Alternative Classical Mechanics

Alejandro A. Torassa

(2013) Buenos Aires, Argentina
atorassa@gmail.com

Abstract

This paper presents an alternative classical mechanics, which can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

Universal Reference Frame

The universal reference frame is a reference frame fixed to the center of mass of the universe.

The universal position $\mathring{\mathbf{r}}_a$, the universal velocity $\mathring{\mathbf{v}}_a$, and the universal acceleration $\mathring{\mathbf{a}}_a$ of a particle A relative to the universal reference frame $\mathring{\mathrm{S}}$, are given by:

$$\mathring{\mathbf{r}}_a = (\mathbf{r}_a)$$

$$\mathring{\mathbf{v}}_a = d(\mathbf{r}_a)/dt$$

$$\mathring{\mathbf{a}}_a = d^2(\mathbf{r}_a)/dt^2$$

where $\mathbf{r}_a$ is the position of particle A relative to the universal reference frame $\mathring{\mathrm{S}}$.

The dynamic position $\breve{\mathbf{r}}_a$, the dynamic velocity $\breve{\mathbf{v}}_a$, and the dynamic acceleration $\breve{\mathbf{a}}_a$ of a particle A of mass m_a, are given by:

$$\breve{\mathbf{r}}_a = \int\int (\mathbf{F}_a/m_a)\; dt\; dt$$

$$\breve{\mathbf{v}}_a = \int (\mathbf{F}_a/m_a)\; dt$$

$$\breve{\mathbf{a}}_a = (\mathbf{F}_a/m_a)$$

where $\mathbf{F}_a$ is the net force acting on particle A.

General Principle

The total position $\tilde{\mathbf{R}}_i$ of a system of particles of mass M_i $(M_i = \sum_i m_i)$, is given by:

$$\tilde{\mathbf{R}}_i = \sum_i \frac{m_i}{M_i}(\mathring{\mathbf{r}}_i - \breve{\mathbf{r}}_i) = 0$$

Therefore, the total position $\tilde{\mathbf{R}}_i$ of a system of particles is always in equilibrium.

Observations

Applying the general principle to a particle A, it follows:

$$\begin{array}{ccc}
\boxed{m_a\mathring{\mathbf{r}}_a - m_a\breve{\mathbf{r}}_a = 0} & \rightarrow & \boxed{\tfrac{1}{2}\, m_a\mathring{\mathbf{r}}_a^2 - \tfrac{1}{2}\, m_a\breve{\mathbf{r}}_a^2 = 0} \\
\downarrow & & \downarrow \\
\boxed{m_a\mathring{\mathbf{v}}_a - m_a\breve{\mathbf{v}}_a = 0} & \rightarrow & \boxed{\tfrac{1}{2}\, m_a\mathring{\mathbf{v}}_a^2 - \tfrac{1}{2}\, m_a\breve{\mathbf{v}}_a^2 = 0} \\
\downarrow & \nearrow & \downarrow \\
\boxed{m_a\mathring{\mathbf{a}}_a - m_a\breve{\mathbf{a}}_a = 0} & \rightarrow & \boxed{\tfrac{1}{2}\, m_a\mathring{\mathbf{a}}_a^2 - \tfrac{1}{2}\, m_a\breve{\mathbf{a}}_a^2 = 0}
\end{array}$$

Substituting $\breve{\mathbf{r}}_a$, $\breve{\mathbf{v}}_a$ and $\breve{\mathbf{a}}_a$ from page [1] into the above equations, we obtain:

$$\begin{array}{ccc}
\boxed{m_a\mathring{\mathbf{r}}_a - \int\int \mathbf{F}_a\, dt\, dt = 0} & \rightarrow & \boxed{\tfrac{1}{2}\, m_a\mathring{\mathbf{r}}_a^2 - \tfrac{1}{2}\, m_a(\int\int (\mathbf{F}_a/m_a)\, dt\, dt)^2 = 0} \\
\downarrow & & \downarrow \\
\boxed{m_a\mathring{\mathbf{v}}_a - \int \mathbf{F}_a\, dt = 0} & \rightarrow & \boxed{\tfrac{1}{2}\, m_a\mathring{\mathbf{v}}_a^2 - \int \mathbf{F}_a\, d\mathring{\mathbf{r}}_a = 0} \\
\downarrow & \nearrow & \downarrow \\
\boxed{m_a\mathring{\mathbf{a}}_a - \mathbf{F}_a = 0} & \rightarrow & \boxed{\tfrac{1}{2}\, m_a\mathring{\mathbf{a}}_a^2 - \tfrac{1}{2}\, m_a(\mathbf{F}_a/m_a)^2 = 0}
\end{array}$$

Where $\tfrac{1}{2}\breve{\mathbf{v}}_a^2 = \int \breve{\mathbf{a}}_a\, d\breve{\mathbf{r}}_a \rightarrow \tfrac{1}{2}\, m_a\breve{\mathbf{v}}_a^2 = \int m_a\, \breve{\mathbf{a}}_a\, d\breve{\mathbf{r}}_a \rightarrow \tfrac{1}{2}\, m_a\breve{\mathbf{v}}_a^2 = \int \mathbf{F}_a\, d\breve{\mathbf{r}}_a \quad (\breve{\mathbf{r}}_a = \mathring{\mathbf{r}}_a)$

Reference Frame

The universal position $\mathring{\mathbf{r}}_a$, the universal velocity $\mathring{\mathbf{v}}_a$, and the universal acceleration $\mathring{\mathbf{a}}_a$ of a particle A relative to a reference frame S, are given by:

$$\mathring{\mathbf{r}}_a = \mathbf{r}_a + \breve{\mathbf{r}}_s$$

$$\mathring{\mathbf{v}}_a = \mathbf{v}_a + \breve{\omega}_s \times \mathbf{r}_a + \breve{\mathbf{v}}_s$$

$$\mathring{\mathbf{a}}_a = \mathbf{a}_a + 2\,\breve{\omega}_s \times \mathbf{v}_a + \breve{\omega}_s \times (\breve{\omega}_s \times \mathbf{r}_a) + \breve{\alpha}_s \times \mathbf{r}_a + \breve{\mathbf{a}}_s$$

where $\mathbf{r}_a$, $\mathbf{v}_a$, and $\mathbf{a}_a$ are the position, the velocity, and the acceleration of particle A relative to the reference frame S; $\breve{\mathbf{r}}_s$, $\breve{\mathbf{v}}_s$, $\breve{\mathbf{a}}_s$, $\breve{\omega}_s$, and $\breve{\alpha}_s$ are the dynamic position, the dynamic velocity, the dynamic acceleration, the dynamic angular velocity and the dynamic angular acceleration of the reference frame S.

The dynamic position $\breve{\mathbf{r}}_s$, the dynamic velocity $\breve{\mathbf{v}}_s$, the dynamic acceleration $\breve{\mathbf{a}}_s$, the dynamic angular velocity $\breve{\omega}_s$, and the dynamic angular acceleration $\breve{\alpha}_s$ of a reference frame S fixed to a particle S, are given by:

$$\breve{\mathbf{r}}_s = \int\int (\mathbf{F}_0/m_s)\; dt\; dt$$

$$\breve{\mathbf{v}}_s = \int (\mathbf{F}_0/m_s)\; dt$$

$$\breve{\mathbf{a}}_s = (\mathbf{F}_0/m_s)$$

$$\breve{\omega}_s = \left|(\mathbf{F}_1/m_s - \mathbf{F}_0/m_s)/(\mathbf{r}_1 - \mathbf{r}_0)\right|^{1/2}$$

$$\breve{\alpha}_s = d(\breve{\omega}_s)/dt$$

where $\mathbf{F}_0$ is the net force acting on the reference frame S in a point 0, $\mathbf{F}_1$ is the net force acting on the reference frame S in a point 1, $\mathbf{r}_0$ is the position of the point 0 relative to the reference frame S (the point 0 is the center of mass of particle S and the origin of the reference frame S) $\mathbf{r}_1$ is the position of the point 1 relative to the reference frame S (the point 1 does not belong to the axis of rotation) and m_s is the mass of particle S (the vector $\breve{\omega}_s$ is along the axis of rotation)

The magnitudes $\breve{\mathbf{r}}$, $\breve{\mathbf{v}}$, $\breve{\mathbf{a}}$, $\breve{\omega}$, and $\breve{\alpha}$ are invariant under transformations between reference frames.

A reference frame S is inertial if $\breve{\omega}_s = 0$ and $\breve{\mathbf{a}}_s = 0$, but it is non-inertial if $\breve{\omega}_s \neq 0$ or $\breve{\mathbf{a}}_s \neq 0$.

In this paper it is assumed that the dynamic position $\breve{\mathbf{r}}_{cm}$, the dynamic velocity $\breve{\mathbf{v}}_{cm}$, the dynamic acceleration $\breve{\mathbf{a}}_{cm}$, the dynamic angular velocity $\breve{\omega}_{cm}$, and the dynamic angular acceleration $\breve{\alpha}_{cm}$ of the universal reference frame $\mathring{\text{S}}$ fixed to the center of mass of the universe are always zero.

In addition, the universal position $\mathring{\mathbf{r}}_{cm}$, the universal velocity $\mathring{\mathbf{v}}_{cm}$, and the universal acceleration $\mathring{\mathbf{a}}_{cm}$ of the center of mass of the universe are always zero.

Kinetic Force

The kinetic force $\mathbf{K}_{a|b}$ exerted on a particle A of mass m_a by another particle B of mass m_b, caused by the interaction between particle A and particle B, is given by:

$$\mathbf{K}_{a|b} = \frac{m_a m_b}{m_{cm}} (\mathring{\mathbf{a}}_a - \mathring{\mathbf{a}}_b)$$

where m_{cm} is the mass of the center of mass of the universe, $\mathring{\mathbf{a}}_a$ and $\mathring{\mathbf{a}}_b$ are the universal accelerations of particles A and B.

From the above equation it follows that the net kinetic force $\mathbf{K}_a$ acting on a particle A of mass m_a, is given by:

$$\mathbf{K}_a = m_a \mathring{\mathbf{a}}_a$$

where $\mathring{\mathbf{a}}_a$ is the universal acceleration of particle A.

From page [2], we have:

$$m_a \mathring{\mathbf{a}}_a - \mathbf{F}_a = 0$$

That is:

$$\mathbf{K}_a - \mathbf{F}_a = 0$$

Therefore, the total force $(\mathbf{K}_a - \mathbf{F}_a)$ acting on a particle A is always in equilibrium.

This paper considers that Newton's first and second laws are false, since there is no relation between the acceleration of a particle A and the total force acting on particle A.

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