Goldbach conjecture is false

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1. Introduction and statement of results

Goldbach conjecture is one of the oldest and best-known unsolved problems in number theory. It states: Every even integer greater than 2 can be expressed as the sum of two primes.

Goldbach original conjecture (sometimes called the "ternary" Goldbach conjecture), written in a June 7, 1742 letter to Euler, states "at least it seems that every number that is greater than 2 is the sum of three primes". Note that here Goldbach considered the number 1 to be a prime, a convention that is no longer followed. As re-expressed by Euler, an equivalent form of this conjecture (called the "strong" or "binary" Goldbach conjecture) asserts that all positive even integers can be expressed as the sum of two primes.

The conjecture has been shown to hold up through 4 × 1018 and is generally assumed to be true, but remains unproven despite considerable effort.

Fortunately, this paper has proved Goldbach conjecture is false with set theory and higher mathematics knowledge.

2. Preliminary theorem

To prove Goldbach conjecture, need to prove 2 preliminary theorems. First one states: if an integer can be expressed as the sum of two integers; when the integer trends to infinity, at least one of the other two integers trends to infinity.

Preliminary theorem:

(2.1) W = U + V; U, V, W $\in$ Z;

$$\lim_{n\to\infty} W = \infty \Rightarrow \lim_{n\to\infty} U = \infty \text{ Or } \lim_{n\to\infty} V = \infty .$$

(2.1.1) Suppose when $\lim_{n\to\infty} W = \infty \Rightarrow \lim_{n\to\infty} U \le U\max$ and $\lim_{n\to\infty} V \le V\max$,

$U\max, V\max \in Z$;

Because W = U + V $\Rightarrow$ $\lim_{n\to\infty} W = \lim_{n\to\infty} U + \lim_{n\to\infty} V$

$$\Rightarrow \quad \infty = \lim_{W\to\infty} W \le U\max + V\max \Rightarrow \infty \le U\max + V\max .$$

It's self-contradictory. So the suppose (2.1.1) is false and preliminary theorem (2.1) is true.

Preliminary theorem:

(2.2) When an odd number trends to infinity, this odd number must be an odd composite number.

First to define a prime set P = {p|p=1 × p; {p/(p-k)} $\neq$ 0; p>1,k≥1,k<p; p, k∈N}. x = [x]+{x} is Gaussian function. [x] expresses the maximum integer but not above x . Set [X] = {[x]| [x]≤x, [x]>x-1; x∈R, [x]∈Z}; {x} expresses the non-negative decimal fraction. Set {X} = {{x}| {x}≥0, {x} < 1, {x} = x – [x]; x∈R, [x]∈Z }

(2.2.1) Suppose when an odd number trends to infinity, there is at least one odd number is prime.

i.e. Exist p1 is an odd number and $\lim_{p1\to\infty} p1 \in$ P

Because p1 is an odd number $\Rightarrow$ $\lim_{p1\to\infty}(p1/(p1-[p1/2]))$ = $\lim_{p1\to\infty}(p1/(p1-(p1-1)/2))$ = $\lim_{p1\to\infty}(p1/(p1/2+1/2))$ = $\lim_{p1\to\infty}(2p1/(p1+1))$ = $2\lim_{p1\to\infty}(1-1/(p1+1))$=2 $\Rightarrow$ {2} = { $\lim_{p1\to\infty}(p1/(p1-[p1/2]))$ } = 0 (2.2.1.1)

Because p1∈P and p1 trends to infinity $\Rightarrow${p1/(p1-[p1/2])} $\neq$ 0 and p1 trends to infinity $\Rightarrow${ $\lim_{p1\to\infty}(p1/(p1-[p1/2]))$ } $\neq$ 0.

It's self-contradictory with (2.2.1.1). So p1 is not a prime, because we have found a divisor [p1/2] beside p1 and 1. According to prime definition (p = 1×p), p1 does not belong to prime set.

Because $\lim_{p1\to\infty} p1 = \infty \Rightarrow \lim_{p1\to\infty} p1 \neq 0$ and $\lim_{p1\to\infty} p1 \neq 1$.

$\lim_{p1\to\infty} p1 \neq 0$, $\lim_{p1\to\infty} p1 \neq 1$ and $\lim_{p1\to\infty} p1$ $\notin$ P $\Rightarrow$ $\lim_{p1\to\infty} p1$ is an odd composite number. Preliminary theorem (2.2) is true.

Preliminary theorem (2.2) is a very key theorem. To explain clearly, let me talk from a Series $X_k = p_k/(p_k-1)$, k∈N, $p_k \in$ P. i.e. X_k =2/1, 3/2, 5/4, 7/6, 11/10, 13/12, 17/16,…. It's easy to calculate the limitation of X_k.

$\lim_{pk\to\infty} pk/(pk-1) = 1$. Similarly, $\lim_{pk\to\infty} pk/(pk-[pk/2]) = 2$. It's strictly "equal to".

But we have found 2 divisors (p_k -1 and p_k -[p_k /2]) of p_k , according to prime definition (p = 1×p), p_k does not belong to prime set. It has become a composite number.

Just like limitation of polygon becomes a circle, that is a qualitative change. The limitation of prime becomes a composite number, that is also a qualitative change.

3. Prove Goldbach conjecture

(3.1) Every even integer greater than 2 can be expressed as the sum of two primes.

(3.1.1) Suppose Goldbach conjecture is true, i.e. 2n = p1 + p2; p1, p2 $\in$ P; n $\geq$ 1, n $\in$ N.

When $n \to \infty \Rightarrow \lim_{n\to\infty} 2n = \infty$; from (3.1.1) $\Rightarrow \infty = \lim_{n\to\infty} 2n = \lim_{n\to\infty} p1 + \lim_{n\to\infty} p2$; because of preliminary theorem (2.1) $\Rightarrow \lim_{n\to\infty} p1 = \infty$ or $\lim_{n\to\infty} p2 = \infty$.

Because p1, p2 is arbitrary, assume $\lim_{n\to\infty} p1 = \infty$.

Because of preliminary theorem (2.2) $\Rightarrow$ p1 is an odd composite number, not a prime.

It's self-contradictory. So the suppose (3.1.1) is false,

Goldbach conjecture being false is proved completely.

4. Theorem verification

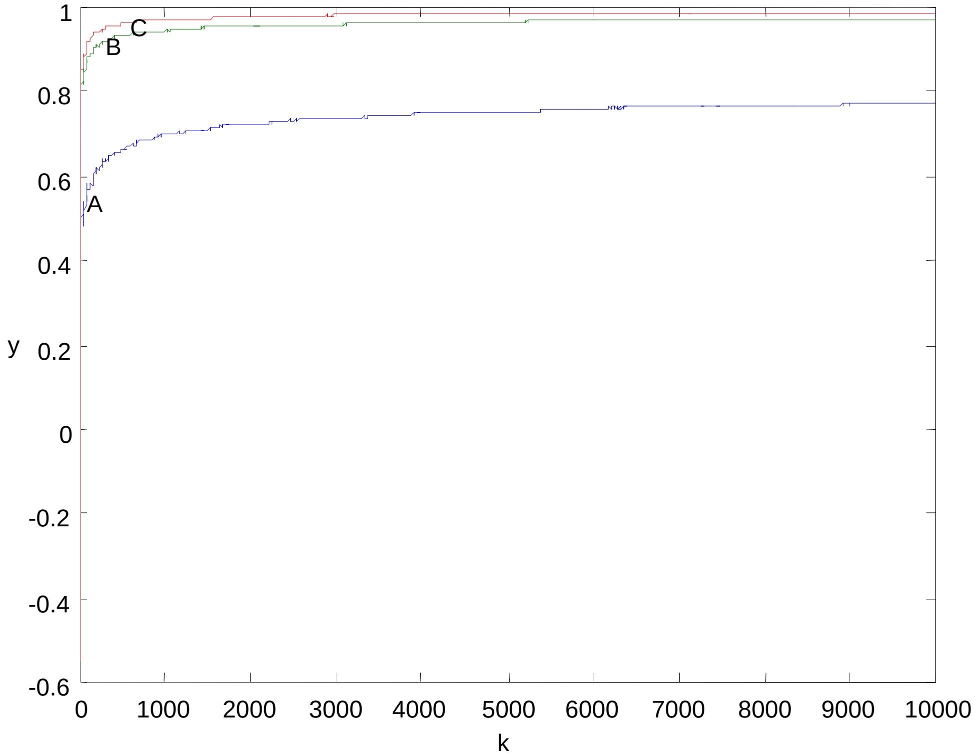
A program on computer to verify preliminary theorem (2.2). Series X_k = 2×k + 1, k $\in$ N; sum(k) equals to odd composite number count; k equals to odd number count.

This program has calculated ln(sum(k))/ln(k) and ln(ln(sum(k)))/ ln(ln(k)) for all odd composite numbers. The output data is in table below:

k	**2×k+1**	**ln(sum(k))/ln(k)**	**ln(ln(sum(k)))/ ln(ln(k))**
4	9	0. 0000	-∞
7	15	0. 3562	-0.5505
10	25	0.4771	0.1128
12	33	0.5579	0.3588
13	35	0. 6275	0.5052
16	39	0. 6462	0.5719
17	45	0. 6868	0.6393
…	…	…	…
77	155	0. 8605	0. 8977
79	159	0. 8608	0. 8983
80	161	0. 8636	0. 9007
82	165	0. 8638	0. 9013
…	…	…	…
14155732	28311465	0. 9919	0.9971
14155733	28311467	0. 9919	0.9971
14155734	28311469	0. 9919	0.9971
14155735	28311471	0. 9919	0.9971
…	…	…	…
16503321	33006643	0.9921	0.9972

16503322	33006645	0.9921	0.9972
16503324	33006649	0.9921	0.9972
16503325	33006651	0.9921	0.9972
…	…	…	…

The function plot like below:



Curve A expresses y = sum(k)/k;

Curve B expresses y = ln(sum(k))/ln(k);

Curve C expresses y = ln(ln(sum(k)))/ ln(ln(k)), which trends to 1 the most fast.

We have found that when $k \to \infty$, ln(ln(sum(k)))/ ln(ln(k)) $\to 1; \Rightarrow$

When $k \to \infty$, sum(k) $\to k$. I.e. $\lim_{k\to\infty}(\ln\ln(sum(k)/\ln\ln(k)) = 1$

It means that when k trends to infinity, the odd composite numbers become more and more and the primes become fewer and fewer. When ln(ln(sum(k)))/ ln(ln(k)) reach limitation, there are all odd composite numbers and no prime.

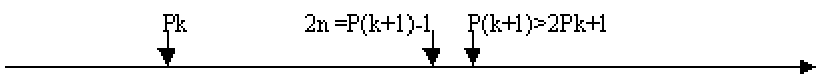
5. Conclusion

When an even integer trends to infinity, an even integer can be expressed as the sum of two odd numbers. But when one odd number trends to infinity, it is only an odd composite number, not a prime. The conclusion is that when an even integer does not trend to infinity, Goldbach conjecture is unproven; when an even integer trends to infinity, Goldbach conjecture become false. At least, when an even integer reaches infinity, Goldbach conjecture is false

6. Prediction

In history, every new finding is often followed by a new prediction. The prediction has become the evidence to verify the finding being true or false.

From preliminary theorem (2.2) and verification function $\lim_{k\to\infty}(\ln\ln(sum(k)/\ln\ln(k))$ =1. I predict that there are 2 sequence primes Pk and P(k+1), no any else prime between Pk and P(k+1), P(k+1)>(2×Pk) + 1, even number 2n=P(k+1) - 1, this even number can't express as the sum of two primes. Moreover, the density of prime become lower and lower with prime increment, the 2 sequence primes with long distance are always existed.



In brief, we will finally find a very big even number with the super computer, which can't express as the sum of two primes.

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