

# Horizon Problem Resolution

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## Abstract

We present a model that offers a resolution to the Horizon Problem of cosmology and eliminates the need for Inflation. It also suggests a possible new origin for the Cosmic Microwave Background Radiation. In addition, this model eliminates the need to invoke Dark Energy and Dark Matter to explain the accelerated expansion of the universe. In essence, it implies that there is no accelerated expansion by fitting the model to Type 1a Supernovae and Gamma Ray Burst data with a reduced chi-square (goodness-of-fit) of 0.99, using only the Hubble constant as a parameter.

## 1 The Horizon Problem

In cosmology, it is believed that regions of space on ‘opposite’ sides of the universe are too far apart to have ever been causally connected. That is, they are outside each other’s ‘particle horizon’. Consequently, it is difficult to explain the apparent similarities in their characteristics as evidenced by COBE results. Inflation theory has been offered as a way to overcome this ‘Horizon Problem’. However, it is my intention, here, to show that it is unnecessary to postulate Inflation in order to insure that *all* regions of spacetime are now, and have always been, causally connected.

## 2 A Resolution to the Problem

Fig. 1 is a spacetime diagram in which the time coordinate (not shown) extends radially from the origin in all directions and space exists on an expanding circle centered at the origin. The ‘radius’ of space  $r$  is a function of time. At the present time, the radius is  $r_0$ , and the event  $P_0$  represents here and now. Imagine that a light pulse is emitted by a source and travels along the expanding circle reaching  $P_0$  at the present time. The event  $P$  represents the position of the pulse at an earlier time along the inner circle of radius  $r$ . The angle  $\theta$ , which is also a function of time, is the angular separation between events  $P_0$  and  $P$ .

We will assume that the speed of a light pulse is  $c$  relative to its location on the circle, everywhere along the circle. If the speed of the pulse slowed as it progressed along the circle, light from more distant sources would propagate more slowly than light from nearer sources. Experience indicates that this is not the case. We also assume, for now, that the expansion rate

$\dot{r} = dr/dt$  is constant. The light pulse vector  $\vec{c}$  and the expansion rate vector  $\vec{r}$  are tangent and normal to the circle, respectively, as shown in the figure.

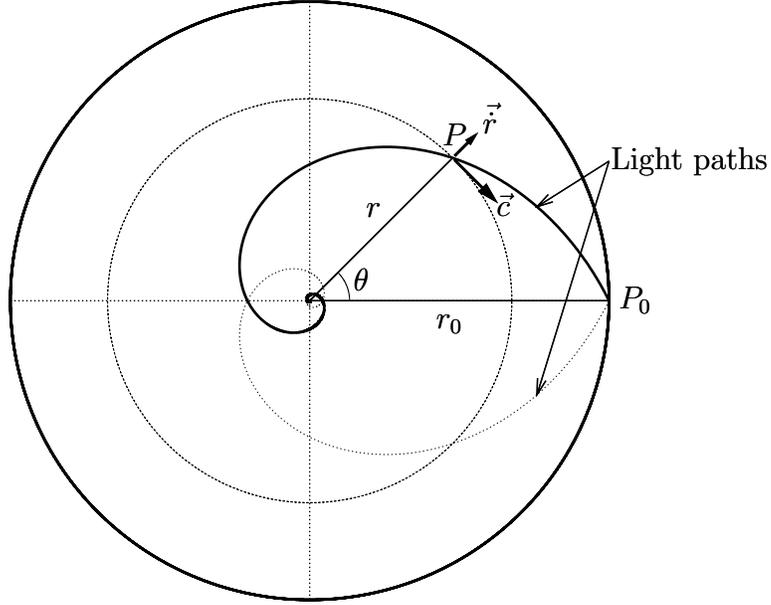


Figure 1: Light paths

A curve describing the light path, subject to the properties above, is known as a logarithmic spiral, which we derive here.

We first imagine that a light pulse travels the distance  $ds$  along the circle with radius  $r$  at the constant speed  $c$  in the time  $dt$ , so that

$$c = \frac{ds}{dt} \quad (1)$$

The circle, along which the pulse travels, expands at a rate of  $\dot{r}$ , which is assumed to be constant for the purposes of this derivation. In the time  $dt$ , the radius of the circle changes by  $dr$ , so

$$\dot{r} = \frac{dr}{dt} \quad (2)$$

We note, using (1) and (2), that

$$\frac{dr}{ds} = \frac{dr/dt}{ds/dt} = \frac{\dot{r}}{c} \quad (3)$$

where  $\dot{r}/c$  is constant. The angle that intercepts  $ds$  is  $-d\theta$ , since the pulse is traveling in the negative  $\theta$  direction, so that

$$ds = -rd\theta \quad (4)$$

Using (4), we can write (3) as

$$-\frac{dr}{rd\theta} = \frac{\dot{r}}{c} \quad (5)$$

Now, separating variables and integrating, we get

$$\int \frac{dr}{r} = -\frac{\dot{r}}{c} \int d\theta \quad (6)$$

$$\ln r = -\frac{\dot{r}\theta}{c} + k \quad (7)$$

$$r = e^{-(\dot{r}\theta/c)+k} \quad (8)$$

$$r = e^k e^{-\dot{r}\theta/c} \quad (9)$$

Setting  $\theta = 0$ , we find that  $e^k = r_0$ , so that we finally get the equation for the light path

$$r = r_0 e^{-\dot{r}\theta/c} \quad (10)$$

which is represented by the upper light path in Fig. 1. We must, of course, allow for a light pulse traveling toward  $P_0$  from the opposite direction, which we represent by the lower light path in the figure. The points of convergence of the upper and lower light paths occur at  $\theta = n\pi, n = 0, 1, 2, 3, \dots$ .

The recession velocity  $v$  between two comoving points on the expanding circle is

$$v = \dot{r}\theta \quad (11)$$

Therefore, inserting (11) into (10), we get for the radius in terms of the recession velocity,

$$r = r_0 e^{-v/c} \quad (12)$$

A point on the circle at an angular separation of  $\theta = 1$  would have a recession velocity, from (11), of  $v = \dot{r}$  along the circle. Assuming for the moment, that  $\dot{r} = c$  and  $r = \dot{r}t = ct$ , any comoving light source at a greater angular separation than  $\theta = 1$  would have a recession velocity greater than  $c$ , but would still be visible. This visibility is due to the fact that the speed of light is equal to  $c$  relative to each point in space along its path.

For example, assume that a source in a region of space at an angular separation of, say,  $\theta = 1.1$  (which has recession velocity of  $1.1c$ , relative to  $P_0$ ) emits a pulse toward  $P_0$ . This pulse would surely reach a region of space at an angular separation of, say,  $\theta = 0.9$  (which has recession velocity of  $0.9c$ , relative to  $P_0$ ), since the two regions have recession velocities of  $0.2c$  relative to each other. And since the pulse is traveling at speed  $c$  relative to space in this region, it will reach  $P_0$ .

Unfortunately, a logarithmic spiral never actually reaches the origin. Consequently, we would never be able to see the origin of the universe. However, we would apparently be able to see our own region of space as it appeared at earlier times ( $\theta = 2n\pi, n = 1, 2, 3, \dots$ ), providing that light maintains its integrity as it passes through the convergence points.

If the expansion rate  $\dot{r}$  varies, the light curve no longer fits the definition of a logarithmic spiral. However, since  $c$  is nonzero, its form would remain similar to a logarithmic spiral.

Equation (10) assures that all regions of space are causally connected at all times. Therefore, there is no particle horizon, thus no ‘Horizon Problem’, in this model.

### 3 Cosmological Redshift

The cosmological redshift,  $z$ , is

$$z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_0 - \lambda}{\lambda} = \frac{\lambda_0}{\lambda} - 1 \tag{13}$$

where  $\lambda$  is the wavelength of light at emission and  $\lambda_0$  is the wavelength at reception. And since, using (12),

$$\frac{\lambda_0}{\lambda} = \frac{r_0}{r} = \frac{r_0}{r_0 e^{-v/c}} = e^{v/c} \tag{14}$$

we can insert (14) into (13) to get

$$z = e^{v/c} - 1 \tag{15}$$

### 4 Velocity Redshift and Distance Redshift Relations

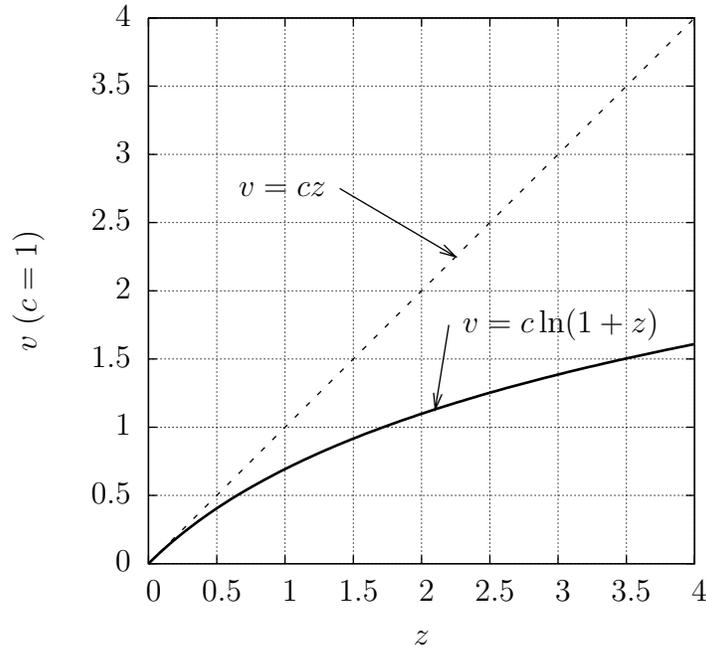


Figure 2: Recession Velocity vs. Cosmological Redshift

By rearranging (15), we find the the recession velocity as a function of redshift

$$v = c \ln(1 + z) \tag{16}$$

This would not be subject to a relativistic correction since the source is not traveling through space, but is comoving with it. Fig. 2 shows the relationship between redshift and recession velocity.

In order to find the proper distance  $d$  between two comoving points on a circle with radius  $r$ , measured along the circle, as a function of redshift, we first note that

$$d = r\theta \quad (17)$$

Rearranging (11) and inserting (16), we have

$$\theta = \frac{v}{\dot{r}} = \frac{c \ln(1+z)}{\dot{r}} \quad (18)$$

Plugging (18) into (17), we get

$$d = r \left( \frac{c \ln(1+z)}{\dot{r}} \right) = \frac{r}{\dot{r}} c \ln(1+z) \quad (19)$$

The present proper distance  $d_0$  between two comoving points on the circle with radius  $r_0$ , as a function of redshift, is

$$d_0 = \frac{r_0}{\dot{r}} c \ln(1+z) \quad (20)$$

If we take the Hubble constant  $H_0$ , for constant  $\dot{r}$  and present radius  $r_0$

$$H_0 = \frac{\dot{r}}{r_0} \quad (21)$$

and insert (21) into (20), assuming constant  $\dot{r}$ , we have for the present distance as a function of redshift,

$$d_0 = \frac{c}{H_0} \ln(1+z) \quad (22)$$

Combining (16) and (22), we get

$$v = c \ln(1+z) = H_0 d_0 \quad (23)$$

Equation (23) is my version of Hubble's Law ( $v = cz = H_0 d_0$ ).

It is important to note that this model is *not* an empty universe model, but rather a model in which the path of light is unaffected by matter (other than through direct contact with matter). The path of light may *appear* to bend in response to the gravitational field of a massive body, according to an observer at rest on the body. However, this 'illusion' of bending is a result of the observer's acceleration (or rather deceleration) in *time*, due to gravity. This is analogous to the illusion of the bending of light experienced by an observer at rest in a rocket, due to the rocket's acceleration in space.<sup>1</sup>

## 5 A New Explanation for the CMBR?

At  $\theta = \pi$ , if  $\dot{r}$  is made to equal  $c$ , we get

$$r_\pi = r_0 e^{-c\pi/c} = r_0 e^{-\pi} \quad (24)$$

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<sup>1</sup>Please see "New Transformation Equations and the Electric Field Four-vector" at <http://www.softcom.net/users/der555>

This is about 5% of the present radius or 5% of the present age of the universe.

Radiation emerging from this period shares the property of omnidirectionality with the cosmic microwave background radiation (CMBR). Furthermore, the possibility exists that it may share *all* of the properties of the CMBR. At this time, these properties have not yet been investigated. However, if these properties match those of the CMBR, the tantalizing possibility exists that the radiation emerging from the convergence point at  $\theta = \pi$  could *be* the CMBR.

## 6 The Limits of Observation

If the radiation emerging from the convergence point at  $\theta = \pi$  is the CMBR, then the era associated with the CMBR would be much later than commonly believed. The cosmological redshift associated with this era (found by inserting  $\dot{r} = c$  into (11), and (11) into (15)) is  $z = e^\pi - 1 \approx 22$ . This value for  $z$  is much less than the current value associated with the CMBR in standard cosmology ( $z \approx 1088$ ).<sup>2</sup>

The possibility exists that radiation passing through the convergence point at  $\theta = \pi$  originated from luminous bodies<sup>3</sup> nearer the origin (as well as standard CMBR sources). However, if radiation does not maintain its integrity (for example, through interference) as it passes through the convergence points, it is conceivable that those bodies may no longer be discernable *as bodies* to an observer at  $P_0$ . (This, however, is not to say that *radiation* originating from those bodies could not be observed at  $P_0$ .) If this is the case, an upper limit of  $z \approx 22$  would be placed on the observation of *discernable* luminous bodies.

## 7 Do We Need Dark Energy or Dark Matter?

The concepts of dark energy and dark matter arose from the attempt to reconcile a discrepancy between theory and observation. It is my intention to show that there is no such discrepancy when my model is compared to observation. The Type 1a Supernovae data used for comparison here is the ‘gold’ sample from Table 6 of Riess et al. 2007 (astro-ph/0611572). This is combined with Gamma Ray Burst (GRB) data from Table 6 of Schaefer 2006 (astro-ph/0612285v1). In order to get the best values of  $H_0$ , a magnitude of 0.32 has been subtracted from the distance modulus  $\mu$  of both sets of data.

We need to compare my values for the distance modulus, which I will call  $\mu_1$ , to the values for  $\mu$  in the data, as a function of redshift  $z$ . To find the equation for  $\mu_1$ , we first find the luminosity distance  $d_L$ .

The flux density  $F$  of the light emitted isotropically from a source and passing through a spherical surface centered on the source is

$$F = \frac{P}{A} \tag{25}$$

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<sup>2</sup>It is important to note that radiation originating from sources nearer the origin than the convergence point may have already undergone significant redshift prior to its arrival at the convergence point.

<sup>3</sup>By “luminous bodies”, I am referring to luminous objects with finite extension in space, such as stars, galaxies, supernovae, etc..

where  $P$  is the total power of the light passing through the surface and  $A$  is the proper area of the surface. The proper area of the surface at reception, in my model, is

$$A = 4\pi d_0^2 \quad (26)$$

where  $d_0$  is my proper distance  $d_0 = (c/H_0) \ln(1+z)$ . However, the power of the light passing through the surface at reception is reduced, compared to its power at emission, due to the expansion of space between emission and reception. The energy of each photon at reception is redshifted by a factor of  $1/(1+z)$ , and photons emitted at time intervals  $\delta t$  arrive at time intervals  $\delta t(1+z)$ . Taking both of these factors into account, the total power  $P$  passing through the surface at reception is the total power  $L$  (absolute luminosity) emitted by the source multiplied by the factor  $1/(1+z)^2$ , or

$$P = \frac{L}{(1+z)^2} \quad (27)$$

So the flux density at reception in my model, after inserting (26) and (27) into (25), is

$$F = \frac{P}{A} = \frac{L}{4\pi d_0^2 (1+z)^2} \quad (28)$$

Another way to write  $F$  is through the flux-luminosity relationship

$$F = \frac{L}{4\pi d_L^2} \quad (29)$$

Comparing (28) and (29), we see that my luminosity distance is, therefore,

$$d_L = d_0(1+z) \quad (30)$$

Recalling from (22) that

$$d_0 = \frac{c}{H_0} \ln(1+z) \quad (31)$$

we then combine (30) and (31) to get

$$d_L = \frac{c}{H_0} (1+z) \ln(1+z) \quad (32)$$

The distance modulus  $\mu_1$ , using (32), is

$$\mu_1 = 5 \log_{10}(d_L) + 25 \quad (33)$$

Finally, in Fig. 3, we compare  $\mu_1$  from (33) with  $\mu$  from the two sets of data, above. According to the graphing utility Gnuplot<sup>4</sup>, an optimum fit of (33) to the data occurs for  $H_0 = 69.245 \text{ km s}^{-1} \text{ Mpc}^{-1}$  with reduced chi-square (goodness-of-fit) of 0.99.

The curve for  $\mu_1$  in Fig. 3 is a very good fit to the data. However, contrary to the current concordance model, my model does *not* imply an accelerating universe, and requires neither dark energy nor dark matter in order to fit observation.

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<sup>4</sup>All graphs were created using Gnuplot (<http://www.gnuplot.info/>). Thanks very much to its creators.

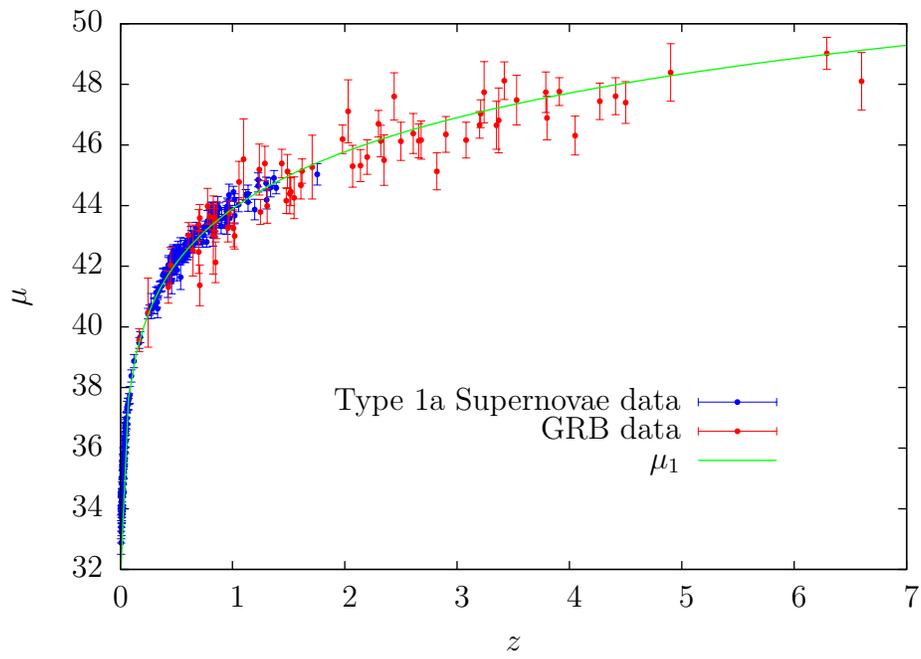


Figure 3: Distance modulus  $\mu$  vs. Redshift  $z$