not. Because the limit of $\nu_s(p)$ is one without any zero divisors,

$$N(t) \sim N_0. \blacksquare$$

$\nu_s(p)$ and the Fundamental Theorem of Arithmetic. We first prove the following result.

Proposition 3. *By means of the fundamental theorem of arithmetic, ν for all $s < 0$ becomes the numerator of the set of elements of the field of fractions $Quot(R_\sigma)$ of the integral domain R_σ, containing all numbers factored out of the set of by a lowest common multiple (lcm). Said $Quot(R_\sigma)$ for a given s then parameterizes the denominator $\delta_s(p)$, where $\lim \nu_s(p)$ demands $\lim \delta_s(p) = 1/Quot(R)$, the prime numbers then considered the atomic elements which, when combined together, make up composite number $-s$.*

In order to prove the proposition, we consider a means to factor numbers $\nu_s(e)$ out of a set having some lcm. Given a parameterized denominator $\delta_s(p) \neq N_0$ (the divisor of $Quot(R_\sigma)$ is not constant for the same reasons $\nu_s(p)$

is the ratio of $N(t)/N_0$, where N_0 is constant, the value of the quantity at $t = 0$; this of course is due to reducibility of fractions in that $a(b/c) = d/c$), we find that for all s the ratio $Quot(R_\sigma)$ is expressible in closed form,

$$Quot(R_\sigma) = \frac{(\ 2\sigma + 1)^{-1} - 4}{4},$$

where σ is the real part of s, such that

$$\frac{\nu_s(p)}{\delta_s(p)} = Quot(R_\sigma).$$

Because the limit of $\nu_s(p)$ equals one, we get a limit of the denominator $\delta_s(p)$ for any given real part of s, which can be calculated without knowledge of the prime numbers, using

$$\lim \delta_s(p) = \left(\frac{(2\sigma + 1)^{-1} - 4}{4}\right)^{-1}.$$

We then inspect various arbitrarily chosen integers for arguments of s. For instance, $s = -18$, where $-s = 2 \cdot 3 \cdot 3$, we get $\nu_{-18}(2) = 512 = e^{18\ln(2)/2} = 2^{3\cdot 3}$. At the next prime, we get $\nu_{-18}(3) = 729 = e^{18\ln(3)/3} = 3^{2\cdot 3}$. For all remaining prime numbers, as $\nu_{-18}(p) \to 1$, no other whole numbers exist as values of $\nu_{-18}(p)$. We write $lcm(512{,}729) = 2^{3\cdot 3} \cdot 3^{2\cdot 3} = 373248 = lcm(512{,}729{,}18)$.

This gives whole numbers for all negative composite numbers s. When s is a minus prime number, the value of the sequence $\nu_s(p)$ at said prime number equals the corresponding prime number itself ($s = -11$: $\nu_{-11}(11) = 11$), which provides a rudimentally simple means of primal testing (if $\nu_s(p_n) = p_n$, then $-s$ is prime). Also, a method for calculating the prime counting function $\pi(n)$: $n \to \infty$ presents itself, though prime numbers would still need to be stored in an array as primes are generated in order to calculate the next $\nu_s(p)$, by no means a complicated requirement). This method works, however, by beginning with known $p = 2$, because the equivalent classes $s = -p_n$: $\nu_s(p_n) = p_n$ occurs at a slower rate than occurrences of $n = p$ in $\pi(n)$, resulting in $\nu_s(p)$ convergence to one at a rate fast enough to limit the numbers of whole elements $\nu_s(e)$ in $lcm(\nu_s(e_1), \nu_s(e_2), \dots \nu_s(e_N))$ finitely, allowing for

$$lcm(\nu_s(e_1), \nu_s(e_2), \dots \nu_s(e_N)) = lcm(\nu_s(e_1), \nu_s(e_2), \dots \nu_s(e_N), -s)\ \forall\ s <> 0,$$

where there exists not only a finite number of natural elements $\mathbb{N}$ for any value of $s < 0$, but also for $s > 0$ (those applicable for exponential decay), becoming rational elements $\mathbb{Q}$, such that the numbers of fractional elements for s are the same as whole elements of $-s$, as

$$\mathbb{Q}\ \nu_e = \frac{1}{\mathbb{N}\ \nu_e}\ \forall\ s.$$

For example, in terms of $s = -6, s = 6$, we get finitely just two elements in each, $8{,}9$ and $1/8$, $1/9$ respectively. This is applicable to all complex and real numbers s as well, but if the real number is irrational or either part of a complex s is irrational, then the finite number of elements is necessarily zero (having zero elements that can be expressed as a fraction).

To prove the proposition we construct a Hermitian matrix such that $z_s(p)$ in

$$\delta_s(p) = \frac{4(z_s(p)+1)}{(-4z_s(p))^{-1}} - 4 = \frac{\nu_s(p)}{Quot(R)},$$

out of consideration of $z_s(p)$ in

$$0 = z_s(p)^2 + z_s(p)(\delta_s(p)+1) + \frac{\delta_s(p)}{-16},$$

the limit of $z_s(p)$, becomes complex for all arguments s in

$$z_s(p) = \frac{-(\delta_s(p)+1) \pm \sqrt{\left(1+\delta_s(p)\right)^2 + \delta_s(p)4^{-1}}}{2}.$$

$z_s(p)$ then become the elements of a Hermitian matrix, as each eigenvalue $z_s(p)$ is the complex conjugate of its reflection in the lead diagonal [14]. Essentially this expression of exponential decay of time itself in $\ln(p)/p$ equivalently defines propositions 30 and 32 in Book VII, of Euclid's Elements [15]. We get a correlation between the finite number of elements $\nu_s(e)$ in sequential eigenfunction $\nu_s(p)$ and the factors pertaining to the fundamental theorem of arithmetic, in that they are equivalent. This is as corollary as the equivalence classes $s = -p_n: \nu_s(p_n) = p_n$, in that if the fundamental theorem of arithmetic is true, then so too are these finite classes of elements true. In other words, if $\nu_s(p_n) = p_n$ for any argument s, then $-s$ is prime. ■

We call then the above expression of exponential decay (the decay of time or space itself) the "root of natural decay", or "Riemann decay", as its degeneracy should reduce to zero. And we will prove that the degeneracy factor indeed does reduce to zero after this root is fully constructed in the next sections. And now that the above is defined and proven, we can now move forward toward the root.

3. The Riemann Probability Density Function

Enumeration of prime numbers starts with one, in that the so-called $zero^{th}$ prime, or n^{th} prime $p(0)$, may be anything that is not equal to a prime, e.g., zero [16]. This could be important while considering an initial quantity N_0 at $t = 0$ when field time t is measured in $\ln(p)/p$. However, because n of the Riemann zeta function in terms of the left hand side of

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \prod_p \left(1 - \left(\frac{N(t)}{N_0}\right)^p\right)^{-1}$$

is undefined at $n = 0$, there too could be no meaning of $\pi(0)$,where again $\pi(n)$ is the prime counting function. However, there is and must be meaning to some initial value N_0 at $t = 0$, so it then must exist at the infinite prime number $\pi(\infty)$, and not at $\pi(n < 1)$. A solution to the value of N_0 though could be determined upon a condition that a condition is true if and only if N_0 is at a particular value.

The same can be said for

$$1 = \int_0^\infty c \cdot N_0\, e^{-\lambda t}\, dt = c \cdot \frac{N_0}{\lambda}\ ,$$

in that if c above, the normalizing constant, were to be shown to be a partition function having a denominator that contained a value $n = 0$ for $t = 0$, the function is considered normalized if for all other values it is normalized outside the undefined value [17]. A common case is the log integral function $Li(x)$ [18], the area under the curve of $1/\ln(m)$ from zero to x, where $Li(x)$ is undefined at $x = 1$, yet summed from zero to infinity (note: the symbol t is most typically used instead of m, but we express it here as m, so as to reserve t always for time in $\ln(p)\,/p$). A solution still exists for the entire sum $Li(x)$, calculable by considering the fractions of arguments x approaching the limit of $x = 1$, yet providing the entirety of the area under the curve. In this case, we solve for the sum $N(t)$ backward in time where the quantity is already fully decayed (note: doing so has no correlation to time reversal, as any chronological moments already-occurred can be mapped backward). And if the quantity had zero degeneration, then we state that we are solving for its value from the beginning of time itself.

That said; the far right hand side of the equation above would be undefined at the root of natural exponential decay (the singularity of exponential decay). It would have no meaningful solution in the event of a partition function having a denominator equal to zero if that single undefined value corresponded to the initial quantity N_0. It can still be expressed in terms of exponential decay, but the root could have no degeneration in and of itself, thus its degeneration factor would be equal to zero, and most intuitively its partition function equal to zero as well. The sum of $N(t)$ would still amount to the same by solving backward in time $\ln(p)\,/p$. Thus, if c above could be shown to be a partition function Z, and there were an $n = 0$ in the denominator of Z at $t = 0$, then $n = 0$ would represent the end of natural time (which would result in an undefined value of the integral above) and $t = 0$ would represent the beginning of field time (the beginning of time itself, a universal singularity).

Again, if the root of natural exponential decay could decay itself, then nothing else could decay based on it, as all exponential decay occurs over time (or distance, which would amount to the same in terms of amplitude A). If time were to be able to be expressed as having degeneration greater than zero, then there would be no meaningful solution to decay itself, as it would degenerate such that the denominator of all other decay becomes equal to zero—thus undefined. Decay itself then should become undefined in all cases, as the normalizing integral above would become undefined for all values. Fortunately, this it is not the case for physics or mathematics.

The root normalizing integral would become

$$1 \neq \int_{0}^{\infty} 0 \cdot N_0 \, e^{-\lambda t} \, dt = 0 \cdot \frac{N_0}{\lambda} \, ,$$

as

$$1 \neq \frac{0}{0} = undefined.$$

Proposition 4. *Let partial sum function $\{c_k\}$ converge absolutely, whereby the definition of the series $\sum_{n=0}^{\infty} a_n$ converges to a limit Z if and only if the associated sequence of partial sums$\{c_k\}$ converges to Z, written as*

$$Z = c_k = \sum_{n=0}^{\infty} a_n \Leftrightarrow Z = \lim_{k \to \infty} c_k,$$

or when $\{c_k\}$ is undefined due to $n = 0$, and if continuity exists between $n, n+1$, then

$$Z = c_k = \sum_{n=1}^{\infty} a_n \Leftrightarrow Z = \lim_{k \to \infty} c_k,$$

such that

$$1 = \int_{1}^{\infty} c_k \cdot N_0 \, e^{-\lambda t} \, dt = c_k \cdot \frac{N_0}{\lambda} \, .$$

The root of exponential decay, its degeneration being zero, imposes

$$Z = c = 0.$$

In consideration of a non-degenerating time continuum and the prime numbers, because

$$\frac{N_n}{N_0} = -\frac{s \ln(p_n)}{p_n} \; \forall \, s > 0, \forall \, p \colon n \to \infty,$$

we simplify and express

$$\nu_s(p) \; = \frac{N(t)}{N_0} \; \forall \, s > 0.$$

Again, p in $\nu_s(p)$ are all the prime numbers as n maps to infinity, the same meaning of p in $\prod_p (1 - \nu_s(p)^{\;p})^{-1}$. With that and the limit of $\nu_s(p)$ known for all s, we have proven above that the sum of the discrete elements

$$N(t) \sim N_0 \; \forall \, p, s > 0,$$

as two values x, y are asymptotically equivalent when the limit of their ratio x/y equals one. And again, the $s > 0$ requirement follows from the study of exponential decay. Thus, trivial zeroes of the Riemann zeta function whose $s < 0$ are not applicable to Riemann decay, and will later be shown to be mathematically independent from our final equations in that the final proof will functions that hold true only for the non-trivial zeros. This is the point where we begin to fully diverge from similarity with Conne's operator, whose result occurs for any zero of the Riemann zeta function and not just the roots of it [11]—even though his Riemann resonant operator still will exist (in form) when we take all of the above and place it in terms of decaying wave amplitudes.

Proposition 4.1. *The Riemann zeta function s is the eigenvalue of a system that forms the root of all natural exponential decay out of consideration of natural numbers and the primes in that domain where s becomes the eigenvalue of the opposite of the differentiation operator with $N(t)$ as the corresponding eigenfunction;*

$$-s\,N(t) - \lim v_s(p) = 0 \;\forall\; s > 0.$$

In order to prove these two propositions, we begin with the second. Using the definition of a power for e with the infinite series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots,$$

we write

$$\frac{-s\ln(p_n)}{p_n} = -s\ln(p_n)\,{p_n}^{-1} = v_s(p_n).$$

apply $v_s(p_1)$, which corresponds to prime number two, to the exponential series, which is 100% of the quantity N without any decay at field time $t = \ln(2)/2$, where two is the first prime number. We get

$$v_s(2) = 1 - s\ln(2)\,2^{-1} + \frac{(-s\ln(2)\;2^{-1})^2}{2!} + \frac{(-s\ln(2)\;2^{-1})^3}{3!} + \frac{(-s\ln(2)\;2^{-1})^4}{4!} + \frac{(-s\ln(2)\;2^{-1})^5}{5!} \ldots.$$

For $v_s(p_2)$ for corresponding prime number three, we get

$$v_s(3) = 1 - s\ln(3)\,3^{-1} + \frac{(-s\ln(3)\;3^{-1})^2}{2!} + \frac{(-s\ln(3)\;3^{-1})^3}{3!} + \frac{(-s\ln(3)\;3^{-1})^4}{4!} + \frac{(-s\ln(3)\;3^{-1})^5}{5!} + \cdots.$$

For $v_s(p_3)$, we get

$$v_s(5) = 1 - s\ln(5)\,5^{-1} + \frac{(-s\ln(5)\;5^{-1})^2}{2!} + \frac{(-s\ln(5)\;5^{-1})^3}{3!} + \frac{(-s\ln(5)\;5^{-1})^4}{4!} + \frac{(-s\ln(5)\;5^{-1})^5}{5!} + \cdots.$$

Next, let the reciprocals of the factorials of $n > 0$,

$$\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$$

be referred to from here on as $m!^{-1}$. Because $1/x(y + y^2 + y^3 + \cdots) = y/x + y^2/x + y^3/x + \cdots$, we write

$$v_s(p_n) = 1 + \int_1^\infty m!^{-1} \left(\frac{-s\ln(p_n)}{p_n}\right)^m dm,$$

or

$$v_s(p_n) = 1 + \iota : \iota = \int_1^\infty m!^{-1} \left(\frac{-s\ln(p_n)}{p_n}\right)^m dm,$$

which is a partial sum of $v_s(p)$ at p_n. Considering the infinite sequence of integrals $\iota(s)$ for all primes to infinity,

$$\iota(s) = \int_1^\infty m!^{-1} \left(\frac{-s\ln(2)}{2}\right)^m dm, \int_1^\infty m!^{-1} \left(\frac{-s\ln(3)}{3}\right)^m dm, \int_1^\infty m!^{-1} \left(\frac{-s\ln(5)}{5}\right)^m dm, \ldots$$

we get the entire sum over all primes in terms of sequence

$$\iota(s) = \int_1^\infty m!^{-1} \left(\frac{-s\ln(p)}{p}\right)^m dm.$$

Because

$$\zeta(s) = \prod_p (1 - v_s(p)^p)^{-1},$$

the rearrangement becomes

$$\zeta(s) = \prod_p \left(1 - (1 + \int_1^\infty m!^{-1} \left(\frac{-s\ln(p)}{p}\right)^m dm)^p\right)^{-1}$$

or

$$\zeta(s) = \prod_p (1 - (1 + \iota(s))^p)^{-1}$$

or

$$\sum_{n=1}^{\infty} n^{-s} = \prod_{p} (1 - (1 + \iota(\,s))^{p})^{-1},$$

where n is natural time greater than zero. These exponential decay relationships are then tied to the Riemann zeta function, and in as simple summation as the expression of the Riemann zeta function on the left hand side of the equation above, involving natural time n. Thus, we can see how the partial sums of the ratio of

$$v_s(p) = \frac{N_n}{N_0}$$

is intricately linked to partial sums of the Riemann zeta function.

The final substitution then, $N_0 = e^C$, is obtained by evaluating the equation at field distance $t = 0$, as N_0 is defined as being the quantity at $t = 0$ [8]. The decay constant $s = \lambda$ from the last section was our description of the decay. But while we are measuring time in $\ln(p)\,/p$, there is no prime number solution in this that equals zero. Yet, the product of the ratio of $\ln(p)\,/p$ does converge to zero even though all prime numbers, all n and all s are greater than zero in this system of exponential decay. For any given $s > 0$, we find that

$$\prod_{p} v_s(p) = 0,$$

which of course does not mean that there need exist a prime number equal to zero; rather simply that time in $\ln(p)\,/p$ converges absolutely.

Though, finding an indefinite integral of a function $f(x)$ is the same as solving the differential equation $dy/dx = f(x)$. Any differential equation will have many solutions, and each constant represents the unique solution of a well-posed initial value problem, as is what we have with N_0 and our initial value problem. Thus, for our many unique solutions C with respect to natural n, the only possible value for N_0 for any $s > 0$ that makes

$$N_n = -N_o \frac{s\ln(p)}{p} = e^{-st+C} \;\forall\; s > 0$$

true is

$$N_0 = \frac{1}{-s} \colon N_n = t,$$

where all the unique solutions C are less than zero for all N_n and s greater than zero. This is what one should expect in a system of the exponential decay of time itself (but where its degeneration would equal zero).

We move further toward the proof of Proposition 4.1 by rearranging and solving for the number of elements in the discrete set $N(t)$, and taking $N(t)$ to its sum where $v_s(p)$ converges on one as it approaches the infinite prime number, being that $N(t) \sim N_0$, where p are all the prime numbers. Sum $N(t)$ is obtained by taking all the primes $p_n: n \to \infty$ and adding the partial sums $\nu_s(p_n)$ in the earlier used form of

$$\nu_s(p_n) = 1 + \int_1^{\infty} m!^{-1} \left(\frac{-s\ln(p_n)}{p_n}\right)^m dm.$$

For the infinite sequence $\iota(s)$ of integrals for all prime numbers toward infinity, we get

$$N(t) = \frac{1}{-s} + \frac{1}{-s}\iota(s),$$

such that $N(t) = 1/-s$ for all s, as the second term simply reduces to zero when the sequence of $\iota(s)$ is considered for all the prime numbers.

$$\frac{1}{-s}\int_1^{\infty} m!^{-1} \left(\frac{-s\ln(p)}{p}\right)^m dm = \frac{1}{-s}\iota(s) = 0.$$

This gives

$$N(t) = N_0\, \nu_s(p) = \frac{1}{-s}\ \forall\, s\ > 0,$$

and as one should now expect, the final result of the proof of Proposition 4.1;

$$-s\, N(t) - \ \nu_s(p) = 0\ \forall\, s > 0. \blacksquare$$

In this sense, our form being the root of natural exponential decay becomes more intuitive (becoming fully intuitive only after proof of Proposition 4.0), considering the above, it being formed exclusively on natural time n and the fundamental theorem of arithmetic in terms of the factors relationship in $\nu_s(p)$.

Construction of the Riemann Probability Density Function. To make an everywhere non-negative function correspond to a normalizing constant so that it becomes a probability density function or probability mass function [19], we consider the area under its graph

$$1 = \int_0^{\infty} c \cdot N_0\, e^{-\lambda t}\, dt = c \cdot \frac{N_0}{\lambda},$$

where constant c becomes a multiplicative constant in integral function $\varphi(n)$, where in our case $\lambda = s$. We find that the Riemann zeta function s involves a series of partial sums $\{c_k\}$ whose limit is zero (therefore also involving a partial sum of the Riemann zeta function), whereby the definition of the series $\sum_{n=0}^{\infty} a_n$ converges to a limit Z if and only if the associated sequence of partial sums$\{c_k\}$ converges to Z, written as

$$Z = c_k = \sum_{n=0}^{\infty} a_n \Leftrightarrow Z = \lim_{k \to \infty} c_k.$$

Again, if $\{c_k\}$ is undefined at $n = 0$, and if continuity exists between $n, n+1$ (such as a function having the domain of natural numbers greater than zero), then we instead consider

$$Z = c_k = \sum_{n=1}^{\infty} a_n \Leftrightarrow Z = \lim_{k \to \infty} c_k,$$

such that

$$1 = \int_1^{\infty} c_k \cdot N_0 \, e^{-\lambda t} \, dt = c_k \cdot \frac{N_0}{\lambda} \; ,$$

as there is no area under the curve from $t = 0$ to $t = 1$ (i.e. no $t = 1/2$). Considering the Boltzmann distribution for the fractional number of particles $N_n / N(t)$ [1], noting that the numerator of the fractional number of particles is more typically referred to as N_i and the denominator N, but changed so as to not confuse with the variables in the preceding proofs (reserving i for notating only ever $\sqrt{-1}$), occupying a set of states n possessing energy E_n, whereby

$$\frac{N_n}{N(t)} = \frac{g_n \, e^{-E_n/(k_B \, T)}}{Z}$$

and

$$N(t) = \sum_n N_n,$$

such that

$$Z = \sum_n g_n \, e^{-\frac{E_n}{k_B \, T}},$$

where k_B is the Boltzmann constant, T is temperature (assumed to be a well-defined quantity), g_n is our degeneracy factor (the number of levels having energy E_n) and $N(t)$ is the total number of particles.

By applying any known energy and temperature values, expressing Boltzmann's Consant equal to 1 in Planck units ($c = G = \hbar = k_B = 1$, where c is the velocity of light, G is the gravitational constant, $\hbar$ is Planck's reduced constant and k_B is the Boltzmann Constant), and $E = 1/2T$, to the following arbitrary, but directed function

$$q = 1 + \int_1^{\infty} m!^{-1} \left(\frac{-E_1}{k_B T}\right)^m dm + 1 + \int_1^{\infty} m!^{-1} \left(\frac{-E_2}{k_B T}\right)^m dm + 1 + \int_1^{\infty} m!^{-1} \left(\frac{-E_3}{k_B T}\right)^m dm + \; \dots,$$

where E_n are the values of energy at each state in the system (at any given moment of natural time n), and all values of $-E/k_B T$ remain constant for any value of T, equal to $-1/2$, such that k converges. In terms of Planck units, $q > 0$, but we get convergence for any multiplicative factor of T to E when $k_B = 1$ even outside of Planck units.

Using a form similar to that of Planck units, we are interested in the multiplicative factor that reduces function q to zero, and that factor is not $1/2$, rather is 9—thus we are interested in $E = 9T$, and $k_B = 1$. This gives

$$0 = 1 + \int_1^{\infty} m!^{-1} (-9)^m dm + 1 + \int_1^{\infty} m!^{-1} (-9)^m dm + 1 + \int_1^{\infty} m!^{-1} (-9)^m dm + \ \dots$$

for all any and all values of E_n and T.

Proposition 4.2. *Riemann decay involves a Boltzmann distribution, where its partition function* $Z = c_s(p) = 0$*, such that* $q = Z/g_n$.

In order to prove Proposition 4.2, which would also prove Proposition 4.0, we change the notation c from here to $c_s(p)$, as it becomes more precise to our particular system having prime number arguments, as well as s, so long as it is noted that $c_s(p)$ is a sequence of c. We then consider a constant $k_c = 1.1291(76)^{-1}$ ..., constant for all $s \neq 0$, but only relevant to exponential decay when $s > 0$, $c_s(p)$ becomes a sequential function for a given decay constant s and a given prime number p_n and the natural number n it corresponds to, such that

$$c_s(p_n) = \frac{-s\, k_c\, e^{\ln(s^{p_n}\, p_n{}^s) \div p_n}}{n^{p_n}} \ \forall\, P_n\!: n \to \infty,$$

or for the entire sequence of partial sums over all prime numbers to the infinite prime

$$c_s(p) = \frac{-s\, k_c\, e^{\ln(s^p\, p^s) \div p}}{n^p} \ \forall\, s > 0,$$

makes the following true:

$$1 = \int_1^{\infty} \frac{-s\, k_c\, e^{\ln(s^p\, p^s) \div p}}{n^p} \cdot N_0\, e^{-\lambda t}\, dn \ \forall\, s > 0 \iff N_0 = \frac{1}{-s}.$$

Note that the normalizing integral begins at natural time $n = 1$ rather than the more typical zero, as the end of natural time $n = 0$ is undefined being in the denominator, where $e^{-\lambda t}$ (equal to $\nu_s(p)$) becomes a probability density function corresponding to the quantity sum N_n, and $e^{\ln(s^p p^s)/p}$ becomes a probability density function corresponding to the sum $N(t)$ of all the quantity sums N_n. Upon inspection of $c_s(p)$, we then therefore show that by taking it to the infinite prime number for any s,

$$\lim_{p} c_s(p) = 0.$$

Then taking $e^{\ln(s^p p^s)/p}$ to the infinite prime number, we get

$$\lim e^{\frac{\ln(s^p p^s)}{p}} = s\colon e^{\frac{\ln(s^{p_n}\, p_n^s)}{p_n}} \sim s.$$

upon inspection of $e^{\ln(s^p p^s)/p}$ we also find

$$\frac{s}{e^{\ln(s^{p_n} p_n{}^s)\, p_n^{-1}}} = \nu_s(p_n)\ \forall\ p_n, s$$

and generally

$$\frac{s}{e^{\ln(s^p p^s) p^{-1}}} = \nu_s(p) = \frac{N(t)}{N_0}\ \forall\ s.$$

Thus

$$\frac{s\, N_0}{N(t)} = e^{\frac{\ln(s^p p^s)}{p}}.$$

Returning then to

$$N(t) = \sum_n N_n = \frac{1}{-s}\ \forall\ s\ > 0,$$

we get

$$Z = \sum_n g_n e^{-\frac{E_n}{k_B T}} = c_s(p) = \frac{-s^2\, N_0\, k_c}{N(t) n^p} = 0\ \forall\ s > 0,$$

as

$$Z = c_s(p) = \sum_{n=1}^{\infty} a_n \Leftrightarrow Z = \lim_{p\to\infty} c_s(p)\ \forall\ s \neq 0.$$

The above then proves Proposition 4.0 ■, and we can continue forward in this same sense toward proof of Proposition 4.2. Being that $Z = c_s(p)$, where $c_s(p)$ is a parameterized constant that normalizes

$$1 = \int_0^\infty c \cdot N_0\, e^{-\lambda t} = \int_0^\infty c_s(p) \cdot N_0\, e^{\frac{-s ln(p)}{p}} = Z \cdot s\,,$$

we can compute it using integration by parts or considering the form

$$N(t) = \frac{1}{-s} + \frac{1}{-s}\int_{1}^{\infty} m!^{-1}\left(\frac{-s\ln(p)}{p}\right)^{m} dm \,\forall\, s > 0,$$

being that

$$\frac{N_n}{N(t)} = \frac{g_n\, e^{-E_n/(k_B\, T)}}{Z}.$$

We can solve for the values of $g_n\, e^{-E_n/(k_B\, T)}$, now defined in $Z = c_s(p)$ because of

$$\int_{1}^{\infty} \varphi(n)\, dn = \int_{1}^{\infty} \frac{-k_c\, s\, e^{\ln(s^{p_n}\, p_n{}^{s}) \div p_n}}{n^{p_n}} \cdot N_0\, e^{-st}\, dn = 1 \,\forall\, s > 0.$$

And thus, because $e^{\ln(s^{p_n}\, p_n{}^{s}) \div p_n} = \frac{s}{\nu_s(p)}$, we can express it as

$$\int_{1}^{\infty} \varphi(n)\, dn = \int_{1}^{\infty} \frac{-k_c\, s\, N_0}{n^{p_n}}\, dn = 1 \,\forall\, s > 0.$$

Because $-s = \frac{1}{N_0}$, this reduces to the very simple sum of a constant divided by all natural numbers greater than zero to the power of the primes, such that

$$\int_{1}^{\infty} \varphi(n)\, dn = \int_{1}^{\infty} \frac{k_c}{n^{p_n}}\, dn = 1 \,\forall\, s > 0.$$

We believe this is exactly what one should expect from the root of natural decay, as also it provides an expression of the fundamental theorem of arithmetic in terms of $\nu_s(p)$, and the result of this fundamental relationship, as well as the continuing proofs below, will begin now to trend toward a meaningful, rigorous proof of the Riemann Hypothesis (albeit progressively).

Now Z, the left hand side of

$$\frac{-s^2\, N_0\, k_c}{N(t)\, n^p} = \sum_n g_n e^{-E_n/(k_B T)},$$

can too be expressed as

$$c_s(p) = \frac{-s\, k_c}{n^p} + \frac{-s\, k_c}{n^p}\int_{1}^{\infty} m!^{-1} \ln\!\left(\frac{s\, N_0}{N(t)}\right)^{m} dm \,\forall\, s > 0,$$

being that

$$\frac{s\, N_0}{N(t)} = e^{\frac{\ln(s^p p^s)}{p}},$$

and such that $c_s(p)$ is expressed in terms of a sequence of integrals over all the decaying quantities

$$\iota_c = \int_1^{\infty} m!^{-1}\ \ln(\frac{s\ N_0}{N(t)})^m\, dm.$$

This also can be written as

$$\iota_c = \int_1^{\infty} m!^{-1}\ \ln(\frac{s\ N_0}{N_1})^m\, dm, \int_1^{\infty} m!^{-1}\ \ln(\frac{s\ N_0}{N_2})^m\, dm, \int_1^{\infty} m!^{-1}\ \ln(\frac{s\ N_0}{N_3})^m\, dm, \ldots.$$

Next, because the right hand side of

$$\frac{-s^2\ N_0\ k_c}{N(t)\ n^p} = \sum_n g_n e^{-E_n/(k_B T)},$$

is such that

$$Z = \sum_n g_n + g_n \int_1^{\infty} m!^{-1} \left(\frac{-E_n}{k_B T}\right)^m dm,$$

express the above equation as an infinite series;

$$\begin{aligned} Z = \ & g_1 + g_1 \int_1^{\infty} m!^{-1} \left(\frac{-E_1}{k_B T}\right)^m dm + g_2 \\ & + g_2 \int_1^{\infty} m!^{-1} \left(\frac{-E_2}{k_B T}\right)^m dm + g_3 \\ & + g_3 \int_1^{\infty} m!^{-1} \left(\frac{-E_3}{k_B T}\right)^m dm + \ \ldots \ , \end{aligned}$$

which gives us

$$\begin{aligned} \frac{-s\ k_c}{n^p} + \frac{-s\ k_c}{n^p} \int_1^{\infty} m!^{-1}\ \ln(\frac{s\ N_0}{N(t)})^m\, dm \\ = \ & g_1 \\ & + g_1 \int_1^{\infty} m!^{-1} \left(\frac{-E_1}{k_B T}\right)^m dm + g_2 \\ & + g_2 \int_1^{\infty} m!^{-1} \left(\frac{-E_2}{k_B T}\right)^m dm + g_3 \\ & + g_3 \int_1^{\infty} m!^{-1} \left(\frac{-E_3}{k_B T}\right)^m dm + \ \ldots \ . \end{aligned}$$

Divide both sides by g_n. One gets

$$\frac{-s\;k_c}{g_n\;n^p}+\frac{-s\;k_c}{g_n\;n^p}\int_1^{\infty} m!^{-1}\;\;\ln(\frac{s\;N_0}{N(t)})^m\,dm$$
$$= 1$$
$$+\int_1^{\infty} m!^{-1}\left(\frac{-E_1}{k_BT}\right)^m dm + 1$$
$$+\int_1^{\infty} m!^{-1}\left(\frac{-E_2}{k_BT}\right)^m dm + 1 + \int_1^{\infty} m!^{-1}\left(\frac{-E_3}{k_BT}\right)^m dm + \;\dots\;.$$

Out of consideration of

$$\frac{Z}{g_n} = \; 1 + \int_1^{\infty} m!^{-1}\left(\frac{-E_1}{k_BT}\right)^m dm + 1$$
$$+\int_1^{\infty} m!^{-1}\left(\frac{-E_2}{k_BT}\right)^m dm + 1 + \int_1^{\infty} m!^{-1}\left(\frac{-E_3}{k_BT}\right)^m dm + \;\dots$$

and $E = 9T$, we get an expanded expression of Proposition 4.0 in

$$Z = \frac{-s\;k_c}{n^p}+\frac{-s\;k_c}{n^p}\int_1^{\infty} m!^{-1}\;\;\ln(\frac{s\;N_0}{N(t)})^m\,dm = 0.$$

Thus,

$$\frac{-s\;k_c}{g_n\;n^p}-\frac{-s\;k_c}{g_n\;n^p} = \; 1$$
$$+\int_1^{\infty} m!^{-1}\left(\frac{-E_1}{k_BT}\right)^m dm + 1$$
$$+\int_1^{\infty} m!^{-1}\left(\frac{-E_2}{k_BT}\right)^m dm + 1 + \int_1^{\infty} m!^{-1}\left(\frac{-E_3}{k_BT}\right)^m dm + \;\dots$$
$$= 0$$

in the same way as

$$\frac{1}{-s}\int_1^{\infty} m!^{-1}\left(\frac{-s\ln(p)}{p}\right)^m dm = \frac{1}{-s}\iota = 0.$$

Since the above is valid for all primes and all n, such that

$$\frac{1}{-s}\int_1^{\infty} m!^{-1}\left(\frac{-s\ln(p)}{p}\right)^m dm =$$
$$= 1$$
$$+\int_1^{\infty} m!^{-1}\left(\frac{-E_1}{k_BT}\right)^m dm + 1$$
$$+\int_1^{\infty} m!^{-1}\left(\frac{-E_2}{k_BT}\right)^m dm + 1 + \int_1^{\infty} m!^{-1}\left(\frac{-E_3}{k_BT}\right)^m dm + \;\dots\;,$$

in this form,

$$\frac{-s\,k_c}{g_n\,n^p} - \frac{-s\,k_c}{g_n\,n^p} = Z,$$

where

$$\frac{-s\,k_c}{g_n\,n^p} - \frac{-s\,k_c}{g_n\,n^p} = 0\;\forall\, s > 0,$$

we nearly have our proof of Proposition 4.2. by means of the Buckingham π theorem, since our physically meaningful equation $-s\ln(p)\,/p = \lambda t$ involving a certain number, n, of physical variables, and these variables are expressible in terms of k independent fundamental physical quantities, then the original expression is equivalent to an equation involving a set of $p = n - k$ dimensionless parameters constructed from the original variables, as

$$\frac{-s\,k_c}{g_n\,n^p} - \frac{-s\,k_c}{g_n\,n^p} = 0\;\forall\, s > 0$$

results in dimensionless s, as do the absolute convergence of time in $\ln(p)\,/p$ on the left.

In order to illustrate this on a more fundamental level and fully prove the proposition, we return to the infinite series of integrals equal to Z/g_n, subtract both sides by the intermixed series of integrals and express the integral having the domain of n;

$$\frac{Z}{g_n} - \left(\int_1^\infty m!^{-1}\left(\frac{-E_1}{k_B T}\right)^m dm + \int_1^\infty m!^{-1}\left(\frac{-E_2}{k_B T}\right)^m dm + \cdots\right) = 1 + 1 + 1 + \cdots,$$

which is the same as

$$\frac{Z}{g_n} - \sum_n \int_1^\infty m!^{-1}\left(\frac{-E_n}{k_B T}\right)^m dm = n,$$

explained by

$$\frac{Z}{g_n} = 0 = 1 + \;-1 + 1 + \;-1 + 1 + \;-1 + \cdots,$$

as all

$$\int_1^\infty m!^{-1}\left(\frac{-E_1}{k_B T}\right)^m dm = -1.$$

Thus, Z amounts to nothing more than a sum of zeros;

$$Z = \sum_n g_n + g_n(-1) \; \forall \, s \; > 0,$$

in the same way as

$$Z = \frac{-s \, k_c}{n^p} + \frac{-s \, k_c}{n^p}(-1) \; \forall \, s \; > 0.$$

We then therefore show that the probability density function involves the Riemann zeta function's s, the eigenvalue of the opposite of the differentiation operator with $N(t)$ as the corresponding eigenfunction, and is that of the form of Boltzmann distribution, a described in Proposition 4.2. This system is expressed in terms of exponential decay, but has zero degeneration, as the sequence of partial sums of g_n converge absolutely, proving the proposition, resulting in a dimensionless mathematical framework in order to study the prime numbers themselves, and thus the non-trivial zeros of the Riemann zeta function in terms of periodic dimensionless waves. ■

5. Construction of Riemann Amplitude Proportionality

In Bernhard Riemann's 1859 paper, "On the Number of Prime Numbers Less Than a Given Quantity" [20], he proposed the hypothesis that the roots ρ in

$$J(x) = Li(x) - \sum_\rho x^\rho - \ln(2) + \int_x^\infty \frac{1}{m(m^2-1)\ln(m)} \, dm$$

all have a real part equal to $1/2$, where said roots are the non-trivial zeros of the Riemann zeta function (note: Riemann used symbol t instead of m, but we express it here as m, so as to reserve t always for time in $\ln(p)\,/p$). Said roots can be expressed as

$$\zeta\left(\frac{1}{2} + i\omega_\rho\right)$$

only if the Riemann Hypothesis is true (note: the symbol t is typically used here as well, but we symbolize it with ω for other reasons that will become clearer as we begin to represent our statistical oscillation).

We then express the real part of the roots of the Riemann zeta function as σ_ρ and the imaginary part always as $i\omega_\rho$, such that $s_\rho = \sigma_\rho + i\omega_\rho$, where all real parts of s_ρ have been proven to exist in a region known as the critical strip $(0 < \sigma_\rho > 1)$ [21]. Any expression hereon of ω_ρ simply means the absolute value of $i\omega_\rho$. Our interest then at this point is an expression of some function ω_ρ in terms of the root of exponential decay. Because $-E/k_B T$ was shown to cancel out, we will now begin to discuss decay using symbols in terms of a wave's amplitude A decaying over field distance x, as the following expressions become more familiar for most mathematicians in terms of waves (although these will be dimensionless waves, thus simply periodic functions). It was discussed at the beginning how A is interchangeable for N, but that was really so long as all dimensions are duly accounted for throughout. Now, however, with $-E/k_B T$

eliminated from our study in that $Z = 0$, we need not distinguish any separation or correlation between time and space at all, no other dimensions to account for.

Construction of the Proportionate Zeta Function. We prove the following.

Proposition 5*. Given* $\nu_s(p) = A(x)/A_0$*, all the prime numbers are to the ratio of* $A(x)/A_0$ *as the reciprocal of* $A(x)/A_0$ *is to all the values of the imaginary part of all the non-trivial zeros of the Riemann zeta function;*

$$p\colon \frac{A(x)}{A_0} :: \frac{A_0}{A(x)} \colon \omega_\rho,$$

such that

$$\frac{k_\rho \, \mathrm{A}_0 \ln(\omega_\rho)}{p\, A(x) \ln(A_0\, Ax^{-1})} \sim -A(x) = \frac{1}{s} \;\forall\, s > 0,$$

where k_ρ *is simply the value* $1.1580\ldots$*, and such that*

$$\omega_\rho(p) = e^{\frac{\ln(p)\, A(x)}{k_\rho(p)\, A_0}},$$

in that the roots of the Riemann zeta function become defined in terms of an infinite sequence over the primes and Riemann decay.

In order to prove the proposition, we first consider the form we used sections earlier;

$$\zeta(s) = \prod_p (1 - \nu_s(p)^p)^{-1} = \prod_p (1 - p^{-s})^{-1},$$

such that

$$\zeta(s) = \prod_p \left(1 - p^{\frac{p \ln(\nu_s(p))}{\ln(p)}}\right)^{-1} : \begin{matrix} \gamma = s \\ X = \dfrac{p}{\ln(p)} \\ \dfrac{A(x)}{A_0} = e^{-\gamma x} = \nu_s(p) \end{matrix}.$$

We find that for any given argument s, considering that $\nu_s(p) = \frac{A(x)}{A_0}$,

$$\prod_\omega \left(1 - \omega_\rho^{\frac{A_0}{A(x)}}\right)^{-1} - \prod_\omega \left(1 - \frac{A_0}{A(x)}^{\frac{A_0 \ln(\omega_\rho)}{A(x) \ln(A_0\, A(x)^{-1})}}\right)^{-1} = 0 \;\forall\, s > 0,$$

or more precisely throughout the product (as well as the entirety of the Riemann zeta function, not only those applicable to exponential decay), from any prime number p_n up to any prime number of a given magnitude p_N, we get

$$\prod_{p}^{p_N}\left(1-\omega_\rho^{\frac{1}{\nu_s(p)}}\right)^{-1}-\prod_{p}^{p_N}\left(1-\frac{1}{\nu_s(p)}^{\frac{\ln(\omega_\rho)}{\nu_s(p)^{-1}\ln(\nu_s(p)^{-1})}}\right)^{-1}=0\ \forall\ s\neq 1,$$

which results in the following:

$$\prod_{\omega_\rho}\left(1-\omega_\rho^{\frac{A_0}{A(x)}}\right)^{-1}\ =\ \zeta(s)\Leftrightarrow\zeta(s)=0, s>0.$$

In other words, we find that equivalence can only occur between these proportionalities when s is a non-trivial zero of the Riemann zeta function $\zeta(s)$, as exponential decay is described for decay constants (and propagation constants) $s>0$, considering the relation $1/\nu_s(p)=A_0/A(x)$. Our proof of the Riemann hypothesis will not center alone on the above, but we will use it to provide us with what we believe leads to a more rigorous proof than anything derivable from the above alone, that we additionally believe will provide deeper insight into other important areas of mathematics.

We next consider the simpler term from above to be considered the Riemann zeta function over all ω_ρ, but having arguments $1/\nu_s(p)=A_0/A(x)$ rather than arguments s as a product over all the prime numbers, which we write as

$$\zeta_\rho(A)=\prod_{\omega_\rho}\left(1-\omega_\rho^{\frac{A_0}{A(x)}}\right)^{-1}\ \forall\ s>0.$$

Or, in a more general sense, for the entire Riemann zeta function

$$\zeta_\rho(A)=\prod_{\omega_\rho,p}\left(1-\omega_\rho^{\frac{1}{\nu_s(p)}}\right)^{-1}\ \forall\ s\neq 1,$$

where again

$$\nu_s(p)=e^{\frac{-s\ln(p_n)}{p_n}},$$

such that

$$\zeta_\rho(A)=\prod_{\omega_\rho,p}\left(1-\omega_\rho^{\frac{1}{\nu_s(p)}}\right)^{-1}=0\ \forall\ s\neq 1,$$

where this proportionate Riemann zeta function converges absolutely, corresponding to any given s. In other words, for any given argument s in $\zeta(s)$, $\zeta_\rho(A)$ is a function that always extends proportionately (as the product taken over all ω_ρ), but always maps to one point: zero—from $\zeta(s)$, as the product $\zeta(s)$ is taken over all the primes.

In consideration of constant $k_\rho=1.1580\ldots$, we find that by taking the partial sum

$$\nu_s(p) = 1 + \int_1^\infty m!^{-1} \left(\frac{-s \ln(p_n)}{p_n} \right)^m dm$$

over all prime numbers p and over all the real valued $\omega_\rho(s)$ of the non-trivial zeros of the Riemann zeta function, we get

$$\lim \frac{p \, \nu_s(p) \ln(\nu_s(p)^{-1})}{k_\rho \ln(\omega_\rho(s))} = s \ \forall \ s \ \neq 1.$$

Thus, we can now consider a parameterized constant $k_\rho(s)$ for any given s, such that

$$\frac{p \, \nu_s(p) \ln(\nu_s(p)^{-1})}{s \ln(\omega_\rho(s))} = k_\rho(s),$$

where

$$\lim \frac{p \, \nu_s(p) \ln(\nu_s(p)^{-1})}{s \ln(\omega_\rho(s))} = k_\rho = 1.1580 \ldots.$$

Taking the sum of all the values over all the prime numbers and all the real valued roots of the Riemann zeta function, we get

$$\zeta(s) = \sum_{n \neq 1}^{\infty} n^{\frac{-p \, \nu_s(p) \ln(\nu_s(p)^{-1})}{k_\rho(s) \ln(\omega_\rho(s))}} \ \forall \ s \ \neq 1.$$

Because $\nu_s(p) = e^{-s \ln(p)/p}$, we get the natural logarithm of the inverse of an exponent in the numerator, which is expressed as

$$\zeta(s) = \sum_{n \neq 1}^{\infty} n^{\frac{-p \, \nu_s(p) \ln\left(\frac{1}{e^{-s \ln(p) p^{-1}}}\right)}{k_\rho(s) \ln(\omega_\rho(s))}},$$

which cancels out to the negative of the inverse of $-s \ln(p)/p$, as $\ln(1/e^x) = -x$. This cancels the p in the numerator, giving

$$\zeta(s) = \sum_{n \neq 1}^{\infty} n^{\frac{-s \ln(p) \nu_s(p)}{k_\rho(s) \ln(\omega_\rho(s))}},$$

where

$$\frac{\ln(p) \, \nu_s(p)}{k_\rho(s) \ln(\omega_\rho(s))} = 1 \ \forall \ p, \omega_\rho.$$

This too can also be expressed as

$$\frac{\ln(p)\, e^{-s\ln(p)p^{-1}}}{k_\rho(s)\ln\big(\omega_\rho(s)\big)} = 1.$$

Multiplying both sides by $\ln\big(\omega_\rho(s)\big)$, we get

$$\frac{\ln(p)\, e^{-s\ln(p)p^{-1}}}{k_\rho(s)} = \ln\big(\omega_\rho(s)\big).$$

Give the exponent of both sides and one gets

$$\omega_\rho(s) = e^{\frac{\ln(p)e^{-s\ln(p)p^{-1}}}{k_\rho(s)}}.$$

To apply arguments to our definition, we express the above as a taylor series;

$$\omega_\rho(s) = \sum_{n=0}^{\infty} \frac{(k_\rho(\mathrm{s})^{-1}\ln(p)\, e^{-s\ln(p)p^{-1}})^n}{n!}.$$

Select an arbitrarily given $s = 2$, the first prime number and the first non-trivial zero and apply it to the infinite series. We get

$$14.1347\ldots = \sum_{n=0}^{\infty} \frac{(7.6423\ldots\ \ln(2)\, e^{-2\ln(2)2^{-1}})^n}{n!}.$$

And then we apply the next prime number and non-trivial zero where $s = 2$,

$$21.0220\ldots = \sum_{n=0}^{\infty} \frac{(5.7664\ldots\ \ln(3)\, e^{-2\ln(3)3^{-1}})^n}{n!}$$

Taking all the sums to the infinite prime number and the infinite non-trivial zero, we get the limit of the parameterized constant (the value $5.7664\ldots$ above) equal to the inverse of $k_\rho = 1.1580\ldots$.

Because

$$\frac{A(x)}{A_0} = \nu_s(p) = e^{\frac{-s\ln(p)}{p}},$$

We have our proof of Proposition 5.0;

$$\omega_\rho(s) = e^{\frac{\ln(p)\,A(x)}{k_\rho(s)\,A_0}}.\ \blacksquare$$

Summary of this Proportionality. Difficulty in defining a precise solution for $k_\rho(s)$ independently from $\omega_\rho(s)$ arises from the fact that

$$\frac{\ln(p)\ A(x)}{A_0} = k_\rho(s)\ln(\omega_\rho(s))$$

involves an uncertainty relation of two products of conjugate variables, which we will present more in the next section. In statistics and probability theory, standard deviation (typically represented by the symbol sigma, σ), shows how much variation or dispersion exists from the average (mean, or expected value) [22]. Thus, we may come to understand how $\omega_\rho(s)$ is to shown to correspond to the average of some property of this Riemann system, thus giving insight into the Riemann operator whose eigenvalues are precisely the non-trivial zeros.

The standard deviation of a random variable, statistical population, data set, or probability distribution is the square root of its variance [22]. Thus, from

$$\omega_\rho(s) = e^{\frac{\ln(p)\ A(x)}{k_\rho(s)\ A_0}},$$

being that $\ln(p)\ /p$ is identically useful as a unit of field time to a unit of field analog distance, we can express

$$\frac{A(x)}{A_0} = e^{\frac{-s\ln(p)}{p}} = \nu_s(p),$$

times the natural logarithm of all primes over some parameterized constant $k_\omega(s)$ as an exponent equal to the non-trivial zeros. $\omega \to \infty$ would also equal the average amplitude of the same periodic function $n \to \infty$, provided the phase is constant and the periodic function is continuous, whereby the same values throughout each period would simply repeat to infinity, amounting to the same average.

Mathematically, this is an uncertainty relation between ω, n that arises because the expressions of a periodic function that arises in the two corresponding bases ω, n are Fourier transforms of one another (ω, n are conjugate variables) [23]. The same could be said for $\ln(p)\ /p$ and n, where we find $k_\rho(s)$ is not arbitrary in the least, as the period of the function having amplitude A is tied both to the denominator of the inverse of the real part of s, as well as the denominator of the inverse of the real part of $z_s(p)$, the elements of the Hermitian matrix from earlier;

$$z_s(p)^2 + z_s(p)(\delta_s(p) + 1) + \frac{\delta_s(p)}{-16} = 0.$$

This will be discussed thoroughly in the next section.

A similar tradeoff between the variances of Fourier conjugates arises wherever Fourier analysis is needed, for example in sound waves. The Fourier transform of a sharp spike at a single frequency gives the shape of the sound wave in the time domain, a completely delocalized sine wave [23]. Thus, we then consider, after having fully defined the Riemann system as the root of exponential decay and now defined the roots of the Riemann zeta function itself in

$$\omega = e^{\frac{\ln(p)\,A(x)}{k_{\omega(s)}\,A_0}},$$

expressing s in terms of a single statistical oscillation (similar in concept to Connes [11]) of a saw tooth function, which gives further analysis through a Fourier series. This benefit follows from a rearrangement of the above, out of consideration of $\nu_s(p) = A(x)/A_0$, such that

$$s = \frac{p}{\ln(p)} \cdot \ln\left(\frac{k_\omega(s)\ln(\omega)}{\ln(p)}\right),$$

Where $p/\ln(p)$ is defined as the inverse of field distance in $\ln(p)\,/p$ and $\ln(k_\omega(s)\ln(\omega)\,/\ln(p))$ results in dimensionless values due to $\nu_s(p) = A(x)/A_0$.

Therefore then, because $\zeta_\rho(A)$ extends proportionately from $\zeta(s)$, by magnitude of parameterized constant $k_\omega(s)$, always mapping to an expected value zero, knowing the roots alone up to a certain magnitude, along with the prime numbers to the same magnitude, we can begin to map any array of s (continuous or discontinuous). When one knows the angle between two vectors corresponding to a common point, one can then determine the distance between the two [24]. This third known (or expected) point is the zero $\zeta_\rho(A)$ that always maps to the same place; that point being analogous to field distance $x = 0$ or field time $t = 0$, as Euler had defined the Riemann zeta function in terms of primes, equivalent to the Riemann zeta function in terms of natural arguments n [4]. This results in a corresponding array of the Riemann zeta function also able to be mapped to infinity, which provides the means to find continuity and discontinuity anywhere throughout the Riemann zeta function if and only if either s is constant or its real part is constant, a propagation constant in terms of amplitude decay over distance and a decay constant in terms of decay over time, though we will handle its dimensionless form.

This result then presents $\zeta_\rho(A)$ as a mathematical tool for exploration into points of continuity based on comparison between two or more average values (one with a finite mapping and the other having an infinite mapping), possibly providing applications in 3D computer modeling, where a common axis is redefined for each new 3D object creation [24]. In this case, however, the singularity is always predefined to at least a given magnitude by the roots of the function, and fully manageable based on the Riemann probability density function. In the next section, we will begin to use this device to explore areas for continuity in the Riemann zeta function.

6. Construction of the Riemann Triangle Periodic Function

Background on Triangular Periodic Functions. According to the de Broglie hypothesis, every object in the universe is a wave, a situation which gives rise to phenomenon of uncertainty relations [23]. The position of the particle is described by a wave function $\phi(x,t)$. The time-independent wave function of a single-moded plane wave of wavenumber k_0 or momentum L_0 (p_0 is commonly used to symbolize photon momentum, but we express it here as the more general momentum L, so as to not confuse with the primes p) is

$$\phi(x) \propto e^{ik_0x} = e^{\frac{iL_0x}{\hbar}}.$$

We should interpret the above as a probability density function due to the Born rule [25], as the probability of finding the particle between a and b is

$$\mathrm{P}[a \leq X \leq b] = \int_a^b |\phi(x)|^2 dx.$$

$|\phi(x)|^2$ is a uniform distribution when the wave is single-moded, in that the particle position could be anywhere along the wave packet [23]. Consider a wave function that is a sum of many waves, however, we may write this as

$$\phi(x) \propto \sum_n A_n e^{\frac{iL_nx}{\hbar}},$$

where A_n represents the relative contribution of the mode L_n to the overall total, or the continuum limit over all possible modes;

$$\phi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \hat{\phi}(L) \cdot e^{\frac{iLx}{\hbar}} dL,$$

with amplitude of these modes expressed in $\hat{\phi}(L)$, the wave function in momentum space. $\hat{\phi}(L)$ is the Fourier transform of $\phi(x)$ in that x and L are conjugate variables. The more precise we sum the plane waves, the less precise the momentum becomes, acquiring a multiplicity of momenta [23].

That said, we can use standard deviation ς to quantify the momentum and location [23] and since $|\phi(x)|^2$ is a probability density function for position, we calculate its standard deviation. The limit of the solution exacted is called the "Kennard bound", an expression of the uncertainty principle. Any pair of non-commuting self-adjoint operators representing observables are subject to similar uncertainty limits in terms of quantum mechanics [23].

The normal distribution is a real line continuous probability distribution, having a bell-shaped probability density function, known as the Gaussian function or informally "the bell curve" [26]:

$$gaussian(\omega;\, \mu, \varsigma^2) = \frac{1}{\varsigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{\omega-\mu}{\varsigma}\right)^2},$$

where μ is the location of the amplitude peak and ς^2 is the variance (typically symbolized with σ, but we symbolize it with the sigma variant, in order to retain σ for symbolization of the attenuation constant, the real parts of s). The arguments ω are typically symbolized x, but we will apply real arguments ω of s (the real numerical values of the imaginary parts). This normal distribution is the most implemented statistical distribution [27].

A triangle wave (or similarly a probability triangular periodic function, which we will begin to construct), is a non-sinusoidal waveform named for its triangular shape, similar to a square wave, the triangle wave contains only odd harmonics, a component frequency of the signal that is an integer multiple of the fundamental frequency. This infinite Fourier series converges to the triangle wave [28]:

$$x_{triangle}(t) = \frac{8}{\pi^2}\sum_{k=0}^{\infty}(-1)^k \frac{\sin\big((2k+1)\omega t\big)}{(2k+1)^2}$$

$$= \frac{8}{\pi^2}\left(\sin(\omega t) - \frac{1}{9}\sin(3\omega t) + \frac{1}{25}\sin(5\omega t) - \cdots\right)$$

where ω in terms of a wave is the angular frequency, though the above can also be expressed for dimensionless periodic functions, having dimensionless arguments ω as well.

The triangle wave (or dimensionless triangle periodic function) can also be defined as the absolute value of the sawtooth wave. For instance, a triangle wave, with range from -1 to 1 and period $2a$ is [29]:

$$x_{triangle}(t) = \left|2\left(\frac{t}{a} - \left\lfloor\frac{t}{a} + \frac{1}{2}\right\rfloor\right)\right|,$$

where

$$\left\lfloor\frac{t}{a} + \frac{1}{2}\right\rfloor$$

is the floor function. It is this corresponding sawtooth wave that will allow the following proposition and its proof to carry conditions over to any number of waveforms, as well as any number of dimensionless periodic functions involving Fourier series, as the relationships between triangle waves and square waves (and most others waves) too can be drawn—although, we will only consider for the following a range from 0 to 1, thus the assignment of the probability.

Unlike the symmetric triangle wave, a sawtooth wave contains all the integer harmonics, and is one of the best waveforms to use for subtractive synthesis of musical sounds, as it can be constructed using additive synthesis [29]. The infinite Fourier series

$$x_{sawtooth}(t) = \frac{2}{\pi}\sum_{k=1}^{\infty}(-1)^k \frac{\sin(2\pi k f t)}{(2k+1)^2}.$$

Our interest in the remainder of this paper are all the asymmetric triangle waves (or dimensionless periodic asymmetric functions) between the symmetric triangle and the sawtooth, although we will certainly not perform Fourier transforms of the great magnitude between or present them in terms of their Fourier series, as the result of the following will shed insight into the Fourier series convergence by Laplace transforming at most a few general functions.

Construction of the Riemann Statistical Oscillation. We prove the following.

Proposition 6. *The period of triangle periodic function in its Laplace transform*

$$\hat{\phi}_\sigma(\omega) = \frac{1}{h} - \frac{1}{g} = \frac{\chi_\sigma(\omega)}{\sigma^2},$$

is equal to the greatest common divisor of the reciprocal of the real part σ of the argument s (the eigenvalue of the root of natural decay) and the reciprocal of the product of the real part σ of s and the real part ξ of the corresponding elements $z_s(p)$ of the Hermitian matrix derived from s;

$$P = \gcd\left(\frac{1}{\sigma}, \frac{1}{\sigma \lim \xi}\right): \begin{matrix} \sigma = Re(s) \\ \xi = Re\big(z_s(p)\big) \end{matrix}.$$

Recalling from elementary arithmetic, a numerator ν of a common fraction (symbolized here as ν instead of the more typical n, as we will be using the same $\nu_s(p)$ as in the previous sections) represents a number of equal parts and the denominator δ (the value $\delta_s(p)$, not to be confused with the Dirac delta, which we will express simply as the Dirac function throughout this paper) indicates how many of those parts make up a whole [30].

Now, we have described earlier an assembly of elements, the number of which decreases to zero (being the roots of the set), the exponential distance constant X is the expected value of the amount of distance before an amplitude-object is removed from the assembly, where the assembly is the analog wave itself. When the amplitude decays over analog distance, if the individual analog distance of an element of the assembly is the analog distance traveled between the starting location and the removal of that element from the assembly, the mean analog distance X is the arithmetic mean of the individual analog distances. Thus, we make an assumption here (one that has no bearing on the outcome of any proof, as none are based on the assumption—only provided for illustrative purposes) that said element of the assembly (consider it perhaps the quanta itself, a photon) of the wave contributes to the amplitude of the wave, and as these elements tend to leave the system over the course of the wave's travel, the amplitude diminishes.

For one grasping for meaning to our construction of the root of exponential decay (Riemann decay) involving a probability space in that no conditions amount to the removal of any element from the assembly, consider simply the definition of a probability space [9]; it is a mathematical construct that models a real-world process consisting of states that occur randomly, consisting of three parts: a sample space, Ω, which is the set of all possible outcomes, a set of events $\mathcal{F}$, where each event is a set containing zero or more outcomes and the assignment of probabilities to the events, $\hat{\phi}_\sigma(\omega)$, from events to probability levels. Our construction provides the means to supply any number of different sample spaces and events, and all of which reduce to our assignment of probabilities $\hat{\phi}_\sigma(\omega)$.

That said, fluorescence lifetime, which is a bit more exacting representation of our construction, refers to the average time the molecule stays in its excited state before emitting a photon [31][32]. Fluorescence follows first-order kinetics in our construction as well:

$$[S_1] = [S_1]_0 \, e^{-st}$$

where $[S_1]$ is the concentration of excited state molecules at time t, $[S_1]_0$ is the initial concentration and s is the decay rate or the inverse of the fluorescence lifetime, an instance of exponential decay. Thus, we believe this construction represents the mathematical process of all of these forms, in that these forms decay over the mathematics presented herein. In this sense, our set of events $\mathcal{F} = 0$, which does have a meaningful solution. The sample space Ω really can be applied in several ways, and does not change the outcome of the rest of the paper, so expressing it in terms of specific samples is unnecessary. In fact, in doing so could even make the mathematics to follow less clear than if we simply allowed the reader to apply his or her own model to what will be presented from here.

Due to our dimensionless root of exponential decay, and the fact that this system has zero degeneracy even though it can be expressed in terms of exponential decay, we can draw dimensionless parallels in order to treat the number of equal parts as $\nu_s(p)$ or A_n/A_0, where $\nu_s(p)$ is not only a ratio of partial sums to the initial amplitude, but also the numerator of the far earlier described $Quot(R_\sigma)$ that is always less than zero. The denominator $\delta_s(p)$ discussed earlier tells us how many $\nu_s(p)$ are needed to add up to the whole amplitude.

Being that $Quot(R_\sigma)$ is the set of equivalence classes of pairs $(\nu_s(p), \delta_s(p)), \nu_s(p),\ \delta_s(p) \in R_\sigma$ and $\delta_s(p) \neq 0$, such that $(\nu_s(p), \delta_s(p))$ is equivalent to (α, β) if and only if $\nu_s(p)\beta = \alpha\delta_s(p)$. This generalizes the property from the rational numbers that $\nu_s(p)/\delta_s(p) = \alpha/\beta$ if and only if $\nu_s(p)\beta = \alpha\delta_s(p)$ [33]. The sum of the equivalence classes of $(\nu_s(p), \delta_s(p))$ and (α, β) is the class of $(\nu_s(p)\beta + \alpha\delta_s(p), \delta_s(p)\beta)$ and their product is the class of $(\alpha\nu_s(p), \delta_s(p)\beta)$. The pairs $(\nu_s(p), \delta_s(p))$ from $Quot(R_\sigma)$ are written $\nu_s(p)/\delta_s(p)$ [33].

In the case of $\nu_s(p)$ having the limit one, being continuous at $s = 0$, as we want to construct the analog of an attenuation constant of s, its real part, we can begin by studying a constant $Quot(R_\sigma)$ by taking the denominator $\delta_s(p)$ times $\nu_s(p)$ until convergence occurs at the limit of $\delta_s(p)$ where we know from earlier that the denominator becomes the reciprocal of $Quot(R_\sigma)$ toward infinity. We chose earlier then to compare the continuity at $s = 0$ where $Quot(R_\sigma)$ is a guaranteed value, and then studied that result in a manner such that it could be expressed for any given value of s.

We now let $Quot(R_\sigma)$ be then the ratio to numerator α having the domain of the real part of s and denominator β too having the domain of the real part of s, referring to the real part of s from here on as σ (the dimensionless analog to an attenuation constant, which is why we use the same symbol). We find that

$$Quot(R_\sigma) = \frac{(2\sigma + 1)^{-1} - 4}{4} = \frac{\alpha(\sigma)}{\beta(\sigma)} = \frac{\lim(\nu_s(p))}{\delta_s(p)},$$

and

$$\beta(\sigma) = \frac{\sqrt{\sigma^2}}{Quot(R_\sigma) + 1},$$

and $\alpha(\sigma)$ is the constant product

$$\alpha(\sigma) = \beta(\sigma)\, Quot(R_\sigma).$$

We take $Quot(R_\sigma)$ and construct a triangular periodic function, whereby the same values throughout each period simply repeat over time to infinity, bounded by zero and one, a probability in that we are working from a probability distribution involving the eigenvalues s.

In the same way that we constructed $Quot(R_\sigma)$ from the limit of $\nu_s(p)$, we construct our triangular periodic function from $Quot(R_\sigma)$ to the power of n in

$$\begin{matrix} \infty \\ h(s) \\ n=1 \end{matrix} = \frac{(Quot(R_\sigma)^n + 1)(Quot(R_\sigma) + 1)}{\sigma + |\omega|},$$

(note: the derivation of $h(s)$ is not provided, but comes from a rearrangement of the Riemann zeta function in its original summation form) where ω is the imaginary part of any given s (not necessarily the non-trivial zeros) and $Quot(R_\sigma)$ is solved directly from s. We symbolize the inverse of the limit of $h(s)$ simply as

$$\begin{pmatrix} Lim\, h(s) \\ n \to \infty \end{pmatrix}^{-1} = \frac{1}{h},$$

as we will only work with the limit from here on, whose value has the property of being all whole positive even number values for all positive, whole real arguments s.

We then take

$$Quot(R_\sigma) \bmod h,$$

which is the cycle used to encode the ramification data for the extensions of a global field [34]. We then consider what we will refer to as the "neutronic equation", from a sequence of equations resulting in integers for all rational arguments, such that for any constant h, and any given magnitude $Quot(R_\sigma)$, its sequence converges (containing only whole numbers for rational arguments $Quot(R_\sigma), h$) absolutely for all $Quot(R_\sigma)$ and divisors h:

$$neutronic\ equation(Quot(R_\sigma), h) = \frac{Quot(R_\sigma) - Quot(R_\sigma) \bmod h}{h},$$

and the neutronic sequence is defined as

$$neutronic(Quot(R_\sigma), h) = \frac{Quot(R_\sigma)_n - Quot(R_\sigma)_n \bmod h}{h}.$$

For a numerical example of this function, let $neutronic(41,3)$ (completely arbitrarily and independent of anything proceeding). We get

$$neutronic(41,3) = \frac{41 - 41 \bmod 3}{3} = 13, \frac{13 - 13 \bmod 3}{3} = 4, \frac{4 - 4 \bmod 3}{3} = 1, \frac{1 - 1 \bmod 3}{3} = 0, 0, 0, \ldots.$$

In this form, add $Quot(R_\sigma)$ to $Quot(R_\sigma)\ mod\ h$ ($Quot(R_\sigma)$, as we find it is always negative for any given arguments; thus, a change of signs only a matter of simplified convenience) and then divide by h, writing it as

$$\frac{(Quot(R_\sigma) + Quot(R_\sigma)\ mod\ h)}{h},$$

such that

$$\frac{1}{g} = \frac{-Quot(R_\sigma) - Quot(R_\sigma)\ mod\ h}{h} + \sigma + |\omega| = \frac{1}{h} \Leftrightarrow \sigma, \omega \in \mathbb{N}.$$

In doing so, we have limited the equivalence of g, h to exist only for natural numbers and have contained all positive (positive, as arguments less than zero are not applicable for exponential decay) integer, rational, real and complex possibilities to between zero and one, such that the field of fractions $Quot(R_\sigma)$ of the integral domain is the smallest field in which it can be embedded, the probability in which it is embedded. The algebraic probability periodic function then becomes

$$\hat{\phi}_\sigma(\omega) = \lim\left(\frac{(Quot(R_\sigma)^n + 1)(Quot(R_\sigma) + 1)}{\sigma + |\omega|}\right)^{-1} - \frac{-Quot(R_\sigma) - Quot(R_\sigma)\ mod\ h}{h} + \sigma + |\omega|.$$

Or we simplify it as

$$\hat{\phi}_\sigma(\omega) = \frac{1}{h} - \frac{1}{g},$$

such that

$$\frac{1}{h} = \lim\left(\frac{(Quot(R_\sigma)^n + 1)(Quot(R_\sigma) + 1)}{\sigma + |\omega|}\right)^{-1}$$

and

$$\frac{1}{g} = \frac{-Quot(R_\sigma) - Quot(R_\sigma)\ mod\ h}{h} + \sigma + |\omega|.$$

The manner in which we use this periodic function is to first consider our constant σ, the real part of s (fixed, not parameterized), and then apply arguments to ω chronologically, the imaginary part of s, from zero to infinity. Where the cycle repeats to infinity producing an average μ.

For rational $\sigma > 1$, the waveform is the same as its fractional arguments, just that the initial amplitude at $\omega = 0$ begins elsewhere in the oscillation. Also, there are a limited number of rational points in this function that at first appear to have continuity when $0 < \sigma < 1$. But by applying rational increments of ω (for example, increments of ω $1/8$-at-a-time, such that $\sigma = 1/2$, we see that periodicity does exist between the natural arguments in the following graph. We can also apply increments of any real number, so long as the progression is considered infinite. By applying the values (most likely irrational values) of the non-trivial zeros ω_ρ, such that $\sigma = 1/2$ shown in *Figure 1*,

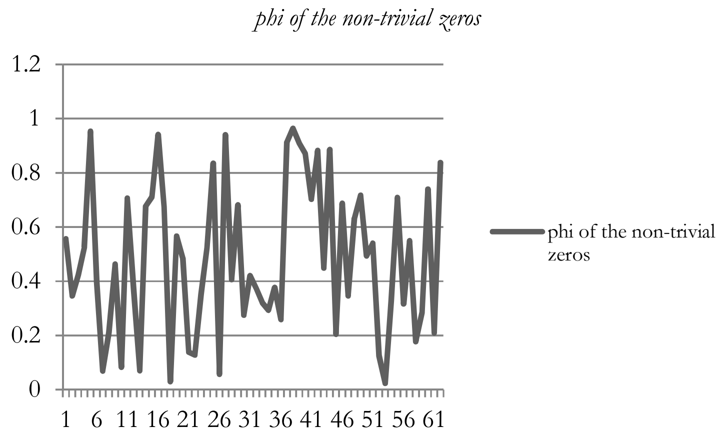

Figure 1, graph of $\hat{\phi}_{\sigma_\rho}(\omega_\rho)$ over its corresponding positive integer

we get that the oscillation average converges on $\mu = 1/2$ almost immediately, as shown in Figure 2.

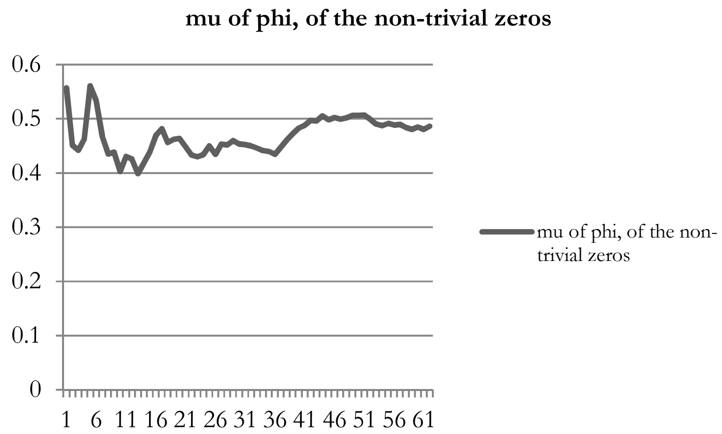

Figure 2, graph of μ of $\hat{\phi}_{\sigma_\rho}(\omega_\rho)$ over its corresponding positive integer

This, however, is not the only set of arguments one can apply to get an average equal to one half, but it will become an important value to consider later.

Our construction coincides in a sense with Broughan and Barnett's Riemann flow in terms of periodic orbits (trajectories), which come back to the initial point after a finite time interval [35]. They explain that mapping $\tau \to \gamma(s_0, \tau)$, the solution is called an orbit or trajectory, where s is our complex variable (arguments of the zeta function), and τ interpreted as time (in our model it is the inverse of $t = \ln(p)/p$, in theirs it is referred to as the maximal domain of existence). The ensemble of orbits they then called a holomorphic flow. A graphical representation of a subset of the orbits is called a phase portrait or phase diagram. They show that through each point s_0 there is a unique solution $\gamma(s_0, \tau)$, with $\gamma(s_0, 0) = s_0$. Our model provides the unique solutions, such that $s = \gamma, \tau = \mathrm{p}/\ln(p)$.

Consider also the case of a black body radiation experiment [36]. If a wave of photons enter the black body through a cavity (all at the same initial trajectory), when they meet the interior baffles (typically carefully placed angled iron oxide coated steel plates), they are either deflected (reflected) or absorbed into the black body material. The artificial black body is ideally constructed in such a way that the waves are angled to never escape from the same cavity they enter, therefore increasing the probability that eventually they will be absorbed. When this occurs, the black body radiates at a lower frequency at which they entered, and at a resonant frequency corresponding to the temperature of the black body. It is this operation that originally allowed Planck to formulate the concept of discrete wave packets (quanta) [37].

It can be thought similar to the Riemann operator of Connes construction of Riemann flow in terms of absorption or fluorescence, a spectral interpretation of the critical zeros of the Riemann zeta function as an absorption spectrum, while eventual noncritical zeros appear as resonances. Those two possible outcomes are opposite (so opposite however that analogies can still be made) to the black body experiment: if it is absorbed into the baffles, the black body radiates, if it is reflected, the body does nothing. We get the consequence of photons passing through a material in that in order to maintain a constant speed, the speed of light, the wavelength itself changes, we also get a change in relativistic mass and thus photon momentum [38]. We show a diagram.

Let $s = 1/3$ (arbitrarily) and apply natural arguments ω to $\hat{\phi}_\sigma(\omega)$. We get the following graph.

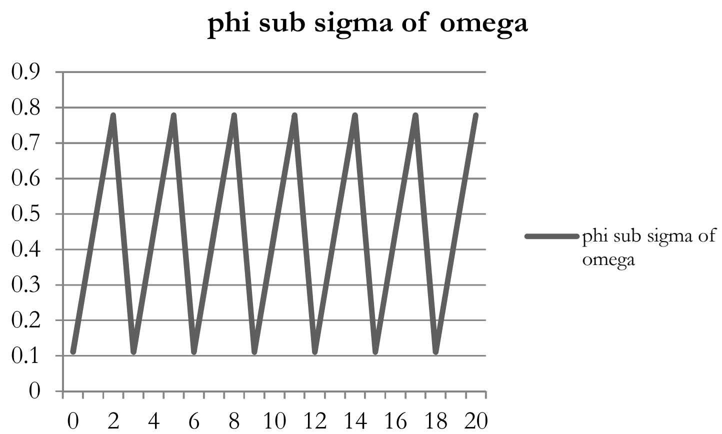

Figure 3, graph of $\hat{\phi}_{1/3}(\mathbb{N})$ over the natural numbers

Then consider the above alongside (alongside in distance for now) the same function in terms of distinct rays of light, save some amplitude offset of $2/3$ beginning at a precise different points in time so as to induce interference. This would give an offset with coinciding points of equality, thus interaction, shown in the following graph.

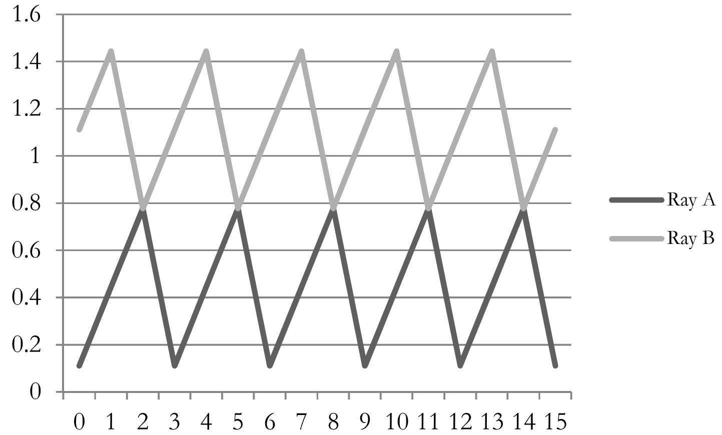

Figure 4, graph of $\hat{\phi}_{1/3}(\mathbb{N})$ and one multiplicative factor over the natural numbers

Consider then three (or more for instance).

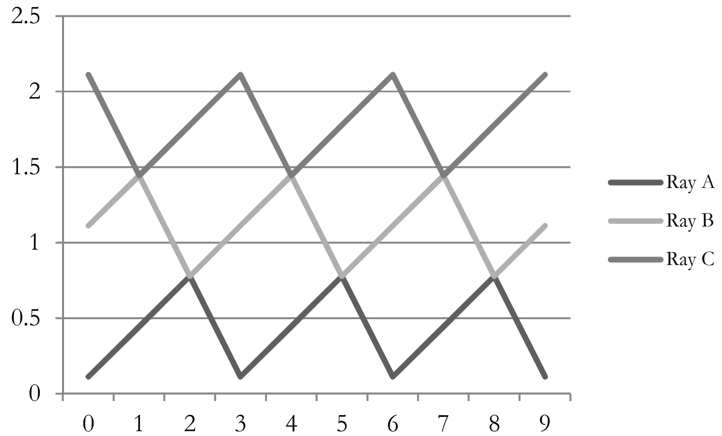

Figure 5, graph of $\hat{\phi}_{1/3}(\mathbb{N})$ and two multiplicative factors over the natural numbers

Now, we can express the above in terms of time, orbits or trajectories, including our rough sketch of the above trajectories (rotated thirty three degrees clockwise) of three rays entering a black body cavity below.

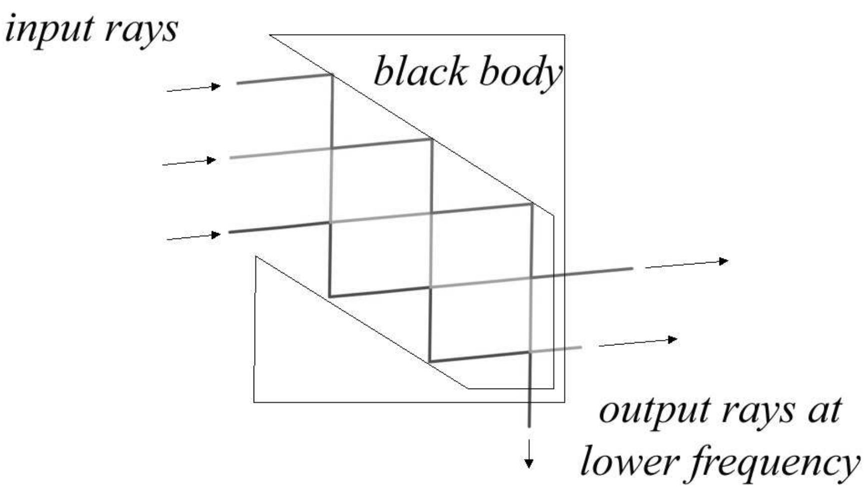

Figure 6, black body representation of $\hat{\phi}_{1/3}(\mathbb{N})$ in terms of Rayleigh scattering

Upon entering the black body through a cavity so that the rays are absorbed before their direction is reversed (to exit the cavity of which it entered), by the individual interactions between the rays (having relativistic mass, force between them exists) and the baffles of the black body the photons relativistic mass is reduced along with its frequency [38]. The rays eventually exit (are removed from the assembly) at a resonant frequency corresponding to the temperature of the black body [37]. If the above represented objects or particles having rest mass, they would eventually cease their travel, velocity reducing due to inertia [39]. However, in terms of discrete quantities having only relativistic mass, they have no *rest*; while their travel may be delayed (such as passing through a prism or blackbody); it is only over a factor of time, not a reduction in velocity [38]. The above provides a conjectured representation (independent of experiment) of Rayleigh scattering [40], though many (if not the majority) physicists differentiate between Rayleigh scattering and fluorescence for some considerable reasons [31].

We understand that the standard representation of black body radiation is that rays are absorbed, the atoms of the black body become excited and the body emits the radiation, not that the rays eventually exit the body (as this would be at odds with black holes, black bodies of which no light can escape due to gravity) [41]. But we do not put preference either way here on theory, as our above sketch is an abstraction and we only present the emission of light in terms of trajectories because it illustrates our function. Whether the light is emitted from atoms uniformly because light was absorbed into the atoms, or if the black body emits light uniformly because the individual trajectories exit in all directions, increasing to uniform emission over time due to interference, we are not concerned in terms of the study of this paper.

Construction of the Period and Phase of the Riemann Statistical Oscillation.

Upon inspection of $Quot(R_\sigma)$ in

$$Quot(R_\sigma) = \frac{(2\sigma + 1)^{-1} - 4}{4} = \frac{\lim \nu_s(p)}{\delta_s(p)}$$

given all arguments σ (inclusive even of $\sigma = 0, \sigma = 1$) to plus or minus infinity,

$$Quot(R_\sigma) = \frac{(2\sigma + 1)^{-1} - 4}{4} = undefined \Leftrightarrow \ \sigma = -\frac{1}{2}.$$

And because the number of finite elements in

$$lcm(\nu_s(e_1), \nu_s(e_2), \dots, \nu_s(e_N)) = lcm(\nu_s(e_1), \nu_s(e_2), \dots, \nu_s(e_N), -s) \ \forall \ s <> 0,$$

where the rational elements $\mathbb{Q}$ are such that the numbers of fractional elements for s are the same as whole elements of $-s$, we get

$$\mathbb{Q} \ \nu_e = \frac{1}{\mathbb{N} \ \nu_e} \ \forall \ s.$$

In other words, if the rational elements originate from the root of natural exponential decay

$$\nu_s(p) = \frac{A(x)}{A_0}$$

when $s > 0$, and they are the inverse of the natural elements when $s < 0$, then the denominator $\delta_s(p)$ of $Quot(R_\sigma)$ must too be equal to zero if and only if $\sigma = -1/2$ (causing $Quot(R_\sigma)$ to become undefined), as $\nu_s(p)$ always converges on one. We believe the fact of this can be deduced simply upon inspection of their earlier definitions;

$$Quot(R_\sigma) = \frac{(\ 2\sigma + 1)^{-1} - 4}{4},$$

$$\frac{\nu_s(p)}{\delta_s(p)} = Quot(R_\sigma) = \frac{A(x)}{A_0 \ \delta_s(p)},$$

and

$$\lim \delta_s(p) = \left(\frac{(2\sigma + 1)^{-1} - 4}{4}\right)^{-1}.$$

We then study the amplitudes of not only the amplitude and modulus of both of s, the eigenvalues of the root of natural exponential decay and $z_s(p)$, the elements of our Hermitian Matrix, but also periodic $\hat{\phi}_\sigma(\omega)$ that imposes the correspondence between the two. In order to solve for the initial phase and period in terms of the proof, which will provide the means to handle this with a Fourier transform for any argument s in $\hat{\phi}_\sigma(\omega)$, we first solve for $\chi_\sigma(\omega)$ for the first value at $\omega = 0$ using the following:

$$\hat{\phi}_\sigma(\omega) = \frac{1}{h} - \frac{1}{g},$$

Where upon inspection of $\chi_\sigma(\omega)$ from $\hat{\phi}_\sigma(\omega)$ for any argument s, taking the function throughout the domain s, and considering σ^2 constant for all $\hat{\phi}_\sigma(\omega)$, having the domain of σ (from which h, g are generated), we find

$$\lim_{\sigma \to 0} \quad \chi_\sigma(0) - \frac{\sigma^{-1} - 2}{\sigma} - 8 - \sigma^{-1} = 0,$$

in that

$$\chi_\sigma(0) \sim \frac{\sigma^{-1} - 2}{\sigma} - 8 - \sigma^{-1} \, \forall \, \sigma,$$

thus

$$\hat{\phi}_\sigma(\omega)^{-1} \left(\frac{\sigma^{-1} - 2}{\sigma} - 8 - \sigma^{-1} \right) \sim \sigma^2,$$

such that

$$\left(\frac{1}{h} - \frac{1}{g} \right) \left(\frac{\chi_\sigma(\omega)}{\sigma^2} \right)^{-1} = 1 \, \forall \, \sigma,$$

so that

$$\frac{1}{h} - \frac{1}{g} = \frac{\chi_\sigma(\omega)}{\sigma^2}.$$

We find that by applying all of the above to the real part of $z_s(p)$, the elements of the Hermitian matrix constructed from

$$\delta_s(p) = \frac{4(z_s(p) + 1)}{(-4z_s(p))^{-1}} - 4 = \frac{\nu_s(p)}{Quot(R_\sigma)},$$

we can determine period P of $\hat{\phi}_\sigma(\omega)$ from the real part of the elements of the Hermitian matrix because

$$-2\alpha(\sigma) \lim \xi - \frac{1}{\sigma} = 0 \, \forall \, s,$$

where again ξ is the real part of $z_s(p)$. Thus, we can use

$$\frac{1}{\sigma \lim \xi},$$

for the solution of the period of this triangular periodic function for any argument s, finding

$$P - \gcd\left(\frac{1}{\sigma}, \frac{1}{\sigma \lim \xi}\right) = 0 \;\forall\; s.$$

The frequency then becomes the inverse of the period, such that

$$f = \gcd\left(\frac{1}{\sigma}, \frac{1}{\sigma \lim \xi}\right)^{-1}. \blacksquare$$

Solving for the periodic function phase is straight forward by calculating $\hat{\phi}_\sigma(\omega)$ over the course of one period with natural arguments, determine A_{max}, A_{min} over that period, where we get a right triangle with points A, B and C, where A is the initial amplitude value, B is the point at which the amplitude begins its first decline and C is the right angle between A, B. We get

$$BC = A_{max} - A_{min},$$

AC is then the value of ω_{Amax}, the first value of the imaginary part of s where the maximum amplitude of $\hat{\phi}_\sigma(\omega)$ occurs (in the above graph where the real part of s equals $1/5, \omega_{Amax} = 2$). We then solve for

$$AB = \sqrt{(AC)^2 + (BC)^2},$$

then

$$\frac{BC}{AB} = sine\ theta,$$

which gives

$$\theta = arcsine\ \frac{BC}{BA}.$$

Knowing the amplitude, the value of $\hat{\phi}_\sigma(\omega)$, the period P (whose reciprocal is the oscillation of the function) and the phase for any given s, we can perform a Fourier transform to express this periodic function in terms of its oscillations to infinity [42]. To prove the pending propositions, however, one must yet universally define the maximum and minimum amplitudes (the ones and zeros).

Necessity of Fourier Convergence in the Riemann Statistical Oscillation. The conditions in determining convergence are the Dirichlet Conditions of which for a real-valued, periodic function $\phi_\sigma(\omega)$ to be equal to the sum of its Fourier series at each point where $\phi_\sigma(\omega)$ is continuous and the Fourier series at points of discontinuity is determined as the midpoint of the values of the discontinuity [42]. The Dirichlet Conditions are sufficient but not necessary, in that if the conditions are met, convergence is guaranteed. However, if they are not met, convergence still may occur [42]. While these conditions are generally satisfied for cases arising in science or engineering, prior to

$$\hat{\phi}_\sigma(\omega) = \lim\left(\frac{(Quot(R)^n + 1)(Quot(R) + 1)}{\sigma + |\omega|}\right)^{-1} - \frac{-(Quot(R) + Quot(R) \bmod h)}{h} + \sigma + |\omega|,$$

there exists no known necessary and sufficient conditions for the conditions of the Fourier series convergence [42]. It should be noted that $\phi_\sigma(\omega)$ is satisfied in all four conditions of convergence through a limit, the real part of s and greatest common denominator between the two,

$$P = \gcd\left(\frac{1}{\sigma}, \frac{1}{\sigma \lim \xi}\right) \ \forall\, s,$$

of which those Dirichlet conditions are

- $\phi_\sigma(\omega)$ must be absolutely integrable over a period.
- $\phi_\sigma(\omega)$ must have a finite number of extrema in any given interval
- $\phi_\sigma(\omega)$ must have a finite number of discontinuities in any given interval
- $\phi_\sigma(\omega)$ must be bounded.

We find the following proposition significant in terms of the problems with a classical interpretation of the dirac function, as expressed fairly recently by Mitrović and Žubrinić;

> "The greatest drawback of the classical Fourier transformation is a rather narrow class of functions (originals) for which it can be effectively computed. Namely, it is necessary that these functions decrease sufficiently rapidly to zero (in the neighborhood of infinity) in order to insure the existence of the Fourier integral." [43]

Proposition 7. *Given that $\hat{\phi}_\sigma(\omega)$ defines the period through a greatest common divisor, a necessary condition is imposed on the Fourier series convergence of the Riemann triangle periodic function. The Dirichlet Conditions may be replaced with a single necessary and sufficient condition: The Fourier series converges because the period is equal to ("or corresponds to", in the case of multiplicative factor cases of the arguments of the function), the greatest common divisor of the reciprocal of the real part of the argument and the reciprocal of the product of the real part of the argument and the real part ξ of its corresponding $z_s(p)$ (i.e. it converges due to reducibility near infinity);*

$$\lim -g\hat{\phi}_\sigma(\omega) + g = dirac(-h) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{im(-h)}\, dm = 0 \ \forall\, \sigma > 0,$$

$$dirac_\sigma(\omega) = -g\hat{\phi}_\sigma(\omega) + g \ \forall\, \sigma > 0.$$

We hypothesize that the above proposition is true in all converging Fourier series, but will only attempt to prove it the case in our specific function. How we claim to hypothesize rather than merely conjecture comes down to the study of the nature of sawtooth waves, their harmonics, which should be able to have the following carried over by some derivation to all convergent Fourier series cases. Such a general proof, however, could be more encompassing than would appropriately serve the benefit of this paper.

That said; the proposition can be proven in at least this individual case by defining a sequential definition of the Dirac function, and then taking it to the limits of the amplitudes in order to show how the convergence occurs. In doing so, we will show that necessity arises for Fourier series convergence for any argument s in $\hat{\phi}_\sigma(\omega)$, as the field of fractions $Quot(R_\sigma)$ whose numerator $\nu_s(p)$ contains the numbers factored out of the set would otherwise amount to numerical paradoxes involving the fundamental theorem of arithmetic. In consideration of but one fact that must mathematically hold, else paradoxes tend to arise, let the Fourier series for $s = -9$ in triangle function $\hat{\phi}_\sigma(\omega)$. We note that the first two values of $\nu_s(p) = \sqrt{512}, 27$, where the only whole element $\nu_s(e)$ factored out of the set by means of prime factorization is 27. A consequence arises because $\nu_s(e) = \nu_s(3)$, the second numerator of $Quot(R_\sigma)$ at $p = 3$, that when $s = -9$, the period of $\hat{\phi}_\sigma(\omega)$ must become complex so that the phase of the greatest common divisor amounts to less than $\pi = 3.14159 \ldots$. If this were not to occur, then the basis of the cosine function in

$$arg(z) = arccosine \frac{|x|}{\sqrt{x^2 + y^2}} = arcsine \frac{|y|}{\sqrt{x^2 + y^2}}$$

would cease to have meaningful solutions.

Performing the inverse Laplace transform of the function to express in terms of distance x (we cancelled out any distance x dimensions sections ago, so the inverse Laplace should amount to zero); whereas the Fourier transform expresses a function or signal as a series of modes of vibration (frequencies), the Laplace transform resolves our function into its moments (field time or field distance) [44]. By solving for the period in terms of $z_s(p)$, as shown earlier, we need not solve for the Fourier coefficients nor any constant, as will be shown. The inverse Laplace of $\hat{\phi}_\sigma(\omega)$ in terms of χ (or in terms of the numerator of any ratio), including

$$\hat{\phi}_\sigma(\chi) = \frac{\chi_\sigma(\omega)}{\sigma^2}$$

becomes simply

$$\phi_\sigma(\chi) = 0,$$

However, if we want a sequential solution for the Dirac function that is precisely accurate for anywhere to infinity, we take the inverse Laplace of

$$\hat{\phi}_\sigma(h) = \frac{1}{h} - \frac{1}{g},$$

in terms of h, which gives

$$\phi_\sigma(h) = \frac{-dirac(h-\varepsilon)}{g} + 1,$$

where ε is one of the Fourier coefficients (more often represented as α, but we already use α for the numerator of $Quot(R)$) [42]. We get

$$\phi_\sigma(h) \geq \phi_\sigma(\chi),$$

though

$$\hat{\phi}_\sigma(h) = \hat{\phi}_\sigma(\chi),$$

Thus, we only get uncertainty between either $\phi_\sigma(h), \hat{\phi}_\sigma(h)$ or $\phi_\sigma(\chi), \hat{\phi}_\sigma(\chi)$ and find that the uncertainty arises only between $\phi_\sigma(\chi), \hat{\phi}_\sigma(\chi)$, as even without knowing ε,

$$\lim -g\,\hat{\phi}_\sigma(h) - g = dirac(h-\varepsilon) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{im(h-\varepsilon)}\,dm = 0\;\forall\;\sigma > 0.$$

Our function $\hat{\phi}_\sigma(h)$ then is tied to a probability distribution pertaining to exponential decay, where the decay constant is greater than zero. The above too works similarly for $\sigma < 0$, except that the above must include the subtraction of one (the function $\hat{\phi}_\sigma(h)$ is not bounded by zero and one for negative arguments s, so the difference between the two becomes one as it is offset by one);

$$\lim g\,\hat{\phi}_\sigma(h) - g - 1 = dirac(h-\varepsilon) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{im(h-\varepsilon)}\,dm = 0\;\forall\;\sigma < 0.$$

In this sense, we can solve for points in the sequence corresponding to the Dirac function of minus h with

$$\lim_{\omega\to\infty} -g\,\hat{\phi}_\sigma(h) + g = dirac(h-\varepsilon) = 0,$$

where the points in the Dirac function can be expressed anywhere between infinity and minus infinity algebraically in terms of arguments s;

$$dirac_\sigma(h) = -g\,\hat{\phi}_\sigma(h) + g.$$

Next take the Laplace of

$$\frac{g\,\hat{\phi}_\sigma(h) - g}{g} + 1$$

and we confirm that everything cancels out in the above except for

$$\hat{\phi}_\sigma(h),$$

as one should expect with $\lim -g\ \hat{\phi}_\sigma(h) + g = dirac(h-\varepsilon) = 0$. We could do the similar with the inverse Fourier transform, but instead, we will now use $\hat{\phi}_\sigma(\omega)$ (and express the study as $dirac_\sigma(\omega)$ and the Fourier series in specific input values rather than approximations of an integral) to examine in a simple manner those regions near infinity (in terms of infinite sums).

First consider $\hat{\phi}_{1/2}(\omega)$, which, as it turns out, does not allow us to examine those regions but allows us to demonstrate why we are able to examine them everywhere else in the domain of $\hat{\phi}_{1/2}(\omega)$. Upon inspection of this argument of the periodic function, we find that the smallest increment applicable to $i\omega$ such that $\omega_a \to \omega_b$ is continuous, is when ω is taken from zero $1/8$ at a time. In such case, we get the first 0 of $\hat{\phi}_{1/2}(\omega)$ at $\omega = 4/8 = 1/2$ and another at $\omega = 12/8 = 1 + 1/2$. We do not get the first 1 until $\omega = 20/8 = 2 + 1/2$, but then, $\hat{\phi}_{1/2}(\omega)$ having a period equal to two, we get that the cycle repeats at $\omega = 24/8$, in that it equals the value at $\hat{\phi}_{1/2}(\omega)$. All other values between those are mixed numbers. Upon inspection of the phase of $\hat{\phi}_{1/2}(\omega)$ we see that it can be constructed from isosceles triangles, $1/2$ being the mid-point between zero and one.

All other real part arguments of $\hat{\phi}_{1/2}(\omega)$ cause the function to tilt away from either zero or one (the phase cannot be determined by isosceles triangles) in terms of rational arguments. In other words, it requires irrational arguments to obtain values of zero for the even reciprocals of σ when $0 < \sigma < 1$ and irrational arguments to obtain values of one for even σ when $0 < \sigma > 1$. The opposite is true in both circumstances for odd values. A detailed example is required to explain this properly.

Consider $\hat{\phi}_{1/8}(\omega)$, where the even reciprocal of σ is eight and $0 < \sigma < 1$. Taking ω also $1/8$-at-a-time, the first 1 occurs at $\omega = 1/8$. The period for $\hat{\phi}_{1/8}(\omega)$, calculable from the real part of s and the real part of z, is 8, but upon inspection of the values, we get no zeros anywhere throughout that period. In fact, taking it at smaller and smaller increments ($1/16, 1/64, 1/1000$, etc.) we see that no rational increments provide zeros. The zeros, however, may still be obtained from irrational values. For instance, the value of $\hat{\phi}_{1/8}(\omega)$, such that $\hat{\phi}_{1/8}(0.12499999999\ldots) \sim 1/8$, does in fact result in $\hat{\phi}_\sigma(\omega) = 0$. Also consider the odd real part of s equal to three, apply then as $\hat{\phi}_3(\omega)$. We get the first one at $\hat{\phi}_3(0)$ and a zero in that same vicinity at $\hat{\phi}_3(0.999999\ldots)$. While there is some debate (often less than professional) in mathematics circles today as to whether or not $0.999999\ldots = 1$ [45], our proof is fortunately aside such discussion, as applying it to the neutronic equation

$$\frac{0.99999\ldots - 0.99999\ldots\ mod\ x}{x},$$

outputs a fraction of $0.99999\ldots$ due to the floor function. The floor function is such that the fractional part sawtooth function, introduced by Gauss [46] by $\{x\}$ for real x, defined by the formula

$$\{x\} = x - \lfloor x \rfloor,$$

for all x

$$0 \leq \{x\} < 1,$$

we get results less than the same fraction of 1. To round to any decimal precision such that $0.99999 \ldots < 1$, for instance, and let $x = 2$ (arbitrarily chosen for a straight forward argument) in the above, we get the result of the Neutronic function as

$$\frac{0.99999 - 0.99999 \, mod \, 2}{2} = .49995 \neq \frac{1}{2}.$$

It is this neutronic equation above that permits inspection of both the Dirac function (and similarly the Heaviside function) by means of $\hat{\phi}_\sigma(\omega)$. It shows how it is not only necessary, but intuitive that these functions decrease "sufficiently rapidly" to zero (in the neighborhood of infinity; for instance, as $0.99999 \ldots \to 1$) in order to insure the existence of the Fourier integral. And while the Fourier transform of such simple functions as polynomials may not exist in the classical sense, we have removed the obstacles by reducing a probability distribution to a polynomial $z_s(p)$, in order to return to the classical interpretation by means of the Neutronic function, consequently eliminating the constants altogether.

We then prove the proposition with the real parts of both $s, z_s(p)$ becoming analog to the attenuation constant of a signal of a propagating wave, corresponding to a Hermitian matrix whose elements are $z_s(p)$, thus a probability distribution based on its intricate links to $v_s(p)$ and the fundamental theorem of arithmetic. The triangular periodic function

$$\hat{\phi}_\sigma(\omega) = \frac{1}{h} - \frac{1}{g} = \frac{\chi_\sigma(\omega)}{\eta(\sigma)},$$

expresses the necessary and sufficient condition for Fourier series convergence in for $\phi_\sigma(\omega)$, in that the Fourier series converges if an only if the period is equal to the greatest common divisor of the reciprocal of the real part of the argument s (the eigenvalue of the root of natural decay) and the reciprocal of the product of the real part of s and the real part of the corresponding elements $z_s(p)$ of the Hermitian matrix. It is this too that allows the above ratio to converge on the function itself. In this case, the necessity for Fourier series convergence is simply due to the reducibility by the greatest common denominator, the period itself anywhere up to infinity, proving Proposition 7. We have also shown how every element of said matrix is calculated from the real part of s alone, independently from ω_ρ, ω_n and independently from the Fourier series without constants (at least constants that cannot be expressed in closed form), from the arguments s and the prime numbers of the Riemann zeta function, which proves Proposition 7.0. ■

Probabilities and the Triangle Function. While we already know our values of $\hat{\phi}_\sigma(\omega)$ algebraically and already that its values are bounded by zero and one for positive arguments s, we want to briefly express them as assignments of probabilities to events in order to derive the sawtooth function from the variance, which in turn produces a symmetric triangle function .

Let $\hat{\phi}_\sigma(\omega)$ be a random variable with mean value μ:

$$E[\phi_\sigma(\omega)] = \mu.$$

Here the operator E denotes the average or expected value of $\hat{\phi}_\sigma(\omega)$. Then the standard deviation of $\hat{\phi}_\sigma(\omega)$ is the quantity

$$\varsigma = \sqrt{E[(\hat{\phi}_\sigma(\omega) - \mu)^2]} = \sqrt{E[\hat{\phi}_\sigma(\omega)^2] - (E[\hat{\phi}_\sigma(\omega)])^2}.$$

Considering μ from earlier,

$$\mu = \frac{1}{2} \forall\, s_\rho,$$

and for $\hat{\phi}_{1/2}(\omega)$ (such that ω are applied in increments of $1/8$, the smallest increment of reducibility, wherein it exposes its ones and zeros in a single period) we get that ς itself takes on a symmetric triangular periodic pattern to infinity, bounded by zero and $1/2$. In this case, $\varsigma_{1/2}(\omega) = |sawtooth|$, having an amplitude of $1/2$, containing all even and odd harmonics.

7. Cancellation Property in Terms of Roots.

Discriminants and Limits. We prove the following.

Proposition 8. *Given an equation in the form of*

$$(\lim g - h)^{-4} - (h)^2 = \sqrt{\Delta_h} \Leftrightarrow h = 0,$$

where Δ_h is the discriminant of h (the function of the polynomial's coefficients that gives information about the nature of its roots) there is only one rational argument h that provides any meaningful solution to the equation.

We seek a definition and solution for h (the same constants arising in Fourier transforms) in order to identify the relationship between the modulus of the complex roots of a polynomial and the continuity of a given function. Any multiplicative or additive rearrangement of g and/or h above will not alter the method used to determine h, as the most elementary relationship g of h to some coefficient a is always such that the $(g - h)th$ coefficient a_{g-h} is related to a signed sum of all possible subproducts of roots, taken h-at-a-time.

Consider Vieta's formulas [47]. Any general polynomial

$$P(x) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$$

(with the coefficients being real or complex numbers and $a_n \neq 0$) is known by the fundamental theorem of algebra to have n (not necessarily distinct) complex roots $z_1, z_1, \ldots, z_n$ [47]. Vieta's formulas relate the polynomial's coefficients $\{a_k\}$ to signed sums and products of its roots $\{zi\}$ as follows:

$$\begin{cases} z_1 + z_2 + \cdots + z_{n-1} + z_n = -\dfrac{a_n - 1}{a_n} \\ (z_1 z_2 + z_1 z_3 + \cdots + z_1 z_n) + (z_2 z_3 + z_2 z_3 + \cdots + z_2 z_n) + \cdots + z_{n-1} z_n = -\dfrac{a_n - 2}{a_n} \ldots \\ z_1 z_2 \ldots z_n = -1^n \dfrac{a_0}{a_n} \end{cases}$$

which gives the sum

$$\sum_{1 \le i1 < i2 < \cdots < ik \le n} z_{i1} z_{i2} \ldots z_{ik} = (-1)^k \frac{a_n - k}{a_n}$$

for $k = 1, 2, \ldots, n$ (where we wrote the indices i_k in increasing order to ensure each subproduct of roots is used exactly once). The left hand sides of Vieta's formulas are the elementary symmetric functions of the roots [47].

However, now with rational g in place of n, solvable exclusively by s, instead of only natural n we can refer back to $\hat{\phi}_\sigma(\omega)$, getting the indices above in increments of g (though it would cease to be a polynomial by definition, the equivalence stills holds), such that

$$\sum_{1 \le i1 < i2 < \cdots < ik \le g} z_s(p)_{i1} z_s(p)_{i2} \ldots z_s(p)_{ik} = (-1)^k \frac{a_g - k}{a_g}.$$

Now more specifically to the heart of the proof, consider $z_s(p)$ of the formation of the period of $\hat{\phi}_\sigma(\omega)$, it being the roots of

$$z_s(p)^2 + z_s(p)(\delta_s(p) + 1) + \frac{\delta_s(p)}{-16} = 0,$$

which is a polynomial, whose values $z_s(p)$ become the eigenvalues of a matrix whose trace is $-(\delta_s(p) + 1)$. We get

$$a_o = \frac{\delta_s(p)}{-16}, a_n = 1\colon\ z_s(p)_1\, z_s(p)_2 = -1^2 \frac{\delta_s(p)}{-16}$$

and find the resilient relationship

$$\frac{1}{\xi} \cdot \frac{a_0}{a_n} = \sigma + \mu, \forall\, s,$$

where μ is the mean variance of the periodic triangular function, such that

$$\mu = \frac{1}{2}\ \forall\, s.$$

Again, ξ is the real part of $z_s(p)$, and

$$\frac{a_0}{a_n} = |z|^2 = r^2, \forall\ s,$$

where $|z_s(p)|$ (or r) is the magnitude (or modulus) of the complex $z_s(p)$. When a complex number is expressed as a matrix the square of the modulus then is equal to the determinant of the matrix [14]; thus, a_0/a_n too becomes the determinant of any matrix expressed by $z_s(p)$;

$$r = |z_s(p)| = \sqrt{\xi^2 + \psi^2} = \frac{a_0}{a_n} : z_s(p) = \xi + i\psi.$$

And we confirm the above for all s by sufficiently defining the limits of $z_s(p)$, the magnitudes of $z_s(p)$, as well as the phases of $z_s(p)$ in closed form from zero to plus and minus infinity:

The limits of $\boldsymbol{z_s(p)}$

$$\underset{s\,=\,0}{z_s(p)} = \frac{1}{6} + \frac{1}{\sqrt{18}} \dots i,$$

$$\lim_{s\ \to\ \pm\infty} z_s(p) = \frac{1}{4} i,$$

The limits of $\boldsymbol{|z_s(p)|}$

$$\underset{s\,=\,0}{|z_s(p)|} = \frac{1}{\sqrt{12}},$$

$$\lim_{s\ \to\ \pm\infty} |z_s(p)| = \frac{1}{\sqrt{16}},$$

The limits of $\boldsymbol{arg(z_s(p))}$

$$\underset{s\,=\,0}{\arg(z_s(p))} = arccosine\left(\sqrt{\frac{1}{3}}\right),$$

$$\lim_{s\ \to\ \pm\infty} \arg(z_s(p)) = arccosine(0) = \frac{\pi}{2}.$$

It is in this same sense we can come to understand now how the neutronic sequence

$$\frac{Quot(R_\sigma)_n - Quot(R_\sigma)_n \ mod\ h}{h}$$

plays out in the earlier construction of the function, as it is a modular rearrangement of Vieta's formula that converges absolutely—though the exact modular expression of Vieta's formula would be

$$\frac{Quot(R_\sigma)_n - h\ mod\ Quot(R_\sigma)_n}{Quot(R_\sigma)_n},$$

which converges either to 1 or becomes undefined for any given argument $Quot(R_\sigma)$ or h, but is always finite. We can then construct an infinite series definition for a single neutronic equation in the sequence as

$$\sum_{1 \leq i1 < i2 < \cdots < ih \leq n} z_{i1} z_{i2} \dots z_{ih} = (-1)^n \frac{Quot(R_\sigma)_n - h\ mod\ Quot(R_\sigma)_n}{Quot(R_\sigma)_n},$$

where above we swap out k for h, which gives now further insight into the formation of

$$\hat{\phi}_\sigma(\omega) = \frac{1}{h} - \frac{1}{g}.$$

While

$$h\ mod\ Quot(R_\sigma)$$

may be constant, we know where it came from algebraically and can always express it in closed form. For instance, letting $s = 1/2 + 0i$, we get $g = 2/7, h = 1/4, Quot(R_\sigma) = -7/8$ and thus $h\ mod\ Quot(R_\sigma) = -5/8$, and even the value of the neutronic equation can be expressed in closed form

$$\left(\frac{1}{4} - \frac{1}{4}\ mod\ -\frac{7}{8}\right) - \frac{8}{7} = -\frac{1}{64}.$$

We consider this significantly convenient for a number of solutions. One of which is, by considering sums and products in the form of

$$(-1)^h \frac{a_n - h}{a_n},$$

is already equal to a sum of the products of the roots, so it too can be made the sum of other products as well.

It is these above expressions that become more useful, as we now have the sequences containing all the algebraic information one may ever require from a given complex argument s in terms of Δ. From this standpoint, one can begin to study

$$(\lim g - h)^{-4} - (h)^2 = \sqrt{\Delta_h}.$$

Any algebraic rearrangement to solve for h (not knowing ahead of time that h is constant) will prove difficult even if we assume simply $\sqrt{\Delta_h} = h$ in

$$(g-h)^{-4}-(h)^2=h,$$

as one finds there are only two possible arguments that make the above true: $g=\infty, h=0$, as the left hand side of the equation converges on the right as g approaches infinity. ∎

The Cancellation Property of the Zeta Function Non-Trivial Zeros.
From the proceeding proof, we have demonstrated a scenario that allows us to illustrate an important rearrangement of it, as will be shown—as we now begin to inspect why the non-trivial zeros reduce the Riemann zeta function to zero. We begin first into a cancellation property of the prime numbers, as Riemann's paper, and the origin of the hypothesis, focused on number of primes up to a given magnitude, and provided its solution in terms of the these Riemann zeta function roots through a Mobius inversion. Thus, these roots are tightly bound to the primes, as is commonly understood. We then consider the mean μ of the assigned random values of our periodic triangle function.

Proof of the Riemann Hypothesis. We prove the following.

Proposition 9. *Let the cancellation property of all prime numbers* p *occur in the subtraction of the mean value of assigned probabilities from the ratio of a multiplicative factor of the field of fractions from its numerator, also the eigenfunction of the root of natural decay containing the elements of prime factors, further subtracted from a continuous real part of the Riemann zeta function;*

$$\sigma-\frac{e^{-\frac{s\,ln(p)}{p}}}{2\,Quot(R_\sigma)}-\mu.$$

If the cancellation property of the product of a multiplicative factor of the field of fractions and the root determinants of a Hermitian matrix expressed by $z_s(p)$*, also required for determining the period of the oscillations* μ *is the mean value of, occurs correspondingly to*

$$-16\,Quot(R_\sigma)\ \ |z_s(p)|^2=1,$$

then continuity exists in σ_ρ *(of the roots of the Riemann zeta function) as* $z_s(p)$ *too correlates to a factor of the ratio of the primes to the roots. Given*

$$\lim\frac{\ln(p)}{\ln(\omega_\rho(s))}=1.22(1)\ldots,$$

where $\omega_\rho(s)$ *are the values of the imaginary parts of the roots of the Riemann zeta function,*

$$\lim\left(\left(\frac{1.22(1)\ldots}{-4\,|z_s(p)|^2}+4\right)^{-1}-1\right)=1\ \Leftrightarrow\sigma=\frac{1}{2},$$

such that

$$\zeta(s) = 0 \Leftrightarrow \sigma_\rho = \frac{1}{2} \ \forall\, 1 > \sigma > 0,$$

where all the real parts of the non-trivial zeros of the Riemann zeta function equal one half.

We derive the form

$$\left(\sqrt{32}(2\sigma - \mu - \delta_s(p))\right)^{-4} = \left(\frac{-\delta_s(p) - 1}{2}\right)^2 + \frac{\sqrt{(1 + \delta_s(p))^2 + \delta_s(p)4^{-1}}}{4}$$

where again $\delta_s(p)$ are the denominators of $Quot(R_\sigma)$. The form is the same as

$$(g - h)^{-4} - (h)^2 = \sqrt{\Delta_h}$$

save some multiplicative factors. We subtract h to the power of two from both sides, which gives.

$$(g - h)^{-4} = (h)^2 + \sqrt{\Delta_h},$$

which puts it in the form our equation is in. Next, because

$$\frac{\sqrt{x}}{4} = \left(\frac{\sqrt{x}}{2}\right)^2,$$

we write

$$\left(\sqrt{32}(2\sigma - \mu - \delta_s(p))\right)^{-4}$$

$$= \left(\frac{-\delta_s(p) - 1}{2}\right)^2 + \left(\frac{\sqrt{(1 + \delta_s(p))^2 + \delta_s(p)4^{-1}}}{2}\right)^2,$$

where the terms on the right are the squares of the values of the real and imaginary parts of $z_s(p)$ respectively. Give the square root of both sides and we get

$$\left(\sqrt{32}(2(\sigma - \mu) - \delta_s(p))\right)^{-2} = |z|,$$

which also has only one argument s that provides any meaningful solution, which is at $s = 1/2 + 0i$, in the same as

$$(g - h)^{-4} - (h)^2 = \sqrt{\Delta_h}.$$

It is this form that allows one to find continuity, along with our proportionate zeta function from earlier, as there is only one possible root magnitude of $z_s(p)$ possible to allow for

$$\frac{1}{\xi} \cdot \frac{a_0}{a_n} = \sigma + \mu, \forall\, s$$

and

$$\frac{a_0}{a_n} = |z_s(p)|^2 = r^2, \forall\ s,$$

for it to be universally true for all arguments s. We will demonstrate why. Rearrange our magnitude of $z_s(p)$ equation up above to

$$\sqrt{8\left(\sigma - \frac{e^{-\frac{s\ln(p)}{p}}}{2\ Quot(R_\sigma)} - \mu\right)^{-1}} = |z_s(p)|,$$

where we now express the equation in terms of s, the prime numbers and $Quot(R_\sigma)$, returning once again to

$$e^{-\frac{s\ln(p)}{p}} = \nu_p = \frac{N(t)}{N_0}.$$

Next, let $8 = \eta(s)$ be a point of continuity in $\eta(s)$ and then solve for $\eta(s)$. We get

$$\eta(s) = \ |z_s(p)|^{-2}\left(\sigma - \frac{e^{-\frac{s\,ln(p)}{p}}}{2\ Quot(R_\sigma)} - \mu\right).$$

In this way $\eta(s)$ has a solution for any arguments s except in the event it becomes undefined. We also can derive a simpler form of $\eta(s)$ from the above, such that

$$\eta(s) = \frac{\sigma + \xi}{\xi^2 + \psi^2},$$

though the proceeding equation gives more insight, as it has the cancellation property at argument $s = 1/2 + 0i$ so that the prime numbers cancel within the parenthesis, dependently on the magnitude of $z_s(p)$, as by taking

$$|z_s(p)|^{-2}\left(-\frac{e^{-\frac{s\,ln(p)}{p}}}{2\ Quot(R_\sigma)}\right) = 8$$

we get

$$e^{-\frac{s\,ln(p)}{p}} = -2\ Quot(R_\sigma)\ (8\ |z|^2) = \nu_s(p) = \frac{N(t)}{N_0},$$

and equally

$$\zeta(s) = \prod_p (1 + (2\ Quot(R_\sigma)\ (8\ |z_s(p)|^2))^p)^{-1}.$$

Thus, if $\nu_s(p)$ is constructed from both parts of s, but always converges to real 1, and the limit of $z_s(p)$ is constructed exclusively from the real parts of s and $Quot(R_\sigma)$, which is also constructed exclusively from the real parts of s, we only should get a single limit of the squared magnitude of $z_s(p)$ for all values of the roots of the Riemann zeta function, as

$$\frac{a_0}{a_n} = |z_s(p)|^2$$

is the determinant of any matrix expressed by $z_s(p)$. In other words, throughout the prime numbers the magnitudes of $z_s(p)$ for all the non-trivial zeros should all map to the same place, all having the same limits. And we have already expressed $z_s(p)$ as the elements of the matrix involving the zeta s in the second section of this paper. Likewise, we should not get the same magnitudes for other shared values throughout the Riemann zeta function. For instance the trivial zeros $(-2, -4, -6,$ etc.) would each have different magnitudes. Additionally, the inverse of the real part of $z_s(p)$ in

$$P = \gcd\left(\frac{1}{\sigma}, \frac{1}{\sigma \lim \xi}\right)$$

could only equal the inverse of the modulus and both would be equal to the minus $\alpha(\sigma)$ in

$$\frac{1}{|z_s(p)|^2} = -\alpha(\sigma) = -\beta(\sigma)\ Quot(R_\sigma) \Leftrightarrow s = \sigma + i\omega_\rho, \Leftrightarrow \sigma = \frac{1}{2}.$$

Similarly, the one half part occurs because $\mu = 1/2$ in

$$\sigma - \frac{e^{-\frac{s\,ln(p)}{p}}}{2\ Quot(R_\sigma)} - \mu,$$

which is not only the mean value of $\phi_{1/2}(\omega_\rho)$, but the mean for all $\phi_{1/2}(\omega)$, in that

$$-2\ Quot(R_\sigma)\ (8\ |z_s(p)|^2) - \nu_s(p) = 0\ \forall\ s <> 0.$$

Considering

$$\beta(\sigma) = \frac{\sqrt{\sigma^2}}{Quot(R_\sigma) + 1} \sim \frac{\sqrt{\eta(\sigma)}}{Quot(R_\sigma) + 1},$$

$$\frac{1}{|z_s(p)|^2} = -\beta(\sigma)\ Quot(R_\sigma)\ \Leftrightarrow \sigma = \frac{1}{2},$$

as

$$\beta(\sigma) = 16\,\forall\,\sigma = \frac{1}{2}.$$

In

$$\frac{\nu_s(p)}{|z_s(p)|^2} = -16\,Quot(R_\sigma)\ \ \forall\, s,$$

the primes always cancel and only cancel if $\sigma = 1/2$. Less generally, they cancel out of

$$8 = \frac{\sigma + \xi}{\xi^2 + \psi^2} \Leftrightarrow s = \frac{1}{2} + i0,$$

whereby the only oscillating value in $\hat{\phi}_\sigma(\omega)$ out of χ, $\alpha(\sigma)$, $\beta(\sigma)$ or σ^2 is χ, as the others are constant, each having the domain of the analog of the attenuation constant, which leaves any encoded information of change into this numerator of the root amplitude

$$\chi_0 \sim \frac{\sigma - 2}{\sigma} - 8 - \sigma^{-1}.$$

Applying then the real part of $s = 1/2 + i\omega$ to the above, we get

$$\chi_0 \sim \frac{\frac{1}{2}^{-1} - 2}{\frac{1}{2}} - 8 - \frac{1}{2}^{-1} : s = \frac{1}{2} + i\omega,$$

$$\chi_0 \sim -10 \colon s = \frac{1}{2} + i\omega,$$

as

$$-\frac{10}{\hat{\phi}_\sigma(\omega)} = -20\colon \frac{-10}{-20} = \frac{1}{2}.$$

This is no trivial result out of consideration of

$$\frac{1}{\xi} \cdot \frac{a_0}{a_n} = \sigma + \mu, \forall\, s,$$

where ξ is the real part $z_s(p)$, which is required to determine the period of $\hat{\phi}_\sigma(\omega)$ along with s, due to the correlation between the finite number of elements $\nu_s(e)$ in sequential eigenfunction $\nu_s(p)$ and the factors pertaining to the fundamental theorem of arithmetic, in that they are equivalent. It is also cannot be considered by any means trivial in that $z_s(p)$ is fundamentally tied to the Riemann zeta function and the primes in

$$\nu_s(p) = -16\, Quot(R_\sigma)\ \ |z_s(p)|^2\ \forall\ s,$$

$$\zeta(s) = \prod_p (1 + (-16\, Quot(R_\sigma)\ 8\ |z_s(p)|^2)^p)^{-1}\ \ \forall\ s,$$

as $z_s(p)$ is intricately linked to

$$Quot(R_\sigma)\ mod\ h,$$

the cycle used to encode the ramification data for the extensions of a global field, such that

$$\frac{\nu_s(p)}{|z_s(p)|^2} = -16\, Quot(R_\sigma)\ \ \forall\ s.$$

We proved this earlier with the infinite series

$$\sum_{1 \le i1 < i2 < \cdots < ik \le g} z_s(p)_{i1} z_s(p)_{i2} \ldots z_s(p)_{ik} = (-1)^k \frac{a_g - k}{a_g},$$

in that a_0/a_n too becomes the determinant of any matrix expressed by $z_s(p)$:

$$r = |z_s(p)| = \sqrt{\xi^2 + \psi^2} = \frac{a_0}{a_n} : z_s(p) = \xi + i\psi.$$

The consequence becomes that all the limits of $|z_s(p)|$ corresponding to all the roots of the Riemann zeta function must map to the same place:

$$\lim \left| z_{\frac{1}{2}+i\omega_\rho}(p) \right| = \frac{1}{\sqrt{14}}\ \forall\ s = \frac{1}{2} + i\omega_\rho.$$

And it must follow that this can only occur because

$$\frac{1}{|z_s(p)|^2} = -\alpha(\sigma) = -\beta(\sigma)\ Quot(R_\sigma) \Leftrightarrow s = \sigma + i\omega_\rho, \Leftrightarrow \sigma = \frac{1}{2}.$$

We can begin to complete this proof by returning to the previous section pertaining to the proportionate Riemann zeta function, where we found

$$\omega = e^{\frac{\ln(p)\, \nu_s(p)}{k_{\omega(s)}}}.$$

Replace $\nu_s(p)$ in the equation with the right hand side of

$$\nu_s(p) = -16\, Quot(R_\sigma)\ \ |z_s(p)|^2,$$

which gives

$$\omega_\rho(s) = e^{\frac{-16\,Quot(R_\sigma)\ |z_s(p)|^2\ \ln(p)}{k_\rho(s)}}.$$

Then rearrange and solve for $k_\rho(s)$. This gives

$$k_\rho(s) = \frac{-16\,Quot(R_\sigma)\ |z_s(p)|^2\ \ln(p)}{\ln(\omega_\rho(s))}.$$

Taking that through all the known non-trivial zeros (of which it is known that at least the first $2{,}001{,}052$ of them have a real part ½ by Odlyzko) [48], if we get convergence in

$$k_\rho(s) = \frac{-16\,Quot(R_\sigma)\ |z_s(p)|^2\ \ln(p)}{\ln(\omega_\rho(s))},$$

before we arrive to the $2{,}001{,}052$th non-trivial zero, we get our proof by definition of the limit of $k_\rho(s)$, as the limit would denote continuity in

$$\left|z_{\frac{1}{2}+i\omega_\rho}(p)\right|^2 = \frac{1}{\sqrt{14}},$$

and thus continuity in

$$\sigma = \frac{1}{2}\ \forall\ \rho.$$

In fact, by taking the above sequence to infinity, we do get exacting convergence. Upon inspection of $-16\,Quot(R_\sigma)\ |z_s(p)|^2$, we see that it reduces to 1 whenever $|z_s(p)| = 1/\sqrt{14}$, such that the limit of $k_\rho(1/2 + i\omega)$ for all the known non-trivial zeros becomes the same as

$$\lim \frac{\ln(p)}{\ln(\omega_\rho(s))} = 1.22(1)\ldots,$$

where we get

$$\omega_\rho(s) \sim e^{\frac{\ln(p)}{1.22(1)\ldots}}.$$

A sequence $\{fn\}$ of functions converges uniformly to a limiting function f if the speed of convergence of $fn(x)$ to $f(x)$ does not depend on x [49]. We see above that $\omega_\rho(s)$ converges to $e^{\ln(p)/1.22(1)\ldots}$ independently from $-16\,Quot(R_\sigma)\ |z_s(p)|^2$. Thus continuity and Riemann integrability, are transferred to the limits of either of our choices $k_\rho(s)$, the limits of the $|z_s(p)|^2$ or the real parts of the non-trivial zeros of the Riemann zeta function, as the convergence is uniform. And we find the above true in our sequence by means of the Uniform convergence theorem [49].

Using said constant, the prime numbers and the herein proposed limits of all the non-trivial zeros (one divided by the square root of fourteen) we fully define the consequence that if

$$Quot(R_\sigma) = \frac{(\,2\sigma + 1)^{-1} - 4}{4},$$

and

$$\frac{(\,2\sigma + 1)^{-1} - 4}{4} = \frac{k_\rho(s)\ln(\omega_\rho(s))}{-16\ \ln(p)\ \ |z_s(p)|^2},$$

and solving for the real part,

$$\sigma = \frac{1}{2}\left(\left(\frac{k_\rho(s)\ln(\omega_\rho(s))}{-4\ \ln(p)\ \ |z_s(p)|^2} + 4\right)^{-1} - 1\right).$$

We then apply constant $k_\rho = 1.22(1)\ ...$ to

$$\frac{1.22(1)\ ...\ \ln(\omega_\rho(s))}{-4\ \ln(p)}$$

to get

$$\lim \frac{1.22(1)\ ...\ \ln(\omega_\rho(s))}{-4\ \ln(p)} = -\frac{1}{4},$$

where $1.22(1)\ ...$ is the constant value that cancels both the primes and the roots of the Riemann zeta function uniformly independent of $|z_s(p)|^2$, thus

$$\left(\frac{k_\rho\left(\frac{1}{2} + i\omega_\rho\right)\ \ln(\omega_\rho(s))}{-4\ \ln(p)\ \ |z_s(p)|^2} + 4\right)^{-1} - 1 = 1\ \Leftrightarrow \sigma = \frac{1}{2},$$

which is the equivalent statement of

$$\lim\left(\left(\frac{1.22(1)\ ...}{-4\ |z_s(p)|^2} + 4\right)^{-1} - 1\right) = 1\ \Leftrightarrow \sigma = \frac{1}{2}.$$

Next, we apply the non-trivial zeros to

$$\frac{1}{2}\left(\left(\frac{1.22(1)\ ...}{-4\ |z_s(p)|^2} + 4\right)^{-1} - 1\right)$$

to get

$$\lim \frac{1}{2}\left(\left(\frac{1.22(1)\ldots}{-4\left|z_{\frac{1}{2}+i\omega_\rho}(p)\right|^2}+4\right)^{-1}-1\right)=\frac{1}{2},$$

in that the real part of the non-trivial zeros is continuous and equal to one half to infinity in the same manner that if any

$$\lim \left|z_{\frac{1}{2}+i\omega_\rho}(p)\right| = \frac{1}{\sqrt{14}},$$

corresponding to the non-trivial zeros of the Riemann zeta function, then all the non-trivial zeros do, as a consequence to the prime numbers cancelling out in

$$\sigma - \frac{e^{-\frac{s\, ln(p)}{p}}}{2\; Quot(R_\sigma)} - \mu$$

and both cancelling in

$$\lim \frac{1.22(1)\ldots \ln(\omega_\rho(s))}{-4\ \ln(p)} = -\frac{1}{4},$$

so that finally

$$\lim\left(\left(\frac{1.22(1)\ldots}{-4\;|z_s(p)|^2}+4\right)^{-1}-1\right)=1\ \Leftrightarrow \sigma=\frac{1}{2}.$$

We find no way around this. Only $\sigma = 1/2$ could cause the cancellation property, and at this most fundamental level, due to the ring of fractions at this value:

$$Quot(R_\sigma) = -\frac{14}{16} \Leftrightarrow \sigma = \frac{1}{2},$$

such that the cancellation property that causes the Riemann zeta function to go to zero when $s > 0$ is a consequence and requirement of its real part being equal to one half, as the reducibility of the primes (the Riemann zeta function converging to zero in the roots) occurs from

$$a_o = \frac{\delta_s(p)}{-16}, a_n = 1\text{:}\ z_s(p)_1\ z_s(p)_2 = -1^2\frac{\delta_s(p)}{-16}$$

at the roots. The primes are intricately bound to the non-trivial zeros through k_ρ, whose limit may be determined by $z_s(p)$, and cancel from

$$\sigma - \frac{e^{-\frac{s\, ln(p)}{p}}}{2\; Quot(R_\sigma)} - \mu,$$

becoming arbitrary values if and only if $\sigma = 1/2$. If this could occur somewhere other than $\sigma = \frac{1}{2}$, then

$$a_o \neq \frac{\delta_s(p)}{-16}, a_n \neq 1\colon\ z_s(p)_1\, z_s(p)_2 \neq -1^2 \frac{\delta_s(p)}{-16}.$$

However, the above is indeed true due to Vieta's formulas and the fundamental theorem of algebra; thus, all the non-trivial zeros of the Riemann zeta function have a real part equal to one half;

$$\zeta(s) = 0 \Leftrightarrow \sigma_p = \frac{1}{2}\ \forall\ 1 > \sigma > 0$$

because

$$\lim\left(\left(\frac{1.22(1)\ldots}{-4\,|z_s(p)|^2} + 4\right)^{-1} - 1\right) = 1\ \Leftrightarrow \sigma = \frac{1}{2},$$

as they both correspond to (and are derived by) the same eigenfunction, roots and matrices comprising exponential decay.

JEFFREY N. COOK
MAUMEE, OH, USA
Email: jnoelcook@yahoo.com

References

[1] Casella, G.; Berger, R. L., *Statistical Inference* (2 Ed.). Pacific Grove: Duxbury, 2002.
[2] Ayto, J., *20th Century Words*. Cambridge University Press, 1989.
[3] http://www.astronomy.swin.edu.au/cosmos/P/Planck+Time
[4] Derbyshire, J., *Prime Obsession*. Plume, 2003.
[5] http://www.microwaves101.com/encyclopedia/propagation.cfm
[6] M.D.Janezic; Jargon, J.A. "Complex Permittivity Determination from Propagation Constant Measures". *Microwave and Guided Wave Letters*. IEEE 9: 76-78.
[7] Thomas, T.G.; Sekhar, S. C., *Communication Theory*. Tata-McGraw Hill, 2005.
[8] Krane, K.S., *Introductory Nuclear Physics*. John Wiley & Sons Inc, 1988
[9] Sazonov, V.V., "Probability space", Hazewinkel, Michiel, *Encyclopedia of Mathematics*. Springer, 2001.
[10] Montgomery, H.L.; Vaughan, R. C., *Multiplicative number theory: I. Classical theory*; *Cambridge Tracts in Advanced Mathematics*, 97. Cambridge University Press, 2007
[11] Connes, A., "Formule de trace en G´eom´etrie non commutative et hypoth`ese de Riemann", *C.R. Acad. Sci. Paris Ser. A-B*, 1996. English Translation, 1998.
[12] Landau, L. D.; Lifshitz, E. M., *Statistical Physics*. 5 (3 Ed.). Oxford: Pergamon Press, 1976. Translated by J.B. Sykes and M.J. Kearsley, 1980.
[13] Ireland, K.; Rosen, M. *A Classical Introduction to Modern Number Theory* (Second Ed.). Springer, 1998.
[14] Brown, W. A., *Matrices and Vector Spaces*. M. Dekker, NY, 1991.
[15] *Euclid's Elements: All Thirteen Books Complete in One Volume*. Green Lion Press, 2002.
[16] http://webhome.cs.uvic.ca/~dmaslov/nthprime.html
[17] Arnold, V. I.; Cooke, R., *Ordinary Differential Equations*. DE, Berlin; Springer-Verlag, NY, 1992.
[18] Abramowitz, M.; Stegun, I. A., eds., *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. Dover, NY, 1965.
[19] Stewart, W. J., *Probability, Markov Chains, Queues, and Simulation: The Mathematical Basis of Performance Modeling*. Princeton University Press, 2011.

[20] Riemann, B., "Über die Anzahl der Primzahlen unter einer gegebenen Grösse". *Monatsberichte der Berliner Akademie*, 1859. *Gesammelte Werke*, Teubner, Leipzig, 1892. Reprinted by Dover, NY, 1953.

[21] Van de Lune, J.; Te Riele, H. J. J.; Winter, D. T., *On the Zeros of the Riemann Zeta Function in the Critical Strip* (4 Ed.). Math. Comp. 46, 1986.

[22] Ghahramani, S., *Fundamentals of Probability* (2 Ed.). Prentice Hall, New Jersey, 2000.

[23] *Quantum Mechanics*. Wiley-Interscience: Wiley, 1996.

[24] Cantrell, C. D., *Modern Mathematical Methods for Physicists and Engineers*. Cambridge University Press, 2000.

[25] Neumann, J.V., *Mathematische Grundlagen der Quantenmechanik*. Springer, Berlin, 1932 (German); English translation *Mathematical Foundations of Quantum Mechanics*, by Robert T. Beyer, Princeton University Press, Pinceton, 1955.

[26] Cover, T.M.; Thomas, J.A., *Elements of Information Theory*. John Wiley and Sons, 2006.

[27] Hazewinkel, M. ed., "Normal distribution", *Encyclopedia of Mathematics*. Springer, 2001.

[28] Weisstein, E. W., "Fourier Series--Triangle Wave", from *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/FourierSeriesTriangleWave.html

[29] Weisstein, E. W., "Sawtooth Wave", from *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/SawtoothWave.html

[30] Galen, L.B., "Putting Fractions in Their Place", *American Mathematical Monthly* 111 (3).

[31] Holler, F.J., *Principles Of Instrumental Analysis*. Douglas A. Skoog & Stanley R. Crouch, 2006.

[32] http://www/micro.magnet.fsu.edu/primer/techniques/fluorescence/fluorescenceintro.html

[33] Hartley, B.; Hawkes, T.O., *Rings, Modules, and Linear Algebra*. 1970.

[34] Cohn, H., *Introduction to the Construction of Class Fields: Cambridge Studies in Advanced Mathematics,* 6. Cambridge University Press, 1985.

[35] Broughan, K.; Barnett, A.R., "The Holomorphic Flow of the Riemann Zeta Function", *Mathematics of Computation*, Vol. 73, No, 246, Nov., 2003.

[36] Landsberg, P.T., *Thermodynamics and statistical mechanics*. Reprint of Oxford University Press 1978 Ed., Courier Dover Publications.

[37] Planck, M., "Über eine Verbesserung der Wienschen Spektralgleichung", *Verhandlungen der Deutschen Physikalischen Gesellschaft* 2: 202–204, 1900. Translated in ter Haar, D., 1967. "On an Improvement of Wien's Equation for the Spectrum". *The Old Quantum Theory*. Pergamon Press.

[38] McGinn, W.D., *Introduction to Relativity*. JHU Press, 2004.

[39] González-Martín, G. R., "A Geometric Definition of Mass", *Gen. Rel. and Grav.* 26: 1177, 1994.

[40] Strutt, J., "On the Transmission of Light through an Atmosphere Containing Small Particles in Suspension, and on the Origin of the Blue Sky", *Philosophical Magazine*, series 5, vol. 47. 1899.

[41] Frolov, V.P.; Zelnikov, A., *Introduction to Black Hole Physics*. Oxford University Press, 2011.

[42] Spiegel, M., *Schaum's Outline of Fourier Analysis with Applications to Boundary Value Problems*. McGraw-Hill, 1974

[43] Mitrović, D.; Žubrinic, D., *Fundamentals of Applied Functional Analysis: Distributions, Sobolev Spaces*. CRC Press, 1998.

[44] Spiegel, M., *Schaum's Outline of Laplace Transforms*. McGraw-Hill, 1965.

[45] http://polymathematics.typepad.com/polymath/2006/06/refutations.html

[46] Hardy, G. H.; Wright, E. M., *An Introduction to the Theory of Numbers* (5 Ed.), Oxford: Oxford University Press, 1980.

[47] Funkhouser, H. G., "A Short Account of the History of Symmetric Functions of Roots of Equations", *American Mathematical Monthly* (Mathematical Association of America) 37 (7). 1930.

[48] Odlyzko, A. M., *The first 2,001,052 zeros of the Riemann zeta function.* http://www.dtc.umn.edu/~odlyzko/zeta_tables/

[49] Hardy, G. H., "Sir George Stokes and the Concept of Uniform Convergence", *Proceedings of the Cambridge Philosophical Society,* 19. 1918.