Title: **Fermat's Last Theorem**

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Abstract: **Recall the theorem states that the equation $a^n + b^n = c^n$ cannot exist if all quantities are positive integers and n>2. Fermat maintained he had a short proof but it has never been found, nor has a short proof been supplied by anyone since. This attempt uses simple mathematics and methods reminiscent of those taught in English grammar schools in the 1950's.**

Fermat's Last Theorem
"Hanson Boys' G. S. Proof"

Statement of the Theorem

Fermat's Last Theorem **(FLT)** states that positive integers {a,b,c} cannot be found satisfying the equation:

$$a^n + b^n = c^n \qquad \textbf{(T)}$$

for any integer value of n greater than 2.

Proof

Assume n is prime.

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If n is not prime, say $n=p_1p_2....p_r$, where the p_i are primes, not necessarily all different, we may rename p_1 to n, and {a, b, c} then become integers raised to the power $(p_{2...}p_r)$.

To clarify, the equation:

$$u^{p1p2....pr}+v^{p1p2....pr}=w^{p1p2....pr} \qquad \{u,v,w \text{ positive integers; } u<v<w\}$$

becomes $u^{n(p2....pr)}+v^{n(p2....pr)}=w^{n(p2....pr)}$

i.e. $a^n + b^n=c^n$ *where* $a=u^{(p2....pr)}$ $b=v^{(p2....pr)}$, $c=w^{(p2....pr)}$

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Assume that all common factors have been cancelled, noting that all or none of {a,b,c} have a common factor. (A)

Assume the theorem is false and n is an integer >2 such that positive integers {a,b,c} **do** exist satisfying the equation:

$$a^n + b^n = c^n$$

Assume a<b, so that a<b<c.

let $a + h = b + i = c$ {h, i positive integers, h>i}

thus $a^n + b^n = (a + h)^n = (b + i)^n = c^n$

We can now rewrite **(T)** in 2 different ways.

(I) Using the Binomial Theorem

$$a^n = (b + i)^n - b^n = nb^{n-1}i + n(n-1)/(2!)b^{n-2}i^2 +......+ i^n$$

$$b^n = (a + h)^n - a^n = na^{n-1}h + n(n-1)/(2!)a^{n-2}h^2 +......+ h^n$$

(II) By factoring

$$a^n = (c – b)(c^{n-1} + c^{n-2} b + ... + b^{n-1})$$

$$= i(c^{n-1} + c^{n-2} b + ... + b^{n-1})$$

$$b^n = (c – a)(c^{n-1} + c^{n-2} a + ... + a^{n-1})$$

$$= h(c^{n-1} + c^{n-2} a + ... + a^{n-1})$$

Let $a = Gy$ {G,y integers>0; G = product of primes **not in** i, y = product of primes **in** i}

and $b = Fx$ {F,x integers>0; F = product of primes **not in** h, x = product of primes **in** h}

thus $x>y$ ($\because$ $h>i$) and {x,y are co-prime $\because$ of (A)}

The equations in **(I)** may now be written:

$$(Gy)^n = i(nb^{n-1} + n(n-1)/(2!)b^{n-2}i + \ldots\ldots + i^{n-1}) \qquad \mathbf{\{i <= y^n\}} \qquad \mathbf{(1)}$$
$$(Fx)^n = h(na^{n-1} + n(n-1)/(2!)a^{n-2}h + \ldots\ldots + h^{n-1}) \qquad \mathbf{\{h<= x^n\}} \qquad \mathbf{(2)}$$

Consider **(1)**, dividing through i by gives:

$$\mathbf{G^n\, Y= nb^{n-1} + n(n-1)/(2!)b^{n-2}i + \ldots\ldots + i^{n-1}} \qquad \mathbf{\{if\ i<y^n\}} \qquad \mathbf{(1a)}$$

where Y is the product of primes remaining from y^n after division
or

$$\mathbf{G^n = nb^{n-1} + n(n-1)/(2!)b^{n-2}i + \ldots\ldots + i^{n-1}} \qquad \mathbf{\{if\ i=y^n\}} \qquad \mathbf{(1b)}$$

Because **(1a)** has factors in Y that are in every i on the RHS but not in b of the first RHS term ($\because$ **of (A)**) n must be a factor of Y and i.

$\therefore$ i and y must have the form:
$y=tn^k$, $i=t^n n^{nk-1}$ {k integer>0, t=product of primes in y other than n}

thus $a^n = (b+t^n n^{(nk-1)})^n - b^n$ and expanding the RHS as per the method in **(II)**

$$a^n = t^n n^{nk-1}[(b+t^n n^{nk-1})^{n-1}+(b+t^n n^{nk-1})^{n-2}b+(b+t^n n^{k-1})^{n-3}b^2 + \ldots. + b^{n-1}]$$

$$a = tn^{(nk-1)/n}[(b+t^n n^{k-1})^{n-1}+(b+t^n n^{k-1})^{n-2}b+(b+t^n n^{k-1})^{n-3}b^2 + \ldots.\ b^{n-1}]^{1/n}$$

and a is not a positive integer and equation **(1b)** applies.

A similar argument may be given for h and we arrive at:

$i=y^n$, $h=x^n$ and the equations in **(II)** may be written

$$(Gy)^n= y^n(c^{n-1} + c^{n-2}b + \ldots.\ b^{n-1})$$
$$G^n= (c^{n-1} + c^{n-2}b + \ldots.\ b^{n-1})$$
$$(Fx)^n = x^n(c^{n-1} + c^{n-2}a + \ldots.\ a^{n-1})$$
$$F^n = (c^{n-1} + c^{n-2}a + \ldots.\ a^{n-1})$$

$\therefore$ **G>F {but Fx>Gy $\because$ Fx=b, Gy=a; a<b}** **(B)**

Now since $a + h = b + i = c$

$Gy + x^n = Fx + y^n = c$

∴ $Fx - Gy = x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y \ldots. xy^{n-2} + y^{n-1})$ {x-y>1}

$= R(x-y)$ {$R = (x^{n-1} + x^{n-2}y \ldots xy^{n-2} + y^{n-1})$ (C)

from **(C)** writing (G-u) for F {u integer>0}:

$(G-u)x - Gy = R(x-y)$

$G(x-y) - ux = R(x-y)$

∴ $G - ux/(x-y) = R$

hence $ux=m(x-y)$ {m integer>0; ∵ all other quantities are positive integers, G>R}

∴ $x(m-u) = my$ and {x,y are not co-prime}

This is a contradiction and the conclusion must be that Fermat's Last Theorem is true.