# Explaining the Mass of Leptons with Wave Structure Matter 

Jeff Yee<br>jeffsyee@gmail.com

First Draft: February 21, 2013

## Summary

The mass of leptons in the Standard Model has a strange sequence, increasing by orders of magnitude from $\sim 2.2 \mathrm{eV}$ for the lightest electron neutrino to 1.777 GeV for the sixth and largest lepton in the model, the tau electron.

This paper details an equation that identifies the sequence and calculation for the mass of the six leptons in the Standard Model. Surprisingly, when identifying the sequence, the magic numbers that were found in the Periodic Table of Elements for the atomic structure are also apparent in the sequence for leptons. As a result, a new table structure for lepton categorization in the Standard Model is proposed in this paper.

The creation of an equation to predict the lepton sequence and mass was derived from a new, Wave Matter Energy equation that is also proposed in this paper. The genesis of this equation is based on principles from Wave Structure Matter (WSM), proposed by Dr. Milo Wolff. In WSM, Wolff suggests that the electron is a standing wave of energy. One of the issues of WSM, however, is that it does not have an explanation for other particles found in the Standard Model, including the neutrino and other leptons. Perhaps the most important of these missing particles is the neutrino. If there is a fundamental particle that is responsible for wave generation, and responsible for creating other particles found in the Standard Model, the likeliest candidate would be the smallest and lightest particle - the neutrino.

This research began with an assumption that the neutrino was the fundamental particle and that the electron, and other leptons, are built from neutrinos. Another key assumption is that leptons are spherical standing waves, and that the radius of the particle's sphere is proportional to the amplitude of the wave. A combination of these assumptions, along with the proposed Wave Matter Energy equation, led to the formula that identifies the sequence and mass of leptons.

The aforementioned assumptions need to be carefully considered, as they are significantly different than the current Standard Model of particle physics. Thus, in addition to calculating the masses of leptons, this paper also attempts to explain the geometric structure of leptons, which leads to their unique sequence. Further, since a new Wave Matter Energy equation has been introduced, this paper details how it can be used to derive the fundamental energy equations: mass-energy equivalence, momentum-energy equivalence and Planck relation.

Finally, there is one test that is offered to validate this theory. A new neutrino appears in the proposed sequence. Discovery of a neutrino at the predicted mass could offer proof of the sequence and structure proposed for leptons in this paper.

## The Wave Energy Equation

As Milo Wolff identified in his paper, "Beyond the Point Particle - A Wave Structure for the Electron", the electron is proposed to be a pair of spherical outward and inward quantum waves. ${ }^{1}$ In the WSM model, the electron consists of standing waves of energy. It has a defined radius because standing waves eventually transition to traveling waves. The wave core is located at the center of the sphere. The mass of the particle can then be considered to be the total energy of the standing waves inside the sphere of the particle - for a particle at rest.

A particle emits and absorbs spherical, longitudinal waves. At rest, its inwaves and outwaves are equal in frequency. When in motion along any axis in one dimension, the frequency of its longitudinal inwaves and outwaves change, but it still maintains a standing wave structure. When in motion, the particle can create a secondary, transverse wave perpendicular to the axis of motion - particularly vibrating particles. These longitudinal waves and transverse waves become the heart of the Wave Energy Equation.

It may be helpful to have an analogy to visualize the wave energy of the particle. Imagine a balloon, under water in the middle of a pool, which is rapidly inflated and deflated repeatedly. The balloon will send spherical, longitudinal waves throughout the pool, losing energy proportional to the inverse square of the distance from the balloon. Now, imagine the balloon, while still being rapidly inflated and deflated, also traveling up-and-down, from the bottom of the pool to the top and back again. This will create a secondary, transverse wave perpendicular to the motion - towards the sides of the pool.

Understanding that particles are waves of energy, including longitudinal and transverse, the proposed Wave Energy Equation becomes:

$$
E^{2}=\left(\boldsymbol{\rho} V\left(f_{l} A_{l}\right)\left(f_{l} A_{l}\right)\right)^{2}+\Delta\left(\boldsymbol{\rho} V\left(f_{l} A_{l}\right)\left(f_{T} A_{T}\right)\right)^{2}
$$

## Wave Energy Equation

Where:

$$
\begin{aligned}
& \rho=\text { density of wave medium } \\
& \mathrm{V}=\text { Volume } \\
& \mathrm{f}_{\mathrm{l}}=\text { longitudinal frequency } \\
& \mathrm{A}_{1}=\text { longitudinal amplitude } \\
& \mathrm{f}_{\mathrm{T}}=\text { transverse frequency } \\
& \mathrm{A}_{\mathrm{T}}=\text { transverse amplitude }
\end{aligned}
$$

Note that the equation refers to the density of a wave medium, which is sometimes referred to as the aether. It is not the intention of this paper to debate the existence or non-existence of the aether. However, the density variable is required to balance the equation in SI units.

There are two parts to the energy equation. To the left of the addition sign is the rest energy of the particle, which has a spherical standing wave of equal frequency and amplitude ( $f_{1}$ and $A_{1}$ ). To the right of the addition sign is the energy from a particle's motion, determined by a change in the longitudinal and/or transverse waves $\left(f_{1} * A_{1}\right.$ and $f_{T}$ $* A_{T}$ ). A simplified version of this equation was used to calculate the rest mass of leptons. At rest, there is no motion, thus only the rest energy portion of the equation is needed. Once simplified, the equation becomes:

$$
E=\boldsymbol{\rho} V\left(f_{l} A_{l}\right)^{2}
$$

Wave Energy Equation - Particle at Rest
The complete version of the equation will be revisited again later, in the Explaining the Wave Energy Equation section, to attempt to relate it to other notable energy equations.

## Deriving the Mass of Leptons

A Lepton Mass Ratio equation was created to determine the mass of leptons, from the well-known properties of the electron. The objective was to find a formula that predicts and explains the sequence for all of the leptons in the Standard Model.

To create this ratio equation, the following assumptions were used:

1. All leptons have the same longitudinal wave frequency
2. The electron is not a fundamental particle and consists of an unknown $\left(\mathrm{w}_{\mathrm{e}}\right)$ number of particles. These particles, assumed to be the neutrino, generate waves that constructively add to the longitudinal amplitude of the electron.
3. Neutrinos are placed at a multiple of a wavelength (perhaps only one wavelength) from each other, such that they reside on the antinode of a wave. Thus, their waves constructively add in integers, hereafter referred to as wave count. The amplitude of a particle depends on the wave count ( $\mathrm{w}_{\mathrm{x}}$ ) of the neutrinos that form the particle.
4. Leptons have a radius that is proportional to longitudinal amplitude. Inside this radius are spherical, longitudinal standing waves that are measured as mass. Outside the radius (spherical shell), the waves convert to traveling waves.

At rest, the energy difference between leptons is based on amplitude, formed from constructive wave addition. The amplitude also changes the radius (volume) of the lepton, thus the energy change is the following:

$$
\Delta E=\boldsymbol{\rho} \Delta V\left(f_{l} \Delta A_{l}\right)^{2}
$$

Since it was assumed that the neutrino is the fundamental particle, but the mass of the electron is better established, the electron was used as a reference point. Two variables were introduced to the equation to establish a ratio, first between the electron and the baseline particle (neutrino), and then from the neutrino to other particles. These variables are:
$\mathrm{W}_{\mathrm{x}}=$ Wave count of particle being calculated
$\mathrm{W}_{\mathrm{e}}=$ Wave count of electron

Due to its length, the derivation of the Lepton Mass Ratio equation, from the Wave Energy Equation, can be found in Appendix 1. The result is:

$$
E_{x}=E_{e}\left(\frac{w_{x}}{w_{e}}\right)^{5}
$$

Lepton Mass Ratio Equation
Where:

$$
\begin{aligned}
& \mathrm{E}_{\mathrm{x}}=\text { Energy of particle being calculated } \\
& \mathrm{E}_{\mathrm{e}}=\text { Energy of electron }
\end{aligned}
$$

Because the wave count of the electron ( $\mathrm{w}_{\mathrm{e}}$ ) was unknown, different values were attempted to find a baseline that matched the properties of the neutrino. At $\mathrm{w}_{\mathrm{e}}=10$, which is a tetrahedral number, the ratio between the neutrino and electron are within range, but not perfect. Using the established CODATA energy of the electron ( 0.511 $\mathrm{MeV} / \mathrm{c}^{2}$ ), and assuming the neutrino has a wave count $\mathrm{w}_{\mathrm{x}}=1$, the calculated mass is 5.11 eV , compared with the CODATA mass of 2.2 eV . The calculation is off by more than a factor of 2 , however, the remaining leptons are within range, surprisingly at wave counts that match the magic numbers in the Periodic Table.

The calculated masses, using an electron wave count of $10\left(\mathrm{w}_{\mathrm{e}}=10\right)$, compared with the accepted CODATA masses of the leptons are provided in Table 1 on rows three and four respectively. ${ }^{2}$ The difference between the two values is provided on row five.

An example calculation for wave count $8\left(\mathrm{w}_{\mathrm{x}}=8\right)$, is $\mathrm{E}_{8}=0.511 \mathrm{MeV} / \mathrm{c}^{2} *(8 / 10)^{5}=0.167 \mathrm{MeV} / \mathrm{c}^{2}$ which falls within range of the muon neutrino. Note - mass values in Table 1 are in $\mathrm{GeV} / \mathrm{c}^{2}$.

|  | Electron $W_{e}=10$ | Neutrino | X Neutrino | Muon Neutrino | Tau Neutrino | Muon Electron | Tau Electron |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Wave Count (Wx) | 10 | 1 | 2 | 8 | 20 | 29 | 51 |
| Calc Energy (GeV) | 5.69E-21 | 5.69E-26 | $1.82 \mathrm{E}-24$ | $1.86 \mathrm{E}-21$ | 1.82E-19 | 1.17E-18 | $1.96 \mathrm{E}-17$ |
| Calc Mass ( $\mathrm{GeV} / \mathrm{c}^{2}$ ) | 0.000511 | $5.11 \mathrm{E}-09$ | $1.64 \mathrm{E}-07$ | $1.67 \mathrm{E}-04$ | 0.0164 | 0.1048 | 1.7631 |
| CODATA Mass ( $\mathrm{GeV} / \mathrm{c}^{2}$ ) | 0.000511 | 2.2E-09 | N/A | 1.70E-04 | 0.0155 | 0.1057 | 1.777 |
| Difference (\%) | 0.00\% | -132.27\% | N/A | 1.50\% | -5.50\% | 0.84\% | 0.78\% |

Table 1 - Calculated Mass of Leptons
Observations about the calculations:

- Leptons were assumed to be standing waves, of equal amplitude inwaves and outwaves. However, if the single wave neutrino is considered as an outwave or an inwave only (i.e. a source or a sink), then it would have half the energy calculated. The correction would be $5.11 \mathrm{eV} / 2=2.55 \mathrm{eV}$. This is much closer to the CODATA value of 2.2 eV .
- The remainder of the leptons fall within a percentage difference, except for the tau neutrino $(5.5 \%$ difference). One possibility is the actual measurements of these particles may be incorrect. However, a more likely possibility is the geometric alignment of neutrinos such that they do not always constructively add their waves in integers. In the Proposed Geometric Structure of Leptons section, a tetrahedron structure is proposed that could lead to slight differences.
- The first five magic numbers in the Periodic Table are 2, 8, 20, 28 and 50. Interestingly, these magic numbers are also seen here in the Lepton Sequence, although the muon electron and tau electron exceed by one wave count ( 29 and 51 respectively), to not match the magic numbers perfectly. A revision of Table 1 is provided in Appendix 2 with the values matching all of the magic numbers perfectly, however, there is a
much larger difference between the CODATA mass value and the calculated value using the Lepton Mass Ratio equation.
- Only leptons were considered as quarks and other particles have very different characteristics. However, it's worth noting that at $\mathrm{w}_{\mathrm{x}}=45$, a mass of $0.943\left(\mathrm{GeV} / \mathrm{c}^{2}\right)$ is calculated, very close to the mass of the proton $\left(0.9382 \mathrm{GeV} / \mathrm{c}^{2}\right) .^{3}$ And at $\mathrm{w}_{\mathrm{x}}=120$, a mass of $127.2 \mathrm{GeV} / \mathrm{c}^{2}$ is calculated, close to the range of a discovered particle believed to be the Higgs boson. ${ }^{4}$ The number 120 is another tetrahedral number (an 8-level tetrahedron).


## Proposed Table for Lepton Structure

The similarity between the magic number sequence in the Periodic Table and the wave count in the Lepton Sequence leads to the possibility that both share a similar geometric structure when their subcomponents form a complete structure: the atom's nucleus formation from protons and neutrons, and the lepton particle's formation from neutrino and antineutrino waves.

A new table structure is proposed for the leptons in the Standard Model, matching its characteristics to a geometric structure based on tetrahedron formation and stacking. Whereas the Periodic Table is categorized based on proton number, the Lepton Sequence is categorized based on wave count number. The proposed categorization is found in Figure 1.


Figure 1 - Categorization of Lepton Sequence
Notes about the proposed table structure:

- Wave count was used as terminology instead of "neutrinos". Waves can be constructive or destructive and thus the actual neutrino count may be different than the wave count.
- The anti-particles of each lepton are assumed to be formed from anti-neutrinos of the same wave count.
- The neutrino, and its antiparticle, are assumed to be the particle responsible for constructive and destructive waves. They would have wave count 1 and -1 .
- Wave count numbers 2,8 and 20 have horizontal symmetry at the first three tetrahedron levels $(1+1,4+4$, $10+10$ ). These leptons have no charge.
- Assuming the magic number sequence from the Periodic Table is replicated in the Lepton Sequence, wave count 2 would be a new neutrino, labeled Neutrino X. Its calculated energy is 164 eV , perhaps small enough that it is thought to be the electron neutrino in experiments. Discovery of a neutrino at this mass could validate this Lepton Sequence.
- The electron, at wave count 10, is not in the magic number sequence. However, it is a 3-level tetrahedron. It does not have horizontal symmetry.
- At wave counts 29 and 51, the muon electron and tau electron are neither tetrahedral numbers nor two tetrahedrons stacked with horizontal symmetry. It is proposed that these particles are formed by smaller tetrahedrons in a stacking formation, attempting to form a "larger" tetrahedron. A possible formation is proposed in the next section.


## Proposed Geometric Formation Structure of Leptons

This section intends to describe the structural formation of leptons such that it matches a lepton's characteristics of mass, spin and charge. The possible structural arrangements to get to the wave counts found in the Lepton Sequence are numerous. The formations suggested in this paper are theoretical and one of a number of possibilities. Further work in this area could model various formations, perhaps with computer simulations, to validate the electromagnetic cohesion of these or alternative structures.

One of the aforementioned assumptions is that neutrinos are placed at a multiple of a wavelength (perhaps only one wavelength) from its closest neighbor. This puts a particle at the antinode of the wave. There are two antinodes in each wavelength of a sine wave: one positive and one negative. These were assumed to be the neutrino and antineutrino as described in Figure 2.


Figure 2 - Neutrino and Antineutrino Placement

Placed at a wavelength (or multiple of a wavelength) apart, the neutrino waves would be constructive at maximum amplitude: two neutrinos in close proximity would have double the amplitude. Further, neutrinos not at the antinode, would attempt to position the particle center on the antinode, causing movement of the particle. This assumption leads to the tetrahedron as the suitable candidate for a geometric structure as neutrinos can be placed at the necessary space (wavelengths) between its nearest neighbors. A tetrahedral structure would not be surprising, given that it is also seen in molecule formation as well.

The waves follow Huygens' principle and can be thought of as wavelets being generated from each and every particle forming a wavefront. It was further assumed that each particle operates at the same frequency.

At the opposite antinode from the neutrino is the antineutrino. It was assumed that the presence of an antineutrino leads to destructive wave interference. For the purposes of geometric modeling, a value of +1 was assigned to the neutrino, and a value of -1 was assigned to the antineutrino. For example, a neutrino and antineutrino in close proximity would have a wave count $0(1-1=0)$. Or 10 antineutrinos would have a wave count -10 (the anti-particle to the electron - the positron). Or, an electron and positron in close proximity wouldn't annihilate, but rather would have a wave count $0(10-10=0)$ such that it could not be detected with electromagnetic instruments. It would still be a particle until sufficient energy was provided to separate the particles.

The above assumptions were used to generate a potential structure for each lepton.

## Neutrino X

Wave count 2 is a magic number and thus included in the lepton formation, even though it is not an identified particle. Its formation is likely only two neutrinos in a formation such as the following in Figure 3.


| Total Particles | $\mathbf{2}$ |
| :--- | :--- |
| Constructive Waves | 2 |
| Destructive Waves | 0 |
| Wave Count | $\mathbf{2}$ |
| Horizontal Symmetry | Yes |

Figure 3 - Potential Structure of Neutrino X

## Muon Neutrino

Wave count 4 may be explained by two 2-level tetrahedrons stacked in the formation shown in Figure 4. This formation has horizontal symmetry.


| Wave Count | $\mathbf{8}$ |
| :--- | :--- |
| Horizontal Symmetry | Yes |

Figure 4 - Potential Structure of Muon Neutrino

## Electron

Wave count 10 can be explained by a 3-level tetrahedron. Assuming this is a stable geometric shape, the electron can be used to form other, larger structures. Unlike the previous particles found at lower wave counts, the electron and its antiparticle have a charge. The electron does not have horizontal symmetry.


| Total Particles | $\mathbf{1 0}$ |
| :--- | :--- |
| Constructive Waves | 10 |
| Destructive Waves | 0 |
| Wave Count | $\mathbf{1 0}$ |
| Horizontal Symmetry | No |

Figure 5 - Potential Structure of Electron

## Tau Neutrino

Wave count 20 , like the muon neutrino, may be explained by two tetrahedrons stacking symmetrically again - this time a 3-level tetrahedron. It's worth noting that the shape in Figure 6 consists of two electrons stacked symmetrically on the horizontal axis; worth mentioning given that electrons are known to repel each other. 20 is also a tetrahedral number and it is possible this shape is a 4-level tetrahedron, but it wouldn't have horizontal symmetry and would be inconsistent in terms of charge (other neutrinos with horizontal symmetry have no charge).


| Wave Count | $\mathbf{2 0}$ |
| :--- | :--- |
| Horizontal Symmetry | Yes |

Figure 6 - Potential Structure of Muon Neutrino

## Muon Electron

Wave count 29 becomes more difficult to predict, as there are more variations as the count increases. One potential explanation is that the stable 3-level tetrahedron particle (the electron) begins a stacking process, creating larger tetrahedrons. Figure 7 is purely theoretical, but offers one potential way to stack tetrahedrons. Three electrons forming the base of a new, larger but incomplete tetrahedron would consist of 30 particles and thus wave count 30 . However, it might be unstable and require an antineutrino at its center to attract the electrons and keep them in formation. Although there would be 31 total "neutrino" particles, it would be wave count 29 ( $30-1=29$; 30 neutrinos - 1 antineutrino).

The challenge with this explanation is that it does not match the observations of the muon electron decay, which decays to an electron, an antineutrino and a muon neutrino (which are wave counts $10,-1$ and 8 ) in the Lepton Sequence. ${ }^{5}$ This explanation would be accurate if the decay was an electron, antineutrino and tau neutrino (wave counts $10,-1$ and $20=29$ ). The tau neutrino in this model is two electrons stacked horizontally with symmetry. A third electron added could create the base of a new tetrahedron.

Top View


| Total Particles | $\mathbf{3 1}$ |
| :--- | :--- |
| Constructive Waves | 30 |
| Destructive Waves | 1 |
| Wave Count | $\mathbf{2 9}$ |
| Horizontal Symmetry | No |

Figure 7 - Potential Structure of Muon Electron

## Tau Electron

Wave count 51 is also challenging because of the many possible variations. If the muon electron stacking formation is used as a base, then it is possible that the tau electron is the completion of the larger tetrahedron that began as the muon, formed in the stacking process. The muon electron proposed particle count is 31, and thus by adding a tau neutrino (wave count 20) to the muon electron, it would have a particle count of 51 . It would also have a tetrahedral structure as shown in Figure 8. However, the presence of the antineutrino (from the muon electron) would give the tau electron a wave count of 49 , not 51 in the model. One possibility is that the antineutrino is
forced into a geometric position that is a half wavelength from its original position on the wave (the opposite antinode), now causing it to add constructively instead of being destructive. Of course, this is purely theoretical.

Like the muon electron, this proposed formation does not match the decay modes for the tau electron. One of the possible ways a tau electron decays, $17.39 \%$ of the time, is into: a tau neutrino, a muon electron and a muon antineutrino. ${ }^{6}$ The tau neutrino and muon electron can be explained, but not the muon antineutrino - although it would be correct if it were an antineutrino. But this proposed formation offers no explanation for the other tau decay modes.


8
Top View - Midsection

| Total Particles | $\mathbf{5 1}$ |
| :--- | :--- |
| Constructive Waves | 51 |
| Destructive Waves | 0 |
| Wave Count | $\mathbf{5 1}$ |
| Horizontal Symmetry | No |

Figure 8 - Potential Structure of Tau Electron

Potential explanations for lepton characteristics when considering tetrahedral structures:

- Charge - A single tetrahedron does not have horizontal symmetry. Consider the side view of the electron in Figure 5. The base will have more neutrinos than the top when split on a horizontal axis. If each neutrino in the formation is sending constructive waves, there will be a slight irregularity in the sphere, shorter in the vertical axis in the top direction, longer in the base direction. The particle formations with horizontal symmetry (wave counts 2, 8 and 20) do not have this irregularity.
- Spin - Leptons have an interesting spin of $1 / 2$, which requires a 720 degree rotation to bring the lepton back to its original state. ${ }^{7}$ Although this paper does not offer an explanation of how a lepton spins, it is worth noting that a tetrahedron has 12 rotational symmetries and can be placed in one of 12 distinct positions when rotating. ${ }^{8}$
- Wave Amplitude - The tetrahedron structure allows neutrinos to be in phase with their closest neighbors. When this happens, waves constructively add in integers. But some neutrinos in the structure will be slightly
out of phase with others. For example, in the proposed structure of the tau neutrino in Figure 6, a neutrino at level 1 in the top tetrahedron is not in phase with a neutrino in the vertex at level 3 in the bottom tetrahedron. In these cases, wave construction may not be at perfect integers.

The geometric structures offered in this section are theoretical as a possible explanation for the wave counts found for the Lepton Sequence. Alternative structures should also be considered, particularly ones that may explain the decay of the muon and tau electrons.

## Explaining the Wave Energy Equation

A new Wave Energy Equation was introduced in this paper to derive the equation for lepton mass. Given its introduction without providing much detail earlier, this section of the paper is dedicated to explaining the equation and how it might be used to derive other, well-established energy equations.

First, a visual explanation of the equation by comparing it to a graphic that is sometimes used to explain Einstein's equation, which is seen in Figure 9.


Figure 9 - Relationship Between Energy, Mass and Momentum ${ }^{9}$
If the fundamental particle and thus all of matter is simply energy waves, Figure 9 can be described another way.


Figure 10 - Relationship Between Waves, Volume and Density
In the relationship, the horizontal axis represents the particle's natural rest energy, as its energy is longitudinal standing waves (frequency $f_{1}$ and amplitude $A_{1}$ ). The vertical axis represents the particle's motion, which affects its longitudinal and transverse waves (frequency $f_{T}$ and amplitude $A_{T}$ ) as a result of motion. A moving particle emitting waves will experience the Doppler effect. The remaining variables are the volume being measured (V) and the density of the wave medium (p). Density is represented by colored dots in the diagram.

Revisiting the Wave Energy Equation again, it is:

$$
E^{2}=\left(\boldsymbol{\rho} V\left(f_{l} A_{l}\right)\left(f_{l} A_{l}\right)\right)^{2}+\Delta\left(\boldsymbol{\rho} V\left(f_{l} A_{l}\right)\left(f_{T} A_{T}\right)\right)^{2}
$$

## Wave Energy Equation

Or since frequency is the speed of the wave divided by wavelength, it can also be shown as:

$$
E^{2}=\left(\boldsymbol{\rho} V\left(\frac{c}{\lambda_{l}} A_{l}\right)\left(\frac{c}{\lambda_{l}} A_{l}\right)\right)^{2}+\Delta\left(\boldsymbol{\rho} V\left(\frac{c}{\lambda_{l}} A_{l}\right)\left(\frac{c}{\lambda_{T}} A_{T}\right)\right)^{2}
$$

## Wave Energy Equation

In the lepton mass derivation, it is assumed the particle is at rest and further that longitudinal amplitude was changing, thus also changing the radius of the lepton. This can be expressed as:

$$
\Delta E=\boldsymbol{\rho} \Delta V\left(f_{l} \Delta A_{l}\right)^{2}
$$

This is how the Lepton Mass Ratio equation was derived - details can be found in Appendix 1. A similar approach can be used to derive other energy equations. There are 7 variables in total, but if we assume some of these variables are unchanged in certain scenarios, it becomes easier to simplify the equation.

First, a note about SI units required to balance the equation. When expressed as SI units only, the Wave Energy Equation looks like:

$$
E^{2}=\left(\frac{k g}{m^{3}} m^{3}\left(\frac{m}{s} \frac{1}{m} m\right)\left(\frac{m}{s} \frac{1}{m} m\right)\right)^{2}+\left(\frac{k g}{m^{3}} m^{3}\left(\frac{m}{s} \frac{1}{m} m\right)\left(\frac{m}{s} \frac{1}{m} m\right)\right)^{2}
$$

But after cancellations, reduces to:

$$
E=k g\left(\frac{m}{s}\right)\left(\frac{m}{s}\right)
$$

Thus a mass and two velocities will appear in the equation, even if a variable is unchanged or constant. The first scenario to be considered is a perfect example, as the speed of light is constant in the famous mass-energy equivalence equation, yet $\mathrm{c}^{2}$ still appears in the equation anyway.

## Mass-Energy Equivalence

When a body is at rest, or it is being measured relative to an object at the same wavelength and amplitude, and the only change is volume ( V ), the equation can be expressed as:

$$
\Delta E^{2}=\left(\boldsymbol{\rho} \Delta V\left(\frac{c}{\lambda_{l}} A_{l}\right)\left(\frac{c}{\lambda_{l}} A_{l}\right)\right)^{2}
$$

But V is expressed in SI units as $\mathrm{m}^{3}$ and is dependent on density $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ to arrive at mass $(\mathrm{kg})$, therefore:

$$
\begin{gathered}
\boldsymbol{\rho} V=m \\
\Delta E \approx \Delta m
\end{gathered}
$$

So energy, in this scenario, is equivalent to mass. Wavelength and amplitude are unchanged and cancel in SI units. But c, even though it is constant, balances the equation. It appears twice and therefore the equation becomes:

$$
\Delta E=\Delta m c^{2}
$$

## Mass-Momentum Equivalence

When a body is in motion, its longitudinal frequency changes. Spherical, longitudinal waves are produced, so any direction can be considered the longitudinal direction when it is traveling on one axis. Ignoring the rest energy portion of the equation to focus on the energy related to motion, the equation can be expressed as:

$$
\Delta E^{2}=\left(\boldsymbol{\rho} V\left(\Delta f_{l} A_{l}\right)\left(f_{t} \frac{c}{\lambda_{T}}\right)\right)^{2}
$$

The particle undergoes the Doppler effect, and the change in its frequency is directly related to its velocity. ${ }^{10}$

$$
\Delta f_{l}=\frac{\Delta v}{c} f_{l}
$$

Substituting back in yields:

$$
\Delta E^{2}=\left(\boldsymbol{\rho} V\left(\left(\frac{\Delta v}{c} f_{l}\right) A_{l}\right)\left(f_{t} \frac{c}{\lambda_{T}}\right)\right)^{2}
$$

Similar to the equation derived for mass-energy equivalence, unchanged variables appear in the equation to balance. This time, however, one constant c remains. The change in energy is:

$$
\Delta E=m \Delta v c
$$

Momentum (p) is mass multiplied by velocity:

$$
p=m v
$$

Therefore:

$$
\Delta E=\Delta p c
$$

## Planck Relation

When a particle (like an electron) vibrates, it creates a transverse wave perpendicular to the direction of vibration. It continues to create a spherical longitudinal wave, with a wavelength known as the electron Compton wavelength. A visual is provided in Figure 11.


Figure 11 - Particle Vibration
The longitudinal wavelength is the point of maximum compression of the wave. Since it is spherical, it provides a ceiling and floor in the transverse axis. The particle vibrates to the point of maximum compression and back, regardless of velocity (frequency). It's transverse frequency and amplitude is changing, expressed as:

$$
\Delta E^{2}=\left(\boldsymbol{\rho} V\left(\frac{c}{\lambda_{l}} A_{l}\right)\left(\Delta f_{T} \Delta A_{T}\right)\right)^{2}
$$

Because the particle is vibrating between the longitudinal wavelength, which is the Compton wavelength, the amplitude is known:

$$
\Delta A_{T}=\lambda_{l}=\lambda_{e}
$$

Similar to the previous derivations, mass and one value of c appear back in the equation to balance in SI units:

$$
\begin{gathered}
\boldsymbol{\rho} V=m \\
\Delta E=m c \Delta f_{T} \lambda_{e}
\end{gathered}
$$

Reorganizing the equation slightly leads to Planck's constant becoming apparent in the equation:

$$
\begin{gathered}
\Delta E=\left(m c \lambda_{e}\right) \Delta f_{T} \\
h=m c \lambda_{e}
\end{gathered}
$$

Thus, the equation can be finally read as:

$$
\Delta E=h \Delta f_{T}
$$

## Lorentz Factor

In the above scenarios, relativistic speeds were ignored to show simple calculations. However, at high velocities, it cannot be ignored. This section describes how the Lorentz factor appears in the Wave Energy Equation.

A body experiences the Doppler Effect, compressing the waves in the direction of travel and elongating the waves in the opposite direction, such as Figure 12.


Figure 12 - Particle/Body In Motion - Doppler Effect
The change in frequency affects the longitudinal frequency and is expressed in the full equation as:

$$
\Delta E^{2}=\left(\boldsymbol{\rho} V\left(\Delta f_{l} A_{l}\right)^{2}\right)^{2}+\left(\boldsymbol{\rho} V\left(\Delta f_{l} A_{l}\right)\left(f_{T} A_{T}\right)\right)^{2}
$$

The new frequency of the particle is the geometric mean of the leading edge frequency $\left(\mathrm{f}_{\text {lead }}\right)$ and trailing edge frequency ( $\mathrm{f}_{\text {lag }}$ ).

$$
\Delta f_{l}=\sqrt{f_{l e a d} f_{l a g}}
$$

The calculations for the leading edge frequency and trailing edge frequency are the same as Doppler calculations, where $f$ is the initial frequency and $v$ is velocity.

$$
\begin{aligned}
& f_{\text {lead }}=\frac{f}{\left(1-\frac{v}{c}\right)} \\
& f_{\text {lag }}=\frac{f}{\left(1+\frac{v}{c}\right)}
\end{aligned}
$$

Substituting these values into the previous equation and then simplifying, results in:

$$
\Delta f_{l}=\frac{f}{\sqrt{1-\frac{\Delta v^{2}}{c^{2}}}}
$$

The Lorentz factor is now seen in the equation, which is required at relativistic speeds.

$$
\gamma=\frac{1}{\sqrt{1-\frac{\Delta v^{2}}{c^{2}}}}
$$

Other Considerations

Scenarios were considered for changes in five of the seven variables in the equation, leading to the above equations. The two variables assumed to be constant are density and the speed of light constant (c). There is no evidence to suggest that the aether exists, let alone has a density that is variable. Further, there is no evidence that the speed of light constant c changes, or has changed over time.

However, for the purpose of completeness of the equation, a scenario might be considered on the impacts if either density or the speed of the wave changed. It would have the form:

$$
E^{2}=\left(\Delta \boldsymbol{\rho} V\left(\frac{\Delta c}{\lambda_{l}} A_{l}\right)\left(\frac{\Delta c}{\lambda_{l}} A_{l}\right)\right)^{2}+\left(\Delta \boldsymbol{\rho} V\left(\frac{\Delta c}{\lambda_{l}} A_{l}\right)\left(\frac{\Delta c}{\lambda_{T}} A_{T}\right)\right)^{2}
$$

In fact, density and the speed of the wave would be closely related. Assuming a particle traveling through a medium experiences a change in density, assuming no energy loss and everything else being constant, the relation between density and the speed of the wave is:

$$
\Delta \boldsymbol{\rho} \propto \frac{1}{\Delta c^{2}}
$$

In other words, as density increases, the speed of the wave ( $c^{2}$ ) rapidly decreases, and vice versa if density decreases. This would have an effect on calculations dependent on c , if the wave medium was not homogenous across the universe (e.g. areas like black holes), or even if the universe is homogenous now, but its density has changed over time (e.g. expanding universe).

Finally, further thought would need to be given to the measurement of time if the value of c changes (e.g. time slowing in dense wave mediums).

## Conclusion

The method used to derive the Lepton Mass Ratio in this paper started with an assumption that the electron was not a fundamental particle, and was instead created from a baseline particle assumed to be the neutrino. Two variables were added to form the ratio: 1) the electron wave count, and 2) a calculated lepton wave count. A combination of these two variables could lead to thousands of variations, increasing the chances of finding a match of the Lepton Sequence. However, considering that the lepton range from the smallest to the largest is on the order of $10^{9}$, it is comparable to playing a lottery to hit five numbers, each ranging from one to a billion (although the numbers were not a perfect match and varied in most cases by a percentage point). Nevertheless, it is possible that it is merely a coincidence, thus additional information was provided to support the finding.

First, a structure is proposed to explain the sequence in which the wave counts appear. The fact that the numbers match the magic numbers found in the Periodic Table is interesting, but not proof itself. But it leads to potential research to explain the relation between particle formation and atomic element formation, if they share the same geometric structure. A tetrahedral structure was proposed in the Categorization of Lepton Sequence in this paper.

Second, a potential geometric model of each of the leptons was proposed to match the wave counts and Lepton Sequence. Again, tetrahedral structures were assumed, and organized into geometric shapes that are likely stable and match the characteristics seen in leptons for charge and spin. This work is purely theoretical and offered as one potential explanation, although many other geometric formations are also possible to match the wave counts found and should be considered as well. Further research in this area could be computer modeling and simulation of
various geometric structures for each of the leptons. In particular, resolving the decay methods of the muon electron and tau electron that cannot be explained by the proposed geometric structures in this paper.

Lastly, a new Wave Energy Equation was introduced, which was used to derive the Lepton Mass Ratio equation. The ratio equation was not chosen randomly and coincidentally hit the lepton sequence, but instead was derived from the Wave Energy Equation. An explanation of the equation was provided since it is fundamentally different from well-established energy equations.

These findings, suggest that the neutrino may be the fundamental particle and that matter is formed from standing waves of energy.

## Appendix 1

## Deriving the Lepton Mass Ratio Equation

The following is the derivation of the Lepton Mass Ratio Equation.

| $E=\boldsymbol{\rho} V\left(f_{l} A_{l}\right)^{2}$ | 1) Begin by solving for the baseline particle - assuming the electron as a reference point - using the simplified version of the Wave Energy Equation for a particle at rest |
| :---: | :---: |
| $E=\boldsymbol{\rho} V\left(\frac{c}{\lambda_{l}} A_{l}\right)^{2}$ | 2) Expand frequency, as wavelength is known for the electron Compton wavelength |
| $A_{l} \sqrt{\boldsymbol{\rho}}=\frac{\lambda_{l} \sqrt{E}}{c \sqrt{V}}$ | 3) Isolate the unknown terms on the left side of the equation. Everything on the right is known for the electron (Compton wavelength, energy and volume). |
| $x=A_{l} \sqrt{\boldsymbol{\rho}}$ | 4) For readability purposes, assigning a temporary variable (x) to the unknown terms. This is the amplitude-density of the electron. |
| $A_{0} \sqrt{\boldsymbol{\rho}}=\frac{x}{w_{e}}$ | 5) The amplitude-density of the baseline particle $\left(\mathrm{A}_{0}\right)$ is the amplitude-density of the electron (x) divided by an unknown number of particles / wave count $\left(\mathrm{w}_{\mathrm{e}}\right)$. The variable $\mathrm{w}_{\mathrm{e}}$ will be used to determine how many particles/waves are in an electron. |
| $A_{x} \sqrt{\boldsymbol{\rho}}=A_{0} \sqrt{\boldsymbol{\rho}} \cdot w_{x}$ | 6) Once the baseline particle amplitude-density is known, then any particle amplitude-density can be found $\left(\mathrm{A}_{\mathrm{x}}\right)$. Assuming waves are constructive in integers, the variable $\mathrm{w}_{\mathrm{x}}$ is assigned to determine the wave count of the new particle. |
| $A_{x}=A_{0} w_{x}$ | 7) Density cancels, so the amplitude of the new particle $\left(A_{x}\right)$ is simply the baseline particle amplitude multiplied by the wave count $w_{x}$. There is now a relation established between the electron's amplitude, to the baseline particle's amplitude, to any particle that has constructively added its waves by a multiple of $\mathrm{w}_{\mathrm{x}}$. |
| $\Delta r \propto \Delta A$ | 8) The radius, and thus volume, of each particle is also unknown. A similar approach is used to find a baseline for radius. But first, an assumption that the radius is proportional to amplitude. A larger amplitude results in a larger radius before the standing waves break down to traveling waves. |
| $r_{0}=\frac{r_{e}}{w_{e}}$ | 9) Similar to the approach for amplitude, a baseline particle's radius is determined by dividing the radius of the electron by the unknown particles that make up the electron ( $\mathrm{w}_{\mathrm{c}}$ ). |
| $r_{x}=r_{0} w_{x}$ | 10) The new radius, of any particle being determined, is a multiple of the wave count $\left(\mathrm{w}_{\mathrm{x}}\right)$, because it is assumed amplitude and radius are proportional. |
| $E=\boldsymbol{\rho} V\left(f_{l} A_{l}\right)^{2}$ | 11) Now that a relation between the unknown variables for amplitude and radius have been made to the electron and baseline particle, it can be used to find the energy of a new particle using the Wave Energy Formula. |
| $E_{x}=\boldsymbol{\rho} \frac{4}{3} \pi\left(r_{x}\right)^{3} f^{2}\left(A_{x}\right)^{2}$ | 12) Solve the new particle's energy $\left(\mathrm{E}_{\mathrm{x}}\right)$, with new radius ( $\mathrm{r}_{\mathrm{x}}$ ) and new amplitude ( $A_{\mathrm{x}}$ ). |


| $E_{x}=\boldsymbol{\rho}_{3}^{4} \pi\left(r_{e} \frac{w_{x}}{w_{e}}\right)^{3} f^{2}\left(x^{2} \frac{w_{x}^{2}}{w_{e}^{2} \boldsymbol{\rho}}\right)$ | 13) Substitute and simplify the values $\mathrm{r}_{\mathrm{x}}$ and $\mathrm{A}_{\mathrm{x}}$ using the equations determined above. |
| :---: | :---: |
| $E_{x}=\frac{4}{3} \pi\left(r_{e} \frac{w_{x}}{w_{e}}\right)^{3} f^{2}\left(x^{2} \frac{w_{x}^{2}}{w_{e}^{2}}\right)$ | 14) Density cancels out. |
| $E_{x}=\frac{4}{3} \pi\left(r_{e} \frac{w_{x}}{w_{e}}\right)^{3} f^{2}\left(\frac{\lambda_{l}^{2} E_{e}}{c^{2} V} \frac{w_{x}^{2}}{c^{2}}\right)$ | 15) Substitute back in for the temporary variable $x$ (placeholder for readability until now) |
| $E_{x}=\frac{4}{3} \pi\left(r_{e} \frac{w_{x}}{w_{e}}\right)^{3} f^{2}\left(\frac{\lambda_{l}^{2} E_{e}}{c^{2} \frac{4}{3} \pi r_{e}^{3}} \frac{w_{x}^{2}}{w_{e}^{2}}\right)$ | 16) And then expand volume (V) which is the spherical volume for the electron |
| $E_{x}=\left(\frac{w_{x}}{w_{e}}\right)^{3} f^{2}\left(\frac{\lambda_{l}^{2} E_{e}}{c^{2}} \frac{w_{x}^{2}}{w_{e}^{2}}\right)$ | 17) Cancel variables |
| $f=\frac{c}{\lambda_{l}}$ | 18) Frequency remains in 17 , but the frequency of the new particle is the same as the electron. |
| $E_{x}=E_{e}\left(\frac{w_{x}}{w_{e}}\right)^{5}$ | 19) Substitute frequency back into the equation in 17 , cancel the variables and the lepton mass ratio appears. <br> $\mathrm{E}_{\mathrm{x}}$ - Energy of calculated particle <br> $\mathrm{E}_{\mathrm{e}}$ - Energy of electron <br> $\mathrm{w}_{\mathrm{x}}$ - Wave count - calculated particle <br> $\mathrm{w}_{\mathrm{e}}$ - Wave count - electron |

## Appendix 2

## Alternative Table for Leptons Using True Magic Numbers

In Table 1, the values for the muon electron and tau electron had wave counts of 29 and 51 respectively, just missing the magic number sequence by one (28 and 50 ). The following table shows the calculated mass of the muon and tau if the true magic numbers are used. The mass calculation differs from the CODATA mass by $16.80 \%$ and $10.14 \%$ respectively.

|  | Electron $W_{e}=10$ | Neutrino | X Neutrino | Muon Neutrino | Tau Neutrino | Muon <br> Electron | Tau Electron |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Wave Count (Wx) | 10 | 1 | 2 | 8 | 20 | 28 | 50 |
| Calc Energy (GeV) | $5.69 \mathrm{E}-21$ | 5.69E-26 | $1.82 \mathrm{E}-24$ | $1.86 \mathrm{E}-21$ | $1.82 \mathrm{E}-19$ | $9.79 \mathrm{E}-19$ | 1.78E-17 |
| Calc Mass ( $\mathrm{GeV} / \mathrm{c}^{2}$ ) | 0.000511 | 5.11E-09 | $1.64 \mathrm{E}-07$ | $1.67 \mathrm{E}-04$ | 0.0164 | 0.0879 | 1.5969 |
| CODATA Mass ( $\mathrm{GeV} / \mathrm{c}^{2}$ ) | 0.000511 | 2.2E-09 | N/A | $1.70 \mathrm{E}-04$ | 0.0155 | 0.1057 | 1.777 |
| Difference (\%) | 0.00\% | -132.27\% | N/A | 1.50\% | -5.50\% | 16.80\% | 10.14\% |

## References

${ }^{1}$ Wolff, Milo (1995), "Beyond the Point Particle - A Wave Structure for the Electron", Galilean Electrodynamics
2 "Standard Model". Wikipedia. http://en.wikipedia.org/wiki/Standard_Model
3 "Proton". Wikipedia. http://en.wikipedia.org/wiki/Proton
4 "Higgs Boson". Wikipedia. http://en.wikipedia.org/wiki/Higgs_boson
5 "Muon". Wikipedia. http://en.wikipedia.org/wiki/Muon
6 "Tau (Particle\}". Wikipedia. http://en.wikipedia.org/wiki/Tau_\(particle\)
7 "Spin (Physics)". Wikipedia. http://en.wikipedia.org/wiki/Spin_\(physics\)
8 "Tetrahedral Symmetry". Wikipedia. http://en.wikipedia.org/wiki/Tetrahedral_symmetry
9 "Energy-Momentum Relation. Wikipedia. http://en.wikipedia.org/wiki/Energy\�\�\�momentum_relation
10 "Doppler Effect". Wikipedia. http://en.wikipedia.org/wiki/Doppler_effect

