Title: **Fermat's Last Theorem**

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Abstract: **Recall the theorem states that the equation $a^n + b^n = c^n$ cannot exist if all quantities are positive integers and n>2.**

Fermat maintained he had a short proof but it has never been found, nor has a short proof been supplied by anyone since.

This attempt uses simple mathematics and methods reminiscent of those taught in English grammar schools in the 1950's.

Furthermore this effort is also unable to trouble a normal margin!

Fermat's Last Theorem
"Hanson Boys' G. S. Proof"

Statement of the Theorem

Fermat's Last Theorem **(FLT)** states that positive integers {a,b,c} cannot be found satisfying the equation:

$$a^n + b^n = c^n \qquad \textbf{(T)}$$

for any integer value of n>2.

Proof

Assume n is prime.

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If n is not prime, say $n=p_1p_2....p_r$, where the p_i are primes, not necessarily all different, we may rename p_1 to n, and {a, b, c} then become integers raised to the power $(p_2...p_r)$.

To clarify, the equation:

$$u^{p1p2....pr}+v^{p1p2....pr}=w^{p1p2....pr} \quad \{u,v,w \text{ positive integers; } u<v<w\}$$

becomes $u^{n(p2....pr)}+v^{n(p2....pr)}=w^{n(p2....pr)}$

i.e. $a^n + b^n = c^n$ *where* $a=u^{(p2....pr)}$ $b=v^{(p2....pr)}$, $c=w^{(p2....pr)}$

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Assume the theorem is false and n is an integer >2 such that positive integers {a,b,c} **do** exist satisfying **(T)**.

Assume that all common factors have been cancelled, noting that all or none of {a,b,c} have a common factor. **(A)**

Assume a<b; thus a<b<c.

let $c = a + h = b + i$ {h, i positive integers, h>i}

if **(T)** is true $a^n + b^n = (a + h)^n = (b + i)^n = c^n$

We can now rewrite **(T)** in 2 different ways.

(I) Using the Binomial Theorem

$$a^n = (b + i)^n - b^n = nb^{n-1}i + n(n-1)/(2!)b^{n-2}i^2 + ...+ i^n$$
$$b^n = (a + h)^n - a^n = na^{n-1}h + n(n-1)/(2!)a^{n-2}h^2 + ...+ h^n$$

(II) By factoring

$$a^n = (c - b)(c^{n-1} + c^{n-2} b + ... + b^{n-1})$$
$$= i(c^{n-1} + c^{n-2} b + ... + b^{n-1})$$
$$b^n = (c - a)(c^{n-1} + c^{n-2} a + ... + a^{n-1})$$
$$= h(c^{n-1} + c^{n-2} a + ... + a^{n-1})$$

Let $a = Ay$ {A, y integers>0;
(A = product of primes **not in** i, y = product of primes **in** i)}

and b = Bx {B, x integers>0;
(B = product of primes **not in** h, x = product of primes **in** h)}

thus $x>y$ (∵ $h>i$) and x, y are co-prime ∵ of **(A)**

The equations in **(I)** may now be written:

$(Ay)^n = i(nb^{n-1} + n(n-1)/(2!)b^{n-2}i + \ldots\ldots + i^{n-1})$ **{$i <= y^n$}** **(1)**
$(Bx)^n = h(na^{n-1} + n(n-1)/(2!)a^{n-2}h + \ldots\ldots + h^{n-1})$ **{$h<= x^n$}** **(2)**

In **(1)** dividing through i by gives:

$A^n\,Y= nb^{n-1} + n(n-1)/(2!)b^{n-2}i + \ldots\ldots + i^{n-1}$ {if $i<y^n$} (1a)

where Y is the product of primes remaining from y^n after the division, or,

$A^n = nb^{n-1} + n(n-1)/(2!)b^{n-2}i + \ldots\ldots + i^{n-1}$ {if $i=y^n$} (1b)

Case **(1a)**
Because Y contains factors that are in every i on the RHS but not in b of the first term, ∵ of **(A)**, n must be a factor of Y and i.

∴ i and y must have the form:
$y=tn^k$, $i=t^n n^{nk-1}$ {k integer>0, t=product of primes in y other than n}

thus $a^n = (b+t^n n^{(nk-1)})^n - b^n$ and expanding the RHS as per the method in **(II)**

$a^n = t^n n^{nk-1}[(b+t^n n^{nk-1})^{n-1}+(b+t^n n^{nk-1})^{n-2}b+(b+t^n n^{k-1})^{n-3}b^2 + \ldots. + b^{n-1}]$
$a = tn^{(nk-1)/n}[(b+t^n n^{k-1})^{n-1}+(b+t^n n^{k-1})^{n-2}b+(b+t^n n^{k-1})^{n-3}b^2 + \ldots. b^{n-1}]^{1/n}$

and a is not a positive integer and equation **(1b)** applies.

A similar argument can be given for equation **(2)** and h and we arrive at:
$i=y^n$, $h=x^n$

Thus we have:
$a = Ay$; $b = Bx$, $c = (Ay + x^n) = (Bx + y^n)$ **{A,B,x,y are co-prime}**

(T) may now be written:
$(Ay)^n + (Bx)^n = (Ay + x^n)^n$ (i)
$(Ay)^n + (Bx)^n = (Bx + y^n)^n$ (ii)

Simple algebra on (i) gives:
$x = (Ay)^{1/n}(\,(1 + (Bx/Ay)^n)^{1/n}) - 1)^{1/n}$
and x is not an integer.

This is a contradiction and the conclusion is that Fermat’s Last Theorem is true.