<u>Zero Division</u>

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<u>Introduction:</u>

This article will be developed through several stages. I'll begin by examining some basic ideas about division and multiplication as well as why those ideas appear to fail when dividing by zero.I will then detail some early attempts to resolve those issues and why they were unsuccessful, before providing a modern definition for the process based upon the Indeterminate Form, and the existence of hidden subspaces.

The *Modern Definition of this article*, based on the presence of subspace directions is unique and has parallels only recently seriously considered in fields such as Quantum Mechanics and String Theory. As the Article then progresses to examples I will show it is necessary the math be inseparably linked to time and what I shall define as subspace, while making a clear distinction between directions and dimensions.

Examples provided will be in 2—space, and 3—space. I will show how these environments develop from a singular point, that they contain additional directions that form the aforementioned subspaces, and how theirinclusion permits division by zero.

The final section of the paper will detail an additional application of zero division, providing a solution to the negative radical before closing with some final thoughts and suggestions for future developments.

<u>Multiplication and Division Basics:</u>

What exactly is a number line? It's really nothing more than a pictographic representation of counting. We assign values by combining two or more number lines, and then define operations based upon gain or losswithin that system. Zero is a transition point along any number line where values flip from negative to positive, with zero itself being both positive and negative simultaneously.

Zero can be several things. Yes, it can represent the idea of nothingness or emptiness. However within the number line, it's meaning is one of balance, not absence. There is no gain or loss represented at this point as an equal amount of positive and negative numbers extending out to infinity exist to its left and right. This idea, that a number line axis represented by a variable is infinite, is important. I'll return to build on it more momentarily.

Division and multiplicationareopposite functions. For any mathematical operation to be determined accurate we must be able to both check that operation by its opposite and be able to use it to accurately portray real world situations. We know that $\frac{10}{5} = 2$because $5 \cdot 2 = 10$. Just as surely we know that a car traveling at 5mph for 2 hours will have moved a total of 10 miles. The math is verifiable and it can be used for predictive results in the real world. Rock solid logic. Problems creep up when a zero exists in the denominator of a fraction. Consider the following two examples.

$$\frac{2}{0} = x \qquad \text{and} \qquad 0x = 2$$

The present understanding of both multiplication and division say there are logic issues and the operations cannot be executed. The first expression,$\frac{2}{0}$ is labeled as undefined because the output value will be infinity. In the second instance there is no definable value in modern mathematics which can satisfy this expression. Any value for *x* will equal *0*, and $0 \neq 2$.

This reasoning is not accurate. It's incomplete and based on an underdeveloped understanding of what multiplication and division by zero imply. 0 is not meaningless and its presence in any such operation will have a verifiable output value. Both of these expressions do have logic issues because they are written improperly. I will show that an expression of this form has output values in a separate axis which though present and implied is hidden. The solution will rely on rewriting the expression to include the existence of that output axis.

For example both of these expressions above are mathematically identical. Using the steps described in this article you'll find that both expressions will take the form $v = \oplus \frac{x}{2}$. The term v is a subspace of y, and will be described in detail later. The circle-plus operator is used here to indicate the expression is simultaneously positive and negative. That will be discussed later as well. When anything is divided, or multiplied by zero there will exist a definable value in a subspace of that system whether or not a real solution can be obtained within the confines of the original expression. These subspaces always exist. An explanation of how to arrive at this understanding must begin with previous attempts and failures to define division by zero.

Early Attempts:

The Brahmasphutasiddhantaof Brahmagupta:

Brahmagupta lived in Ujjain, India from 598 to 668 AD and made notable contributions to mathematics and astronomy. In 628 AD he is reported to have written a text called the Brahmasphutasiddhanta, *The Opening of the Universe* in English, within which is found the earliest known attempt of defining division by zero (Wikipedia: The Brahmasphutasiddhantaof Brahmagupta, *http://en.wikipedia.org/wiki/Brahmasphutasiddhanta*, and *http://www-history.mcs.st-andrews.ac.uk/Biographies/Brahmagupta.html*). Brahmagupta attempts to define division by zero as equaling zero.

$$\frac{a}{0} = 0 \text{ where } a \text{ is any constant.}$$

This assumption is incorrect and appears to be drawn from the idea of defining any function by its opposite. For example we know that $a \cdot 0 = 0$, and $\frac{0}{a} = 0$ are both true statements. It's easy to see how the assumption could be made but it's still wrong. I use the familiar function $y = \frac{1}{x}$ at several locations in the article to illustrate my own examples and it works rather well here too. The output value approaches infinity as x approaches 0. Infinity cannot be defined in standard mathematics but it represents and impossibly huge value—not zero.

If you divide 1 by a very small number you get a very large output. The reasoning is simple logic. The smaller the denominator, the more times a unit of that value can be added to together to get one whole—the numerator. The smaller that denominator gets, the larger the quotient becomes. When *x* equals 0 the reasoning says the output has to reach infinity. Again the answer is clearly not zero.

Something has to be happening at this value that traditional reasoning has failed to explain. If we can divide by other values and obtain real results one must exist for this expression as well, it just requires understanding beyond that of Brahmagupta's time. Saying the answer is infinity does not mean there is no answer. Since the *y* cannot reach the assumed value of infinity it means the output does not lie on the *y*—axis. That's why we express it as ∞; it no longer using this axis at all.

Mahavira and Bhaskara II:

Two others followed Brahmagupta in an attempt to correct his definition of division at zero by asserting their own. In 830 A.D. Mahavira wrote a text titled *Ganita Sara Samgraha* in which he claims *A number remains unchanged when divided by zero* (Wikipedia: Division by Zero—*http://en.wikipedia.org/wiki/Division_by_zero*).

$$\frac{a}{0} = a \text{ where } a \text{ is any constant.}$$

Mahavira was undoubtedly brilliant but still made an error here. If a number remains unchanged after division, it has either been divided by 1 or not divided at all. With the function $y = \frac{1}{x}$ we clearly see this. One divided by one is one. As *x* becomes larger or smaller than one, no matter how slightly so, *y* will become infinitely small or large respectively. So just avoiding the issue of division at zero and claiming no change takes place won't work.

Bhaskara II, another Indian mathematician who made contributions to calculus, tried his hand at the problem too. He is said to have made the assumption that…*when a finite number is divided by zero, the result is infinity* (http://en.wikipedia.org/wiki/Bhaskara_II*and*http://en.wikipedia.org/wiki/Division_by_zero).

$$\frac{a}{0} = \infty$$

This expression is the closest anyone has come to a true understanding of what is implied by this operation. It is true, and verifiable in the function $y = \frac{1}{x}$ that dividing by 0 will result in an infinite value for *y* but it doesn't explain what that means. Saying something is infinite is the same thing as saying you have no answer. Though the presence of infinity is really no different than an expression which is divided by 0 it does provide a clue how to proceed. The answer comes from a close examination of our ideas of the space we use to define points, lines, and curves in a graph. The examination of this real space will show solutions are more complex than previously thought, inseparably linked to time and the presence of extra, hidden directions. Before making this examination let's look at how a subspace equation is obtained, and how this solves the issue of division at zero.

The process requires an equation be multiplied on both sides by a figure which can simultaneously be related to an infinite quantity to a real number. That figure will have to equal infinity to represent and replace the value *y* attempts to reach when dividing by zero, and yet still hold the value of 1 to prevent from changing the value of the equation despite altering its form. There is only one figure which satisfies this requirement, the Indeterminate Form $\frac{0}{0}$.

I'm certain that everyone reading this is appalled, their mind recoiling in horror at the absurdity they have just been presented with. Is it really that absurd? Think about it. If accepted that any value divided by the same value equals one then we can define $\frac{0}{0}$.If I made the substitution that $a = 0$, then the indeterminant becomes $\frac{a}{a}$. If one didn't know what *a* wasthis figure would likely be defined as equaling 1 so long as it did not equal 0. Try to keep an open mind about this. If 0 represents a set of nothingness or *emptiness,* then dividing it into the same size whole would be one; the division process leaves the set unchanged.

Even if we held that to be true another reason the indeterminate is not accepted is becausemore than one value applies to the ratio. Remember that 0 is positive and negative at the same time. This means it is possible for either the numerator or the denominator to have the opposite sign of the other. Hence it also equals -1.Additionally we can say this operation asks how many times can we place nothing into a space holding nothing—an infinite number of times. Since the only real mathematical object which can equal infinity is a number line, the presence of an infinite will allow the substitution of a new variable representing a subspace within the system defined by an equation. We define the expression as$\bigoplus 1 = \frac{0}{0} = \infty$.Think about it. If an infinite indicates that an axis is not used because the value cannot be reached on its range, then the only thing which can equal infinity is an entirely new number line which by its very nature contains an infinite set of numbers.

The Indeterminate Form does not equal one or the other of these answers. Instead it equals all of them at the same time. Though 1 clearly does not equal infinity this not a logic issue but rather a manifestation of two separate ways of interpreting what this specific division process describes.It's not surprising the term is considered undefined in modern mathematics as it does cause mathematical absurdities in most situations. However when used properly the absurdities disappear.

The Indeterminate Form is ideal for making a mathematical transformation into the subspace form of an equation. When we convert to a subspace form we are exchanging a variable of one axis for another. Since an infinite in an equation indicates that an axis listed in an equation is not used, we must multiply that side of the equation by a term to cancel the infinite and replace it with a real number. The other side of the equation will be multiplied by the same term but will reinsert an infinite value in the form of a new subspace axis.

So, whether or not an infinite value will be generated in an equation, we can use the infinite solution of the indeterminate form to remove a given variable and replace it with its subspace term but for a singular exception. I'll cover that in just a second. The variable being removed is always replaced with $\oplus\, 1$ after conversion. The opposite side of the equation which contains the primary input value will always take the new variable. The exception occurs when solving for the subspace of the primary input. Subspace terms will be added and signs of terms will be adjusted but nothing will be removed. The reason is that subspace represent dimensional time which is present for all variables.

This process will fully represent all values of the Indeterminate form. The following statements will collectively provide a definition for division by zero and provide the transformation equations for subspace terms.

Lemma 1:

A subspace is a number line axis paired with but orthogonal to the given defined axis used to plot point positions in an equation. It exists simultaneously with other defined axis, implied by, but not included in the original equation.

Lemma 2:

Any value divided by 0 will result in an infinite value. The only term which can be used in mathematical operations and still represent an infinite value is a new number line, represented by a subspace term.

Lemma 3:

Any term divided by zero is an improper expression, which must be rewritten into the subspace form it implies to determine the output value of the original division expression.

Lemma 4:

Transformation between normal space in a defined equation and the subspace(s) it implies is achieved by multiplying both sides of the equation by $\frac{0}{0}$ such that:

Normal Space Value	=	Indeterminate	=	Subspace term
$\oplus\, 1$	$=$	$\frac{0}{0}$	$=$	∞

Lemma 5:

Information must be conserved in during a transformation so as to not destroy the primary input function. Values for the indeterminate on each side of the equal sign must support this consideration.

<u>Subspace Transformation Equations:</u>

There is a difference between time as a direction and time as a dimension. Time is specified as*T = Dimensional Time*—indicating a presence and flow of time, and *t = Directional Time*—indicatinglength of time and when events occur.

The transformation equations solve for the quality represented in the Time Dimension and will not remove values present in a given equation. The Time Direction *t* is provide by a parametric substitution of*t = x*.

Equation 1a: An expression of the form $y = \frac{a}{x}$ where *a*equals any non-zero constant, and x is a variable, can be rewritten into its subspace form by a.)Multiplying *y* by $\frac{0}{0} = \oplus 1$ and b.)factoring out the constant, before multiplying $a \cdot \left(\frac{1}{x}\right)$ by $\frac{0}{0} = \infty = New\ Subspace\ Term.$

$$\left(\frac{0}{0}\right)y = a \cdot \left(\frac{1}{x}\right)\left(\frac{0}{0}\right) \qquad \left(\frac{0}{0}\right)y = a \cdot \left(\frac{v}{x}\right) \qquad \oplus 1 = \frac{av}{x} \qquad av = \oplus x \qquad v = \oplus \frac{x}{a}$$

Real output values exist in y, but become undefined when x equals 0. At this value the output exists only in the v-axis but is still a real number value; the y-axis is just not used at this value. Thus we are replacing the y-axis with its subspace term. That side, y-axis, must be multiplied by $\oplus 1$ and the new variable introduced on the opposite side.

Equation 1b: The reversal of the process will solve for the *x*-subspace of Dimensional Time. Since *T* is a subspace of *x* the *y* component will not be removed. The indeterminate is set to equal the new variable of *T* on that side. The opposite side of the equation contains the primary input function and cannot be destroyed. Thus the indeterminate will set to equal $\oplus 1$ and then distributed through. This will be the case for any equation when solving for the dimensional time subspace. As you'll see it will always have the same value.

An expression of the form $y = \frac{a}{x}$ where *a* equals any non-zero constant, and x is a variable, can be rewritten to solve for the time subspace by a.)multiplying*y* by $\frac{0}{0} = T$ and b.) factoring out the constant, and multiplying $a \cdot \left(\frac{1}{x}\right)$ by $\frac{0}{0} = \oplus 1$.

$\left(\frac{0}{0}\right) y = a \cdot \left(\frac{1}{x}\right)\left(\frac{0}{0}\right)$ $\quad T \cdot y = \oplus \frac{a}{x}$ $\quad T = \oplus \frac{\frac{a}{x}}{y}$ $\quad T = \oplus \frac{\frac{a}{x}}{\frac{a}{x}}$ $\quad T = \oplus \frac{a}{x}\left(\frac{x}{a}\right)$ $\quad T = \oplus 1$

Equation 2a: An expression of the form $y = \frac{a}{x}$ where *a* equals 0, and x is a variable, can be rewritten into its subspace form bya.)multiplying*y* by $\frac{0}{0} = \infty = v$, b.)factoring out the constant 0 before multiplying $\frac{1}{x}$ by $\frac{0}{0} = \oplus 1$.

$\left(\frac{0}{0}\right) y = 0 \cdot \left(\frac{1}{x}\right)\left(\frac{0}{0}\right)$ $\quad \frac{0}{0} = 0\left(\oplus \frac{1}{x}\right)$ $\quad v = 0$ For all *x*

Equation 2b: Solving for the subspace of time will be forced to use the *y* component. The right side of the equation will equal 0 for all x. The *y* component will be multiplied by $\frac{0}{0} = \infty = T$. The right half of the equation will be multiplied by $\frac{0}{0} = \oplus 1$.

$\left(\frac{0}{0}\right) y = 0 \cdot \left(\frac{1}{x}\right)\left(\frac{0}{0}\right)$ $\quad T \cdot y = 0\left(\oplus \frac{1}{x}\right)$ $\quad T = \frac{\frac{0}{x}}{y}$ $\quad T = \frac{\frac{0}{x}}{\frac{0}{x}}$ $\quad T = \frac{0}{x}\left(\frac{x}{0}\right)$

$T = \frac{0}{0} = \oplus 1$

Remember this is the dimensional component of Time which contains an entire number line in its own right. So it's already infinite. The only sense of the indeterminate which can be used then is this.

Equation 3a: An expression of the form $y = 0x$ can be rewritten into its subspace form by a). *y* will not become infinite for any value of x and will be directly converted to *v* and b.) multiply the right side of the equation by $\frac{0}{0} = \oplus 1$.

$\left(\frac{0}{0}\right) y = 0 \cdot x \cdot \left(\frac{0}{0}\right)$ $\quad v = 0 \cdot \oplus x$ $\quad v = 0$ For all *x*

Equation 3b: The time subspace is again solved by a reversal of the operation and inclusion of the *y* component. Make the conversion by a.) multiplying the *y* component by $\frac{0}{0} = \infty = T$ and b.) multiplying the right half of the equation by $\frac{0}{0} = \oplus 1$.

$$\left(\frac{0}{0}\right) y = 0 \cdot x \cdot \left(\frac{0}{0}\right) \qquad T \cdot y = 0 \cdot \oplus x \qquad T = \frac{0}{0x} \qquad T = \frac{0}{0} = \oplus 1$$

Equation 4a: An expression of the form $= a \cdot x$, where a is any non-zero constant, can be rewritten into its subspace form by a.) distributing *y* through $\frac{0}{0}$ and setting it to equal $\oplus 1$ and, b.) multiplying *ax* by $\frac{0}{0} = \infty = v$.

$$\left(\frac{0}{0}\right) y = a \cdot x \cdot \left(\frac{0}{0}\right) \qquad \oplus 1 = axv \qquad v = \oplus \frac{1}{ax}$$

Equation 4b: The Dimensional Subspace of Time is solved by a direct reversals of Indeterminate values just like before.

$$\left(\frac{0}{0}\right) y = a \cdot x \cdot \left(\frac{0}{0}\right) \qquad T \cdot y = \oplus a \cdot x \qquad T = \oplus \frac{ax}{y} \qquad T = \oplus \frac{ax}{ax} \qquad T = \oplus 1$$

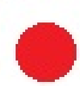
Zero division has not been understood in the past because the output must be expressed in a subspace axis implied by but not defined in the original equation. If this information is not included when an equation implies it must be, it will generate infinites or other mathematical nonsense. Until now with the advance of theoretical mathematics like String Theory which calls for the inclusion of extra dimensions the assumption of the existence of a higher number of space directions would have seemed crazy at best. They are however necessary to solve division by zero.

Saying something equals infinity implies this because it is equivalent to saying the output value cannot exist on that axis anywhere. If we assume every expression has a real output value, then $\frac{x}{0}$ is an improper expression of a subspace term and requires us to accept the presence of extra directions and dimensions which are not directly perceivable to us but needed to provide a proper solution. As can be seen from the subspace transformation equations the subspace directions are dependent upon the x-input value. It would be foolish to assume the subspace outputs would be undefined just because *y* was. The inclusion of the subspace terms through the transformation equations produces usable equation and real answers but how do we know these subspace directions exist? To see how the idea of subspace is developed, and how it must be defined requires a close examination of how we define space and points within it. The following section details this process.

Developing Subspace—Hidden Directions and Dimensions: Before moving forward let's make a clear distinction between two terms—direction and dimension. For the purpose of this article, a direction will describe a number line axis. A dimension will be the space described by any given number of directions which, either alone or in combination, are used to plot point positions. Directions will be denoted by d^{α} where α is the total number of directions in a system. Dimensions will be denoted by D^{α} where α is the total number of space dimensions coexisting within that reference frame. R_m will denote space where *m* is the total number of *perceived* space directions.

R_0 The Point: Zero Space means they are no space directions. Figure 1 at right is a singular point; a dot. Though we may draw a point to represent this and even label it, it cannot convey the reality of the point's existence. Since there is no space for it to exist in, it is infinitely small. All space direction is the same and

Figure 1:



therefore meaningless. The only direction which applies is time. From the perspective of the point there is no space but it can still measured in time and motion in a limited sense. We can measure how long the point exists. If it has spin or any kind of energetic flux we can measure its period from one moment to the next. Both perspectives would allow us to define the point as a function of time. It also means *time* is not only a direction but a dimension on its own. Both concepts must be included in a subspace form. Equations take the form of $P_1 = t$ for directional time (length of time) and $T = \frac{0}{0} = \oplus\, 1$ for dimensional time (presence and flow of time).

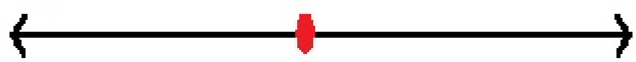
The points length of existence is measured through time. It does not exist at t = 0 because measurement had not yet begun; for the point time does not exist there.. Both statements are equivalent. P_1 may be replaced by a frequency equation or any other aspect the point is determined to have per unit of time.

R_1 The Line: By expanding a point out to infinity in one direction we arrive at one-space. This is best represented by a line; a one dimensional realm with no up, down, left or right. Only forward and backward exist on this line.

Figure 2

All points are identical and the line infinite, preventing measurement from anything. Without something else for a reference point there is literally nothing there. To illustrate this I'll add a single point to the line shown at right in figure 3.

Figure 3



No matter where I place the point no information can be conveyed. There is still nothing else within this domain. Sure I know the point is there but how do I define its location aside from the length of its existence?

In our 3-directional world I may actually label the line with a numerical scale allowing measurement. But from the perspective of the point, it's not possible. This is 1—space. There is only one space direction. That means there is no other direction I can use to mark the graph indicating the point's position. So from the perspective of anything on the line no numerical scale can be created aside from the earlier idea of time. As an example, imagine the dot on the line is the value x = 4. The graph cannot be drawn because there is no Y axis. Because it does not exist the point can be drawn anywhere and the information it conveys is identical to all other points.

Without at least one other direction present those values and positions are not capable of being understood as part of space because there is nothing to define them by. Time is still present and can be used for measurement. This makes Time a subspace of R_1 both directionally and dimensionally. On the line time can be perceived even from the perspective of a singular point dimensionally as how long it exists even without a know definable location, and directionally as the difference in how long it takes for it to move from one position to another. Lets add another point to the line to further develop this idea. See figure 4 at right.

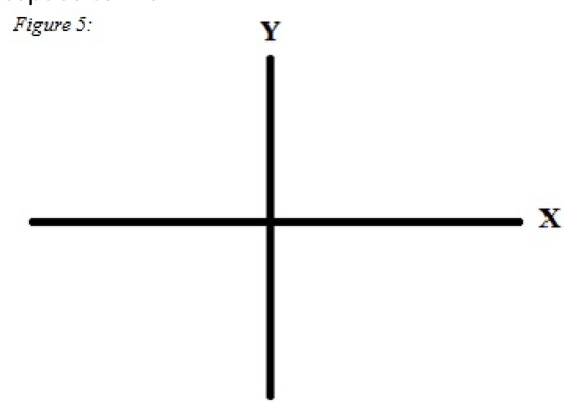
Figure 4

With the addition of even one more point the problem begins to change. These two points can be the same body at two different times or two separate bodies. Though space to the left of the blue point and to the right of the red point is infinite the space between them is finite. That makes it measurable to us 3-directional humans, and at least perceivable from the 1-directional point of view as a function of time where equations will take the form of $x = f(t)$.

If we were to record the positions of any point as it moved back and forth according to the rules of some defining equation we could develop a map of where it was in relation to any and all previous locations based upon the time it took to get to another. The first recorded location of the point is labeled the origin. Positions to the left are negative and to the right are positive. The 1-Space position equation is then a function of time input and distance from the origin as an output. A single line will have one perceived space direction (R_1), and actual space direction (d^1: the $x - axis$), one space dimension (D^1: The line) with the additional of the single subspace of *time (t)* suspended within a temporal dimension *T*.

We can draw the conclusion from the expansion to a line from a point that space is an emergent property of time. It allows all space dimensions to be definable even alone in a single 1-Space. No form of space can exist without this aspect of time, a fact which complicates issues when considering anR_n where $n > 1$ as each perceived direction will have its own organizational and thereby temporal orientation.

There can be only one *time* dimension for a system. That dimension is shared by all directional axis as they are all emergent from it. The primary input for an equation will provide the aspect directional time for all other axis through parametric substitution. However all directional axis an time aspect identical to that described in the R_1 example of the line. Since this aspect of time is a direction it must be defined as a space direction for all subsequent subspace terms.



R_2 The Plane: Figure 5 at right depicts the standard Cartesian plane with *x* and *y*-axis. They share the same aspects of time, both directionally and dimensionally. The addition of the *y*-axis also brings a subspace. Since time is used is already accounted for the new subspace will form a third directional axis in R_2. We will label this axis *v*.

Equations within the plane are of lines and take the form of $y = f(x)$. We have all used the Cartesian Plane and have never noticed the presence of a *v*-axis. So where is it? The answer lies in how we align one axis to another. The x and y axis are displayed orthogonal to each other. They have to be. If we take the dot product of any two corresponding vectors they will equal zero, making them orthogonal.

$x \cdot y = \langle x_n , 0\rangle \cdot \langle 0 , y_n\rangle$ 0= $[(x_n \cdot 0) + (0 \cdot y_n)]$

In other words both the x and y axis are turned 90° to each other. The *v*-axis will be orthogonal to both *x* and *y* axis. From the transformation equations above we see that each *v* will be a function of *x* such that $v = f(x)$. An *x* axis input and a *v* axis output means we can take the dot product of both and see if they are orthogonal.

$x \cdot v = \langle x_n , 0\rangle \cdot \langle 0 , v_n\rangle$ 0= $[(x_n \cdot 0) + (0 \cdot v_n)]$

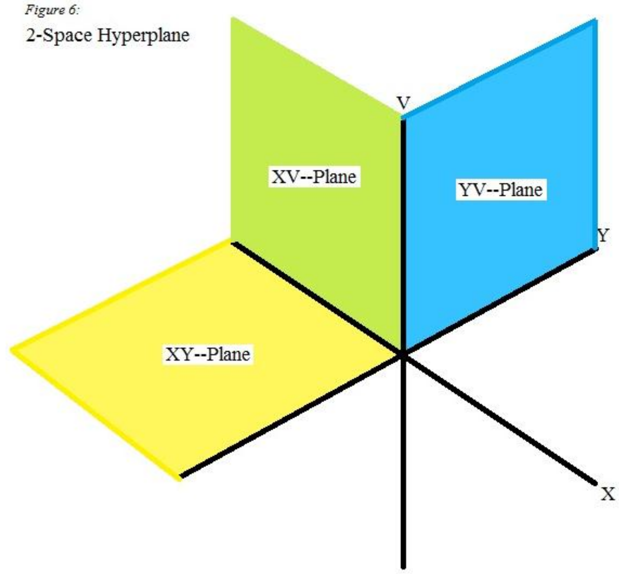
Clearly they are both orthogonal. If we use equation 1a from the transformations and set $a = 1$ then we will have $y = \frac{1}{x}$. The transformation equation will provide that $v = \oplus\, x$ for this subspace. *v* does equal +*x*. If we make that substitution we get $y = f(v)$=$\frac{1}{v}$. Though *v* is an output of a function of *x*, this substitution introduces those values as an input of for $y = f(v)$, relating the two axis to each other. As an input the *v* axis vectors will take the form of $\langle x_v , 0\rangle$. The dot product of these two axis will be 0 making them orthogonal.

$v \cdot y = \langle v_n , 0\rangle \cdot \langle 0 , y_n\rangle$ 0= $[(v_n \cdot 0) + (0 \cdot y_n)]$

Thus we have the following. The y-directional axis is dependent upon the x input value, which is itself dependent upon time. The other subspace direction axis is also dependent upon x but is a subspace of the*y*-axis. It is not seen because this is still 2-Space, not 3-Space. This direction is orthogonal to the plane and viewed edge on looking down on the Cartesian Plane. For every *x* input there is a *y* output in the *xy*-plane and a *v* output in the orthogonal *xv* plane. This means there are an infinite number of these subplanes which make up this surface, which is better described as a hyperplane. It also means that any of the space directions can be used as the time direction and allow *t* to be redefined as a space direction if desired. Figure 6 shows the 2-space hyperplane with its three space dimensions. The following statements will be true.

Figure 6:

2-Space Hyperplane



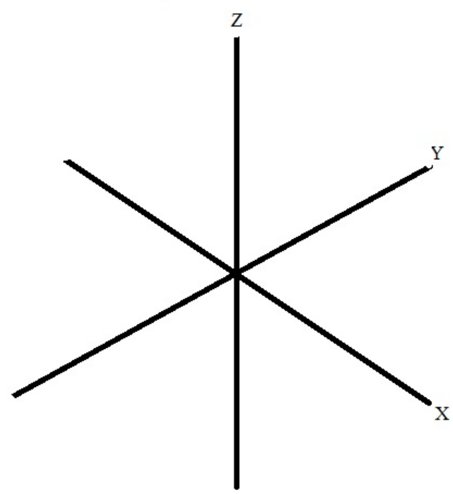
Lemma 6: *y* is dependent upon *x*
Lemma 7: *x* is dependent on *t*
Lemma 8: *t* is a subspace of *x*
Lemma 9: *v* is a subspace of *y*
Lemma 10: *v* must also be dependent upon *x*.

The presence of subspace directions can be assumed for any system in which there is a perceived direction and flow of time. That means all equations will have them. All axis, normal and subspace, will be orthogonal to each other. By acknowledging the inclusion of subspace in any environment it is clear that all perceivable space is actually a hyperspace. In summation a plane is actually a hyper plane and has two perceived space directions (R_2), three total space directions (d^3: x-axis, y-axis and v-axis) three total space dimensions (D^3: XY-plane, XV-plane and YV-plane), directional time (*t*)and dimensional time (*T*).

<u>R_3 The Volume:</u> Any volume will have three perceivable directions and is a direct expansion of a plane orthogonal to its two perceived directions. Figure 7 depicts the standard 3-space volume. The Volume like the plane will share all aspects of time with all of its directional components. The *X*-input will carry the subspace of directional time. The *y*-axis carries the subspace of *v* which has already been covered. The *z*-axis will also bring a subspace direction which will be labeled *w*.

Figure 7:

Standard 3-Space Volume



A volume is actually a hyper volume with five space directions. It cannot be portrayed accurately on paper and will not be shown here due to the large number of subvolume spaces which exist within it. A later example will make clear why and provide a partial view of the hypervolume's form. In summary any volume is a hypervolume having three perceived space directions (R_3) and five total space directions (d^5). As will be shown shortly it has ten total space dimensions (D^{10}), plus directional time *t* and Dimensional time *T*.

Equations are of planes and take the form of $z = f(y) + f(x)$. All directional axis *x*, *y*, *z*, *v* and *w* are all orthogonal to each other. From this we can make the following assumptions.

Lemma 11: *z* is a function of *x* and/or *y*
Lemma 12: *z* is dependent on *x* and/or *y*
Lemma 13: *w* is a subspace of *z*
Lemma 14: *w* is dependent on x and/or *y*

We can also produce the following theorem to calculate the number of subspace axis which will be present in any given normal space.

Theorem 1:

Let d equal the total number of space directions in any hyperspace. Where the number of perceived space directions is m, the number of subspaces is n and $n = m - 1$. *There will be a total of d = m + n space directions in the hyperspace.*

<u>Subspace Properties:</u> The following properties will be true for 2—Space, and 3—Space. No higher examples will be given here due to increasing complexity. These properties can be expanded to any coordinate system to define values previously thought undefinable.

<u>2—Space</u>: Two space will have two extra directions; one in space and the other time ***t***. The subspace will be labeled ***v***. ***v*** is a subspace of *y* but we can also say *y* is a subspace of ***v***. The parameters for 2-Space will be R_2 d^3 D^3: $t = time$ $d_1 = x$ $d_2 = y$ $d_3 = v$.

The Cartesian **XY-Plane** exists within a higher Hyperplane, just as described in the previous pages. The ***XY*** plane is one of three subplane dimensions existing within the Hyperplane. These three subplanes form the basis for the 3-directional space in which we live, a world suspended within a higher Hypervolume. The Hyperplane allows the single **x** and **y** directions to exist independent of each other by combination with ***v***.

Any two subplanes share only one space direction, which isolates that subplane to its own space preventing interaction between objects existing in opposing planes. For every instant of time the subplane dimensions which exist then in the R_2 Hyperplane are D_1: (x , y) , D_2: (x ,***v***) , and D_3: (***v*** , y). All three of these instances will exist for any given form of 2-space at the same time. Occasionally we will see asymptotes, holes or other forms of discontinuity in graphs.

At these points output values are undefined in the primary equation but have real values in an alternate subplane. This will be shown in examples momentarily. It must be this way. If the understanding of division is correct then there must be an output for all inputs to any equation. *If the output is not defined in one subplane it will be in another. Also it is possible for a point to have defined values in all three dimensions of the hyperplane at the same time.* Notation for points will take the form of: $P_n = (x , y|v)$.

<u>3—Space</u>: Three space will have two shared subspace directions with the three primary directions. This results in 10 total sub-volume dimensions (plus one for time). With m = 3, the total number of subspaces will be defined by *d* = 3 + (3 – 1) = 5. We will label the extra subspace directions ***w*** and ***v.*** The Parameters for 3-Space will be R_3 d^5 D^{10}: $d_1 = x$ $d_2 = y$ $d_3 = z$ $d_4 = v$ $d_5 = w$ $time = t$.

Interaction between objects within any subvolume sharing any two directions with any other subvolume will be indirectly perceivable. Objects may seem to have interacted with an unforeseen agent or spontaneously leap in or out of existence as they migrate between shared subvolumes. Interactions of this nature may not be noticed by beings living in a 3-directional world and will likely be confined to subatomic scales.

Three Space is (x , y, z) with subspace directions ***w*** and ***v*** and directional time ***t.*** The five space directions combine to form 10 Dimensions in one Hypervolume:

D1: (x , y, z)	***w*** *and* ***v*** *are present but output values are not defined in their axis.* This is normal 3—space.
D2: (x ,***w*** ,***v***)	X in combination with w and v. Y and Z are undefined.
D3: (y, ***w*** ,***v***)	y in combination with w and v. X and Z are undefined.

D4: (z ,***w*** , ***v***) z in combination with w and v. X and Y are undefined.
D5: (***w*** , y , z) w in combination with y and z. X and V are Undefined.
D6: (x ,***w*** , z) w in combination with x and z. Y and V are Undefined.
D7: (x , y , ***w***) w in combination with x and y. Z and V are Undefined.
D8: (***v*** , y , z) v in combination with y and z. X and W are Undefined.
D9: (x ,***v*** , z) v in combination with x and z. Y and W are Undefined.
D10 (x , y , ***v***) v in combination with x and y. Z and W are Undefined.

It is possible that a given point will have defined values in all parameters. In that instance some part of it will exist in all ten listed dimensions. Notation for points will take the form of: $P_n = (x\,, y|v\,, z|w)$.

Example 1: Evaluate $y = \frac{1}{x}$, with a domain of $X: [-4\,, 4]$ within the Hyperplane.

Solution: 1.) Begin evaluating the equation by labeling all the parameters it contains; those listed and implied. Dimensional Time will possess a flow value of $\oplus 1$. Its presence will be seen in the calculations but may be ignored. Directional Time will be included through parametric substitution with $t = x$. We have two directional space coordinates defining $(x\,, y)$ points. We know the total number of Subspaces will be defined by *d* = *m* + *n,* where *m* = 2 and *n* = 2 – 1. Thus $d = 3$.

The equation $y = \frac{1}{x}$ represents three space directions within a Hyperplane which combine to form three two-directional subplane dimensions in R_2, plus *t* = time. The Hyperplane parameters are listed below R_2, d^3 , D^3 and $t = time$. The hyperplane directions are, d_1: X—the domain, horizontal axis, and input value, d_2: Y—the primary range, vertical axis, primary output value, d_3:V—secondary range, orthogonal axis to X and Y, secondary output value. These combine forming hyperplane dimension subplanes D_1: (x , y), D_2: (x ,***v***) and D_3: (***v*** , y) with a fourth temporal dimension of D4: Time.

2.) Next set up a graphic representation of the R_2 hyperplane. These three directions, though in 2—space, are the basis for the three perceivable directions in 3—space. This allows the graphic representation to be a true depiction of the hyperplane.

3.) When x = 0, y becomes undefined. This is represented by the presence of asymptotes at that x value. The Orthogonal V-axis will have definable output values at x = 0 and all other values of x. The chart below will provide a general solution and specific solution of the subspace equation.

4.)

$y = \frac{1}{x}$ *General Solution*	$y = \frac{1}{0}$ *Specific Solution*
The clue to the solution comes from the asymptote. The solution $y = \infty$ at x = 0 means that the graph in the Cartesian plane has completely left the y axis. Where did it go? The V-axis. So the presence of an ∞ indicates that a subspace is being used at a given value and can be used to convert equations. So let's multiply both sides by the Indeterminate Value $\frac{0}{0}$. This figure has several interpretations which will be valuable to this conversion.	The first step here is identical to the one taken for the general solution. The output value will be undefined as the right side of the equation has no meaning in the standard Cartesian plane. Both sides must be multiplied by the indeterminate, $\frac{0}{0}$.
$\left(\frac{0}{0}\right) y = \frac{1}{x}\left(\frac{0}{0}\right)$ Values for the subspace output of V are dependent upon x just like y. Since the Y value become	$\left(\frac{0}{0}\right) y = \frac{1}{0}\left(\frac{0}{0}\right)$ The steps are the same as those at left but will provide a specific solution at *x* = 0. Distribute the

infinite it must be removed from this equation and related to a real number. To do that distribute the *y* into the indeterminate to remove it. Then set the left side of the equation to equal $\oplus 1$. The right side of the equation has to remove the X value from the denominator. The indeterminate here will convert it to the ∞ before distributing. The infinite represents the subspace term not described in the original equation.	*y,* and set the left side equal to $\oplus 1$. The indeterminate on the right will convert to *v*. and multiply across.
$\left(\frac{0}{0}\right) y = \frac{1}{x}\left(\frac{0}{0}\right)$ $\frac{0}{0} = \frac{1}{x}(\infty)$ $\oplus 1 = \frac{1}{x}(v)$ The X value must now be multiplied out of the denominator on the right side of the equation. $\oplus x = v$ This is the subspace form of the original equation. When x = 0, though y is undefined, v is defined and has a real value. We will express this value with the point notation provided earlier. $(x\ , y\|v)$ At x = 0, we have, $(0\ , \nexists\|0)$ $\nexists$ meaning "*there does not exist*"	$\frac{0}{0} = \frac{1}{0}(v)$ $\oplus 1 = \frac{v}{0}$ The last remaining step is to remove the zero from the denominator by multiplying it out to solve the expression. When *x =0* $v = 0$ This is the same point defined in the general solution at left. $(0\ , \nexists\|0)$ *This point is the 2—space hyperorigin.*

5.) Now we have two equations. Both are dependent upon the *x* input, and though it was never written the second was implied by the first equation: $y = \frac{1}{x}$ and $v = \oplus x$. The second portion of this example asks that we evaluate this single function (*because it really is one function*) over the Domain from X = -4, to +4. Let's first label and chart all values within this domain.

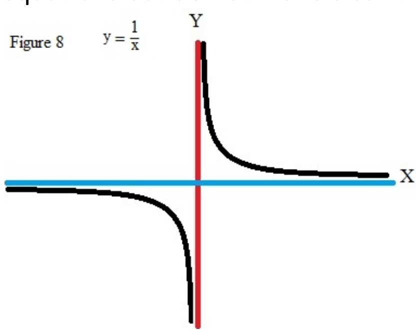
The new V equation will separate each (x , y) point within a separate (x , v) subplane per unit of time. The *v* axis effectively permits us to label events of past, present or future as separate, and permanent places in space which can be revisited should the graph return to that location at some later value of *t*.

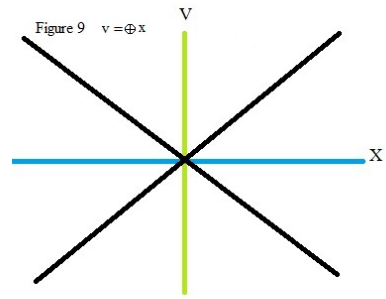
Since $v = \oplus x$ there will be a forward and reverse temporal echo for each input, respectively representing the forward and reverse flow of dimensional time*T*. Yet each echo will preserve the order in which events occur preventing problems with causality. Thus the positive or negative side of *v* may be discarded if desired without loss of information.

t	x	y	v
-4	-4	$-\frac{1}{4}$	$\oplus 4$
-3	-3	$-\frac{1}{3}$	$\oplus 3$
-2	-2	$-\frac{1}{2}$	$\oplus 2$
-1	-1	-1	$\oplus 1$
0	0	$\nexists$	0
1	1	1	$\oplus 1$

2	2	$\frac{1}{2}$	$\oplus 2$
3	3	$\frac{1}{3}$	$\oplus 3$
4	4	$\frac{1}{4}$	$\oplus 4$

The patter here shows that regardless of the direction of Time, forward or backward the order in which events will take place is identical. One is simply a mirror of the other. At X = 0, Y is undefined but a definable output exits at the hyperorigin. Figure 8 and 9 bellow show the separate graphs of the two equations but do not make clear the significance of this information.





Their Combination into a Hyperplane graph is quite different. Observe Figures 10 and 11.

Figure 10:

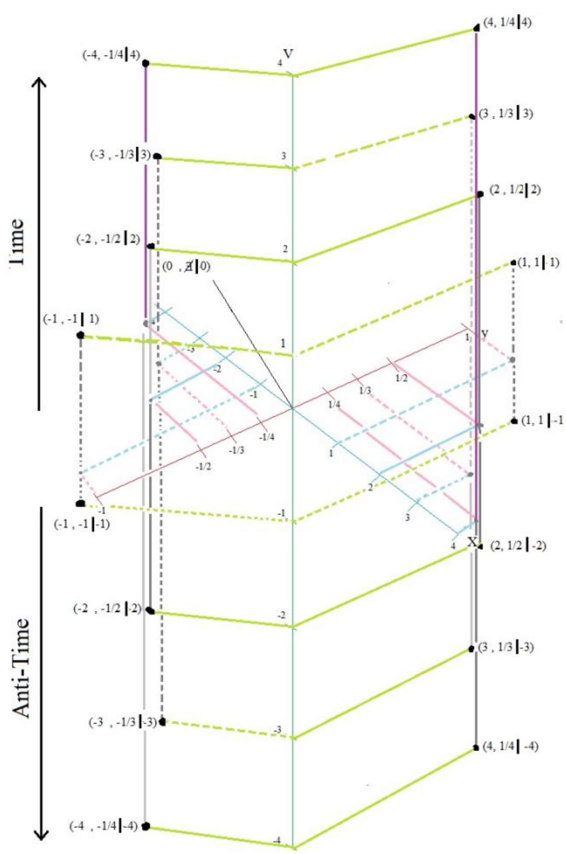
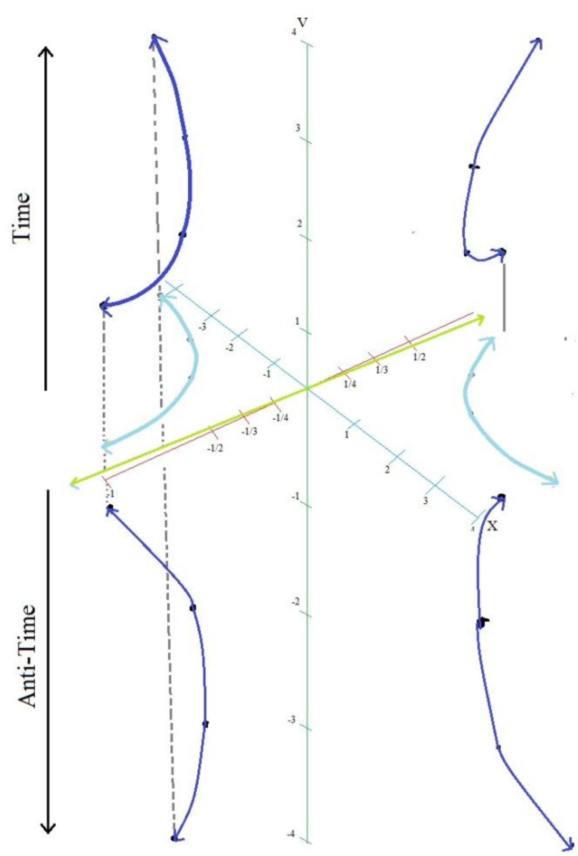


Figure 11:



The graphs show several key features discussed in the previous material. I'll review that information here. The graph on the left shows gird lines with labels for all points. The graph on the right is the same graph but without the labels and gridlines.

1.) Remember this is all 2—space, not 3—space. This means that for every instant in time, the X and Y points in the graph exist in within separate subplanes of V.

2.) The + and – subplanes of V have identical point positions for each instant of directional time, but represent opposite directions of dimensional time. This indicates that time is its own opposite and causality is preserved within it. No matter which way we move through the time dimension events will occur in proper order of perception. A reversal would only appear to occur from the perspective of a point on a graph which returns to an earlier value of *v*. Yet the math would provide a separate and later value of *t ,* which will remain positive in directional time for that location in space. Clearly it is occurring at a later time, and though not literally in the past, that place is identical to it.

3.) Though each subplane of V is independent from every other for each instant of time, they do remain within a specific location in the hyperplane. This means that although the V2 Subplane t = 2, $\left(2\,,\frac{1}{2}|2\right)$is the past from the point of the V3 Subplane t = 3, $\left(3\,,\frac{1}{3}|3\right)$ it does still continues to exist. This means that Time, in aspects of Past and Future, defined as a separate locations within the hyperplane are not distinguishable from the present, from the point of view of a point which returns to this location at a later value of *t.*

4.) Some graphing utilities will provide a line connecting infinity and negative infinity on the graph of $y = \frac{1}{x}$. This is done to illustrate the two separate pictures are of one graph. After all the function is continues expect for one point; the infinite discontinuity where *x* = 0. The graph seems to leap to infinity, wrap around to negative infinity and then continue on. So this line is a visual aid. If we really could draw a line connecting these extremes as real number it's would be nearly ninety degrees along the y-axis and would have to pass through the origin; just what our visual aid shows.

The hyperorigin in 2-space coexists in the same spot as the Cartesian plane origin. It just uses the *v*-axis instead of the *y*-axis. So by plotting the points of the hyperplane equation on its graph one should actually be able to draw that line connecting the two infinite extremes and have it pass through the origin.

The Green line cutting through the XY plane in Figure 11 represents the connection of $-\infty$ with $+\infty$ when x = 0 in that plane of the graph. It passes through the hyper origin located at $(0\,,\nexists|0)$ and satisfies this hypothesis.

5.) Lastly, though mathematically the y-values do extend out toward positive and negative infinity, any such situation will likely have some real world limiting factor giving upper and lower bounds to all possible values of Y and V as time passes and x increases or decreases.

<u>Example 2:</u> Due to extreme complexity of any 3—space representations, full graphic depictions will not be provided. Given the equation $z = 2y + \frac{4}{x}$where $y = \frac{1}{x}$, fully evaluate the system of equations with a domain of $X{:}\,[-4\,,4]$ within a hypervolume.

<u>Solution:</u> We'll begin evaluating this problem the same way as in the previous example. 1.) This time there are three directions defined within the equation; $(x\,,y\,,z)$. We know the total number of Subspaces will be given by *d = m + n*. *m = 3 so n* = 3 – 1. This gives us *d = m + n* = 2 + 3 = 5. The Parameters for the equation will be R_3 , d^5 and D^{10}.

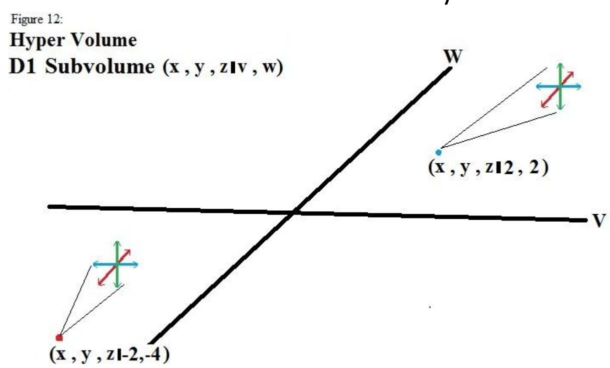
2.) Any given an equation in 3—space, though it holds only three variables for any subvolume dimension, actually represents five space directions within a hypervolume composed of 10 subvolume dimensions plus *t* = time. They are labeled d_1: X—the domain, horizontal axis, primary input, dependant on *t,* d_2: Y—1st range, lateral axis, 1st primary output value and orthogonal to *x,* d_3: Z—2nd range, vertical axis, orthogonal to X and Y axis, 2nd primary output. The remaining directions will be a bit confusing because there is no way we can visualize what they really look like. d_4: V— Y subspace direction, orthogonal to X, Y, and Z, secondary output dependant on X, d_5: W— Z subspace direction, orthogonal to X, Y, V and Z, secondary output dependant on X and/or Y.

Hypervolume consists of the following Subvolume-Dimensions:

D1: (x , y, z) D2: (x , ***w*** ,***v***) D3: (y, ***w*** , ***v***) D4: (z , ***w*** , ***v***) D5: (***w*** , y , z)
D6: (x , ***w*** , z) D7: (x , y , ***w***) D8: (***v*** , y , z) D9: (x , ***v*** , z) D10 (x , y , ***v***)
D11 : Time

3.) The Graph of the Hypervolume….. This isn't easy. In the 2-space example we already exist in a higher 3-space universe which allows us to easily and accurately view the Hyperplane in its entirety. Just like its three space directions, though trapped in 2-space, formed the basis for our three directional existence so too will the 3-space hypervolume form the basis for the existence of higher 5-directional hyperspace. That means to accurately portray what this looks like we would have to literally exist within a universe of 5 perceivable space directions. So we will use a simplification.

Figure 12 at right is one possible way to represent a portion of the hypervolume. This graph is labeled to show this is the D1 subvolume. The ***v*** and ***w*** coordinates have an infinite number of possible pairings, just like a regular Cartesian plane does. The difference in the notation is to denote a dimension instead of a point. Within the D1 subvolume, v and w outputs would not be included in an equation requiring transformation to determine their values.



In Figure 12 each point indicates a separate three directional subvolume of $(x\ , y\ , z)$ coordinates suspended with a subspace defined by a given (v , w) point. There are nine more dimensions with similar properties which would have to be considered to depict the entire hypervolume. The graph above shows two separate points within the D1-Subvolume. This information must be indicated in any graph or set of equations to make clear which subvolume is being dealt with, and where at points are being plotted within that dimension. We could mark the same points values any of the ten dimensions but if it appears in the arrangement of $(\nexists\ , y|v\ , z|\nexists)$ the point would be entirely in the D8 Subvolume where *x* and *w* may be undefined. If all five coordinates have defined values it means the point generated by a given equation will exist in all 10 subvolume dimensions.

3.) In both equations, $z = 2y + \frac{4}{x}$ and $y = \frac{1}{x}$ we know that y, and therefore z as well will become undefined. So we definitely have values within the ***v*** and ***w*** axis of the hypervolume.

$z = 2y + \dfrac{4}{x}$	$y = \dfrac{1}{x}$
Although we will end up with both Y and Z being undefined it is Z which must be converted. The reason is the equation is clearly marked to solve for this value. Though Y is a variable here, an equation is provided which shows it's dependent on X, and therefore so is Z. Substitute the Y equation into the Z equation, multiply both sides by $\frac{0}{0}$ and solve for ***w.***	We'll begin this evaluation the same way we did for the 2-space example earlier. The evaluation will be for the V-axis equation. We know that $y = \infty$ at x = 0 and the output is totally in the V-axis. Like before, multiply both sides by the Indeterminate Value $\frac{0}{0}$.

$\left(\frac{0}{0}\right)z = \left(2y+\frac{4}{x}\right)\frac{0}{0}$ $\left(\frac{0}{0}\right)z = \left(\frac{2}{x}+\frac{4}{x}\right)\frac{0}{0}$ Distribute the Z value through and then set that side to equal $\oplus 1$. On the right side set the indeterminate to equal infinity, as W.	$\left(\frac{0}{0}\right)y = \frac{1}{x}\left(\frac{0}{0}\right)$ Like before we have to remove the X from the denominator and be able to multiply it on the left with a usable value. The left side of the equation will be $\oplus 1$ while the right will become the new variable.
$\oplus 1 = \left(\frac{6}{x}\right)\left(\frac{0}{0}\right)$ $\quad$ $\oplus 1 = \left(\frac{6w}{x}\right)$ Now multiply out the *x* and divide by 6. $w = \oplus\frac{1}{6}x$	$\left(\frac{0}{0}\right)y = \frac{1}{x}\left(\frac{0}{0}\right)$ $\quad$ $\frac{0}{0} = \frac{1}{x}(\infty)$ $\quad$ $\frac{0}{0} = \frac{1}{x}(v)$ $\oplus 1 = \frac{v}{x}$ $\quad$ $\oplus x = v$

4.) There are now 4 equations, and all are dependent upon the x-input. Two were not written but they were implied by the first two.

$$z = 2y + \frac{4}{x} \qquad y = \frac{1}{x} \qquad v = \oplus x \qquad w = \oplus\frac{1}{6}x$$

5.) We set X = t for directional time, and re-label all equations allowing us to evaluate the set from x = -4 to x = 4.

$t = x$	$y = \frac{1}{t}$	$z = \frac{6}{t}$	$v = \oplus t$	$w = \oplus\frac{1}{6}t$
-4	$-\frac{1}{4}$	$-\frac{3}{2}$	$\oplus 4$	$\oplus\frac{2}{3}$
-3	$-\frac{1}{3}$	-2	$\oplus 3$	$\oplus\frac{1}{2}$
-2	$-\frac{1}{2}$	-3	$\oplus 2$	$\oplus\frac{1}{3}$
-1	-1	-6	$\oplus 1$	$\oplus\frac{1}{6}$
0	$\nexists$	$\nexists$	$\oplus 0$	$\oplus 0$
1	1	6	$\oplus 1$	$\oplus\frac{1}{6}$
2	$\frac{1}{2}$	3	$\oplus 2$	$\oplus\frac{1}{3}$
3	$\frac{1}{3}$	2	$\oplus 3$	$\oplus\frac{1}{2}$
4	$\frac{1}{4}$	$\frac{3}{2}$	$\oplus 4$	$\oplus\frac{2}{3}$

From these values we see that we have two echoes of the hypervolume graph. One moving forward in time and the other backward with identical images, each preserving proper causality. For every instant in directional time there is a separate location within the various subvolumes which continues to exist containing the point defined by the equations for that instant. The positions continue to exist within the hypervolume despite the continual flow of dimensional time. And the hyperorigin for this set of equations exists entirely within the D2-Subvolume $(0\,, \nexists|0\,, \nexists|0)$.

<u>Other Uses for Zero-Division, The Negative Radical:</u> Occasionally we run into situations where an equation cannot be solved due to the presence of what is called an imaginary number. We depict these numbers as a multiple of $\sqrt{-1}$, often shortened to i.

This done to at least give a result by plotting points on a complex number line. All of this is done with the current understanding that no value exists, which when squared will equal -1. The same methods used to provide solutions to division by zero can also provide solutions for the negative radical. The indeterminate value does carry one possible interpretation of $\frac{0}{0} = \oplus 1$. If we square this figure we get the same value: $\left(\frac{0}{0}\right)^2 = \frac{0}{0} = \oplus 1$. Therefore the reverse is also unchanged: $\sqrt{\frac{0}{0}} = \frac{\sqrt{0}}{\sqrt{0}} = \frac{0}{0} = \oplus 1$. Thus the indeterminate value must be an answer to the expression $\sqrt{-1}$. As a proof, since $\frac{0}{0}$ also equals +1, it must also be a solution to $\sqrt{1}$. Well it is. $\sqrt{1} = 1 = \frac{0}{0}$. This gives us the next theorem:

Theorem 2: Any Complex number defined as a multiple of $\sqrt{-1}$ is an improper expression. It can be solved by $\sqrt{-1} = \frac{0}{0}$, where output values will exist within some definable subspace.

<u>Example 3:</u> Evaluate $y = x \pm 2i$. Show the equations normal 2-space equation and its subspace form.

<u>Solution:</u> The equation having the complex number *i* in it automatically implies its written incorrectly. Either i has to go, or the equation has to be rewritten in the form of *v* equals something. An equation like this is unique though in that the subspace term is not being generated to take care of an undefined infinite value or to replace another variable. Instead it is actually part of this equation. From Theorem 2 we can rewrite the equation: $y = x \pm 2i \quad \rightarrow \quad y = x \pm 2\sqrt{-1} \quad \rightarrow \quad y = x \pm 2\left(\frac{0}{0}\right)$. The indeterminate can equal several things simultaneously and all of them are implied whenever it is present. That is no less true when dealing with the negative radical.

The indeterminate can be +1 and -1. For the normal 2-space equation the solution must include both values. This will result in two separate equations for the *y*—equation. $y = x \pm 2(\oplus 1)$ $y = x \oplus 2$	The indeterminate can also be seen as an infinite implying the use of a subspace term. We let that equal *v*. The *y* output is still present and will be determined by the equations at left. $y = x \pm 2\left(\frac{0}{0}\right)$ $y = x \oplus 2v$ $v = \oplus \frac{y - x}{2}$
Because the $\oplus$ will produce a mirror echo the $\pm$ signs may be dropped and exchanged for either + or –. The two equations may be paired with each other in a sense; positive with positive, and negative with negative. $v = \frac{y-x}{2}$ *when* $y = x + 2$ and $v = \frac{y-x}{-2}$ *when* $y = x - 2$	

<u>Returning to Normal 2—Space, The XV-Plane Asymptotic Curve:</u> Just because an equation does not have an occurrence of a an asymptote does not mean it lacks output values within its implied subspace directions. It undoubtedly does and as you'll see something interesting happens in the conversion to those equations.

Consider the simple linear equation $y = x$. There are no values for which Y-axis outputs will be undefined. Let's see what happens with the conversion to subspace.

$$y = x \qquad \left(\frac{0}{0}\right) y = x \left(\frac{0}{0}\right) \qquad \oplus 1 = xv \qquad v = \oplus \frac{1}{x}$$

Earlier I said that the presence of an Infinite within an output of an equation indicates that values have moved completely into a subspace parameter. Since ***v*** is a subspace of ***y***, it should make perfect sense that the two are interchangeable; ***y*** is a subspace of ***v***. Whenever we have an equation which does not have the presence of an undefined value, it means the asymptotic curve exists in the subspace of the equation.

This shows we can always convert an equation into its subspace equivalent. It also means we can convert any subspace equation into the normal space form.

$$v = \oplus\frac{1}{x} \qquad \left(\frac{0}{0}\right)v = \oplus\frac{1}{x}\left(\frac{0}{0}\right)$$

The left side of the equation will be set to equal $\oplus 1$. The right portion of the equation set to equal infinity and therefore the subspace value of ***v***, which is ***y.*** Next multiply out the ***x***. The result of the division will yield a positive value. Since both the left side and the divided out x-value are positive and negative ($\oplus$) they can only produce a positive result. It will arrive at the original equation.

$$\left(\frac{0}{0}\right)v = \oplus\frac{1}{x}\left(\frac{0}{0}\right) \qquad \oplus 1 = \oplus\frac{y}{x} \qquad y = x$$

Final Thoughts: At a glance some graphs may appear to fail the vertical line test. However, since we focusing on subspaces, and time is a subspace of the primary input, it must be included. That means that all functions are functions of time. So though outputs may have the same position in space they don't at the same time. For every instant of time a point exists in a separate subspace. This allows the graphs having identical y-outputs for any given *x*-input can still pass the vertical line test and be a function provided those outputs occur for separate values of *t*.

It will be curious to see how the conversion to subspace and its very fluid perspective of time will relate with other forms of mathematics such a string theory, quantum physics, and quantum gravity. It will be a later goal of mine to consider such interests and develop theory upon them.

Conclusions: This article has shown that division by zero is not impossible and real number solutions are obtainable. It also shows us solutions to equations are more complex, existing within subspace dimensions based upon the number of perceivable space directions. Understanding this may not have been possible until now, where modern science, mathematics and physics see the universe having more the three space directions we normally use. We have explored how to convert into these subspace equations and return from them to original form. We have also seen that the subspaces defined in these equations exist independently of advance of the flow of time. With this more complete understanding of multiplication and division it is hoped we will be able to make further advancement in scientific and mathematical endeavors.