ON THE INTERPRETATION OF THE LORENTZ TIME TRANSFORMATION

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Abstract

Einstein gave two conflicting interpretations of the Lorentz transformation for time, $\tau = t\sqrt{(1-v^2/c^2)}$, applied to a moving clock. The first was as a coordinate transformation, which was the basis of its derivation. The second was as a physical slowing effect on the moving clock caused solely by its motion relative to a stationary reference clock. These interpretations are not independent. That is, the Lorentz coordinate transformation cannot be applied during the clock's time of motion without correcting for the lack of synchronization between the moving and stationary clocks resulting from the slowing of the moving clock. Otherwise, the Lorentz transformation gives an incorrect result. In addition, the interpretation as a physical effect has seemingly insurmountable logical difficulties, as it subjects the moving clock to a physical slowing dependent upon an arbitrary inertial reference frame, and which is therefore indeterminable. This interpretation is supported by questionable experimental evidence.

Keywords: Lorentz transformations; Time transformations; Relativity; Special Relativity; Coordinate transformations; Time dilation; Twin paradox.

After deriving the Lorentz transformations as coordinate transformations, Einstein [1], in section 4, “Physical Meaning of the Equations Obtained in Respect to Moving Rigid Bodies and Moving Clocks”, makes the rather astonishing claim that the Lorentz time coordinate transformation,

$$\tau = t\sqrt{(1-v^2/c^2)} = t - (1- \sqrt{(1-v^2/c^2)})t\ , \qquad (1)$$

represents a physical slowing effect on a moving clock due merely to its motion relative to a stationary reference clock.

This interpretation not only has no apparent basis in his derivation, but also contradicts it. Now the clocks are no longer physically synchronized at $t > 0$, so the Lorentz transformation (1) is no longer valid during the time of motion. By Einstein’s argument, the time interval of the moving clock measured in its own coordinate system at $t > 0$ is no longer τ, but is decreased by the amount

$$(1- \sqrt{(1-v^2/c^2)})t.$$

That is, the clocks are synchronized at $t = \tau = 0$, but are out of synchronization at $t > 0$ by this amount, so that the time coordinate measured by the moving clock at $t > 0$ in its own coordinate system is not τ, but rather

$$\tau' = \tau - (1- \sqrt{(1-v^2/c^2)})t. \qquad (2)$$

The correct transformation at $t > 0$ then becomes

$$\tau' = (t- (1- \sqrt{(1-v^2/c^2})t) - (1- \sqrt{(1-v^2/c^2}))t, \text{ or}$$

$$\tau' = t - 2(1- \sqrt{(1-v^2/c^2}))t, \qquad (3)$$

which gives a slowing during the time of motion twice that of the Lorentz transformation (1).

But even then, the interpretation as a physical effect seems to me impossible, because the slowing depends upon an arbitrary choice of inertial reference frame of the stationary clock, and which therefore cannot be uniquely determined. That is, an arbitrary number of inertial reference frames different from the moving clock would subject the clock to an arbitrary number of different slowing effects at the same time, or if we limit the effect to one stationary reference frame, then by simply choosing either of two different stationary reference frames we would produce two different results.

The experimental evidence claiming to support this second interpretation can be divided into two types. The first measures the clock during the time of motion. In this case, if the observed time difference appears to be consistent with the Lorentz transformation, then the experiment would, according to the above argument, support the first interpretation, but not the second. Therefore this type of experiment need not be examined.

The second type of experiment is one where the moving clock is brought to rest and compared with the stationary clock, the elapsed time interval of motion compared, so that only a physical change over the time of travel would be observed. The results claimed by [2] are based on the behavior of synchronized cesium clocks on the Earth's surface and in aircraft flying in opposite directions with respect to the Earth's rotation, and then returning to the point of departure.

The elapsed times of the different clocks over the time of travel were compared with the predicted effect from the Lorentz transformation using the Earth's axis as the stationary reference frame. Clocks flown in planes going in one direction showed a slower rate of time measure during the motion than the earth-bound clock, and those in the planes flying the opposite direction showed an increased rate. The experiment was criticized by [3,4] and was claimed to have been confirmed by [5].

I find the results interesting, as none of the issues discussed here are mentioned. In particular no reason is given why the clocks experienced the effects from the particular choice of stationary reference frame of the Earth's axis, but not from some other, in which case the clocks would have shown different results. An obvious choice would be the reference frame of the ground-based clock, that of the first example given by Einstein. In that case, both traveling clocks would have shown a slowing with respect to the ground-based clock.

[1] A. Einstein, *Annalen der Physik*, 17:891, 1905. 1923 English translation
[2] Hafele and Keating, *Science* Vol. 177 pg 166-170 (1972)
[3] A. G. Kelly, "Reliability of Relativistic Effect Tests on Airborne Clocks", Inst. Engineers Ireland Monograph No. 3 (February 1996)
[4] Schlegel, *AJP* **42**, pg 183 (1974)
[5] National Physical Laboratory, summarized in News from the National Physical Laboratory, Issue 18, Winter 2005