Hilbert Logic

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Abstract

Early in the twentieth century a suitable foundation of quantum physics was found in traditional quantum logic. This idea exploits the lattice isomorphism between the set of propositions of a quantum logic system and the set of closed subspaces of an infinite dimensional separable Hilbert space. Later Constantin Piron restricted the choice for the number systems that can be used for specifying the inner product of the Hilbert space to members of a division ring. A slight refinement of the quantum logic system extends the lattice isomorphism to a topological isomorphism. This new logic system will be called Hilbert logic and it has a striking resemblance with a separable Hilbert space. The paper also provides insight in how fields can be interpreted.

Introduction

The subject of this paper is more elaborately treated in http://vixra.org/abs/1211.0120. Here the subject is restricted to the aspects of Hilbert logic systems. The coupling of fields is not treated here. The same holds for equations of movement for the identified objects and for continuity equations that lead to a fluid dynamics theory of Hilbert propositions. Also the influence of the objects on space curvature is not treated. For these aspects the reader is referred to the specified reference. The corresponding mathematics is treated in http://vixra.org/abs/1210.0111. These papers are part of the Hilbert Book Model project that was started in 2009 by the author. The target of this project is the investigation of the foundations of physics.

Restricting quantum logic

If physics must be based on an axiomatic foundation, then traditional quantum logic emerges as a natural choice. Early in the twentieth century, shortly after the instant that it became clear that physics required a new fundament that could explain the deviating behavior of small particles, the duo Garret Birkhoff and John von Neumann [1] found a useful answer. Birkhoff was an expert in lattice theory and von Neumann was recognized for his expertise on Hilbert spaces. They started from the insights of Schrödinger and Heisenberg and found that Hilbert spaces had the proper structure for supporting this new methodology. The duo also discovered a striking isomorphism between parts of the Hilbert space and the set of propositions of quantum logic. When compared to classical logic, quantum logic is a weakened form of logic. In more detail, the modular law of classical logic is exchanged for the weak modular law of quantum logic. The set of propositions of quantum logic is isomorphic to the set of closed subspaces of an infinite dimensional separable Hilbert space. Still there exists a functional and a structural gap between a quantum logic system and a separable Hilbert space. However, a full isomorphism appears to exist between the set of atomic propositions and a corresponding set of Hilbert base vectors. However, such an relation does not exist between combinations of atomic propositions and linear combinations of Hilbert vectors. With other words, in general exists no functional map between quantum logical propositions and Hilbert vectors.

This gap can be closed by putting some extra restrictions to the quantum logic system. The solution is the introduction of the notion of linear propositions. A linear proposition has a certain strength. This strength can be specified by number that is taken from a division ring. Any linear combination of linear propositions is again a linear proposition that belongs to the new logic system. We will call this new logic system a Hilbert logic. Further we will introduce the notion of relational coupling measure of two linear propositions. The relational coupling measure is the equivalent of the inner product of the Hilbert space. Its value is a number that is taken from the selected division ring. Another name for linear proposition is Hilbert proposition. A Hilbert logic system is closed with respect to the norm that is derived from the relational coupling measure. Quantum logic already contains linear propositions, however they form a subset. A Hilbert logic is a subset of a quantum logic.

As a consequence the Hilbert logic system is lattice isomorphic as well as topological isomorphic to the separable Hilbert space. It also means that it is possible to define linear operators in the Hilbert logic system. When a set of atomic Hilbert propositions is selected such that these atoms act as eigen-propositions, then the eigenvalues of the operator can be used in order to enumerate the propositions.

Since three suitable division rings exist, also three types of Hilbert logics will exist. Further the Hilbert logic can be divided in subsets that are again Hilbert logic systems. Each of these subsets are closed with respect to the relational coupling measure. As is already stated, this also holds for the original Hilbert logic system.

The refinement of quantum logic down to Hilbert logic eases the possibility to consider Hilbert logic as the foundation of Hilbert space based physics.

Examples

Color system

Three well selected base colors can form the atomic propositions of a three dimensional Hilbert logic. Via linear combinations all visible colors can be constructed. This principle is widely used by painters and by color display technologies.

Psycho analysis

A psychoanalytic institute analyses its candidates for a set of mutually independent characteristics that can be present with a given strength. This results in a Hilbert logic that represents a multidimensional classification of the candidates. The same can be done with available tasks. The result of the analysis can be used to match the candidates with each other or with a selected task. The institute preforms its analysis via a questionnaire that contains hundreds of questions, which are linear propositions, while the candidate is requested to add an integer valued strength. The integer may range from –N to +N, where N is usually taken at 5 or 7. The database of responses must be solved such that M mutually independent characteristics will be found. The database and the required solution forms a heavily over-determined set of linear equations that can only be solved when a certain normally distributed random error is accepted in the responses. With that addition the system can be resolved with standard mathematical methods.

Ambient color system

In this system the three mutually independent base colors can be mixed and the mixture can change in time. This can be achieved by using quaternions as color mixture indicators and as color brightness and change indicators. For example the squared modulus can act as color brightness and the imaginary part can act as color change indicator. A second quaternion describes the color mixture. It also acts as the identifier of the color mix. In the last number the real part of the quaternion act as a progression parameter. The Hilbert logic system contains an operator that enumerates and thus identifies the mixed colors. The strength value of the mixed colors indicates the brightness and the direction and speed of change of the mixed color. On a given progression instance the Hilbert logic system gives a full description of the static status quo including the potential next value of the status quo. A sequence of such Hilbert logic systems gives a full dynamic description where the sequence number corresponds to the progression parameter.

Location system

A set of items is considered to occupy a set of potential locations. A probability distribution describes the probability that one of the items is at the considered location. In a dynamic situation the item may move to another potential location. The strength indicator of the location may indicate both the probability of being there and the probability of moving away to a next available location. This can again can be specified with quaternionic numbers. The probability is stored in the squared modulus of the strength indicator. Further, it is possible to indicate the location itself with a quaternionic number. In the last case the real part of the quaternion can be used to indicate progression. The corresponding Hilbert logic system contains an operator that enumerates and thus identifies the potential locations.

Infinite location system

An infinite location system must still be countable. So it is best to enumerate the locations with rational quaternions. This might pose problems when dynamics must be installed, because a dense coverage of space by the set of locations leaves no room for movement. The solution for this dilemma is the introduction of a lowest rational quaternion. If this lowest quaternion is kept fixed then still no dynamics can take place. However, the enumeration process may randomly generate the enumerators with a variance equal to the selected lowest rational. This seems to be the way that nature solves its dynamic location problems. It solves its problems in a way similar to the method used by the psychoanalytic institute. The approach of the continuum delivers a highly over-determined set of linear equations, which can only resolved by accepting a normally distributed error in the approach results.

Dynamic Hilbert logic

As indicated above Hilbert logic systems can only describe a static status quo. That description can include the preparation for the next static status quo. A full dynamic description consists of an ordered sequence of such static descriptions. However, in order not to end in dynamical chaos a correlation vehicle must take care of the cohesion between the subsequent members of the sequence. The cohesion must not be too stiff otherwise no dynamics will take place. The correlation vehicle must be able to identify the atoms of the separate Hilbert logic systems. It can use the available enumeration mechanism for this purpose. The set of enumerators is countable. A continuum can be approached by using rational numbers. Further the correlation vehicle must be able to use some span in order to implement change. This can be achieved by assigning a lowest rational number.

The application of multidimensional enumerators will require a randomized approach for handling the available span. Otherwise, the enumeration process will introduce extra functionality. The embedding continuum can be considered to be an affine space with no origin and no preferred directions. The correlation vehicle may use the embedding continuum as a reference space.

History

A dynamic logic has a history. Its elements cover a compartment of the embedding affine continuum. If somewhere in the history of an infinite dynamic Hilbert logic a state of densest packaging occurred, then that compartment of the embedded continuum can be considered to be a (real) quaternionic number space and the rational quaternionic enumerators form an orderly and dense coverage of this quaternionic number space. In the next sequence of Hilbert logics the ordering is disturbed. The corresponding dynamic enumeration process can be described by a quaternionic enumeration function. This function is the convolution of a continuous sharp function and a spread function. The spread function is implemented by a stochastic process. The sharp part of this function produces an image of a rational quaternionic enumerator (RQE) that image will be called a ***Qpatch***. It is the center location of a stochastic pattern that will be called ***Qpattern***. The Qpattern is the result of a 3D random stochastic enumeration process that produces a 3D normal distribution. In each progression step the enumeration process produces one element of the Qpattern. That element will be called a ***Qtarget***. When the Qpatch moves, then the produced pattern is stretched along the path of the move.

Qpatterns can be interpreted as objects. Due to its quaternionic nature a Qpattern exists in sixteen different discrete symmetry forms. The discrete symmetry set of a given Qpattern is determined by the local properties of the enumerator generation function. Since it is a continuous quaternionic function this enumerator generator also exist in sixteen versions that differ in the discrete symmetry set of the target values.

Quaternionic probability amplitude distributions

As a consequence the enumerators form dynamic density distributions. These distributions can be described by a quaternionic probability amplitude distribution (QPAD). The real part of this function describes the density distribution of the enumerators. It corresponds to a scalar potential field. The imaginary part of the QPAD describes the density of the enumerator currents. It corresponds to a vector potential field.

Remember that the enumerators identify the atomic Hilbert propositions. The QPAD's correspond to descriptions of probable linear combinations of these atoms. Thus there exist a one to one correspondence between a local QPAD and the Hilbert proposition that represents the local Qpatch. On the other hand Hilbert propositions relate directly to Hilbert vectors. QPAD's have their equivalent in the corresponding Hilbert space.

This investigation tells what the fields that are contained in a QPAD stand for. They stand for dynamic distributions of atomic Hilbert propositions.

Fourier picture

So far we only considered pattern generations in configuration space. However, it is also sensible to consider a similar enumeration process in the canonical conjugated partner of the configuration space. The corresponding Qpattern will again be a normal Gaussian 3D distribution. This time the Qpatch will have its location in the canonical conjugated space. The corresponding distribution characteristics are related via an equivalent of Heisenberg's uncertainty relation.

Events

A local event occurs when the enumerator generator switches its mode from locally generating the Qpattern in configuration space into locally generating the Qpattern in the canonical conjugated space or vice versa. The switch also moves the corresponding Qpatch to “the other space”.

In the original space, the object that corresponds to the Qpattern is annihilated while in the new space the transformed object is generated. Since the Qpattern is generated with a Qtarget at each progression step the event has immediate consequences.

References

[1] http://en.wikipedia.org/wiki/John_von_Neumann#Quantum_logics & Stanford Encyclopedia of Philosophy, *Quantum Logic and Probability Theory*, http://plato.stanford.edu/entries/qt-quantlog/