Title: **Fermat's Last Theorem**

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Abstract: **Recall the theorem states that the equation $a^n + b^n = c^n$ cannot exist if all quantities are positive integers and n>2. Fermat maintained he had a short proof but it has never been found, nor has a short proof been supplied by anyone since. This attempt uses simple mathematics and methods reminiscent of those taught in English grammar schools in the 1950's.**

Fermat's Last Theorem
"Hanson Boys' G. S. Proof"

Statement of the Theorem

Fermat's Last Theorem **(FLT)** states that positive integers {a,b,c} cannot be found satisfying the equation:

$$a^n + b^n = c^n \qquad \textbf{(T)}$$

for any integer value of n greater than 2.

Proof

Assume n is prime.

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If n is not prime, say $n=p_1p_2....p_r$, where the p_i are primes, not necessarily all different, we may rename p_1 to n, and {a, b, c} then become integers raised to the power $(p_{2...}p_r)$.

To clarify, the equation:

$$u^{p1p2....pr}+v^{p1p2....pr}=w^{p1p2....pr} \qquad \{u,v,w \text{ positive integers; } u<v<w\}$$

becomes $$u^{n(p2....pr)}+v^{n(p2....pr)}=w^{n(p2....pr)}$$

i.e. $a^n + b^n=c^n$ *where* $a=u^{(p2....pr)}$ $b=v^{(p2....pr)}$, $c=w^{(p2....pr)}$

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Assume that all common factors have been cancelled, noting that all or none of {a,b,c} have a common factor. (A)

Assume the theorem is false and n is an integer >2 such that positive integers {a,b,c} **do** exist satisfying the equation:

$$a^n + b^n = c^n$$

Assume a<b, so that a<b<c.

Let $a + h = b + i = c$ {h, i positive integers, h>i}

Thus $a^n + b^n = (a + h)^n = (b + i)^n = c^n$

We can now rearrange and rewrite **(T)** in 2 different ways:

(I) Using the Binomial Theorem

$$a^n = (b + i)^n - b^n = nb^{n-1}i + n(n-1)/(2!)b^{n-2}i^2 +......+ i^n$$

$$b^n = (a + h)^n - a^n = na^{n-1}h + n(n-1)/(2!)a^{n-2}h^2 +......+ h^n$$

(II) By factoring

$$a^n = (c - b)(c^{n-1} + c^{n-2} b + ... + b^{n-1})$$

$$= i(c^{n-1} + c^{n-2} b + ... + b^{n-1})$$

$$b^n = (c - a)(c^{n-1} + c^{n-2} a + ... + a^{n-1})$$

$$= h(c^{n-1} + c^{n-2} a + ... + a^{n-1})$$

Let $a = Gy$ {G,y integers>0; G = product of primes **not in** i,

y = product of primes **in** i}
and $b = Fx$ {F,x integers>0; F = product of primes **not in** h,
x = product of primes **in** h}

thus x>y (∵ h>i) and {x,y are co-prime ∵ of (A)}

The equations in **(I)** may now be written:

$(Gy)^n = i(nb^{n-1} + n(n-1)/(2!)b^{n-2}i + ... + i^{n-1})$ $\mathbf{\{i <= y^n\}}$
$(Fx)^n = h(na^{n-1} + n(n-1)/(2!)a^{n-2}h + ... + h^{n-1})$ $\mathbf{\{h<= x^n\}}$

Let $i= y^p$ {0<p<=n}and now dividing through by y^p gives:
$G^ny^{n-p} = nb^{n-1} + n(n-1)/(2!)b^{n-2}i + ... + i^{n-1}$

Because y still remains on the LHS and also in the powers of i on the RHS one of the following two cases must be true:

(1) p=n-1 and n=y (since y cannot be in b^{n-1} ∵ of **(A)**), or,

(2) p=n and all occurrences of y have been cancelled out.

If **(1)** is true **(T)** may now be written:

$(An^q)^n + b^n = c^n$ {1<=q; q integer, A=product of all primes in a other than n}

∴ $An^q = (c^n – b^n)^{1/n}$ and n is not prime.

Thus $\mathbf{i=y^n}$ and similarly $\mathbf{h=x^n}$**.**

The equations in **(II)** can now be written

$(Gy)^n = y^n(c^{n-1} + c^{n-2}b + ... b^{n-1})$

$G^n = (c^{n-1} + c^{n-2}b + ... b^{n-1})$

$(Fx)^n = x^n(c^{n-1} + c^{n-2}a + ... a^{n-1})$

$F^n = (c^{n-1} + c^{n-2}a + ... a^{n-1})$

∴ **G>F {but Fx>Gy ∵ Fx=b, Gy=a; a<b}** **(B)**

since $a + h = b + i = c$
$Gy + x^n = Fx + y^n = c$

∴ $Fx - Gy = x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y .... xy^{n-2} + y^{n-1})$ {x-y>1}
$\mathbf{= R(x-y) \quad \{R = (x^{n-1} + x^{n-2}y ... xy^{n-2} + y^{n-1})}$ **(C)**

from **(C)** writing (G-u) for F {u integer>0}:
$(G-u)x – Gy = R(x-y)$
$G(x–y) - ux = R(x-y)$

∴ $G - ux/(x-y) = R$

hence $ux=k(x –y)$ {k integer>0; ∵ all quantities are positive integers, G>R}

∴ $x(k-u) = ky$ and {x,y are not co-prime}

This is a contradiction and the conclusion must be that Fermat's Last Theorem is true.