Flux Divergence Method of solving Einstein equations

Dmitri Martila (eestidima@gmail.com)
Estonian Physical Society
Lääne 9-51, Tartu 50605, Estonia
(Dated: April 18, 2013)

Trying to solve Einstein Equation with new integral of motion (conserved quantity) for system. The integrals of motion are important. Let us say, that discovery of new one means the Nobel Prize. Using them the "unsolvable" problems solve. Several dust collapse solutions satisfy the new formulas. I propose to try them while collapse of perfect liquid sphere and all other questions. Critically discussed known conserved quantities. Revealed the nature of Dark Energy. Cancelled the cyclic Universe hypothesis (World will never be shrinking).

PACS numbers: 04.20.-q, 98.80.Bp, 98.80.-k, 01.55.+b, 10.00.00, 11.10.-z

I. INTRODUCTION

There is Newman-Janis complex transformation method [1], which not allways working. And explained is only for Schwarzschild starting spacetime [2] (and not quite clearly, for my opinion). From Schwarzschild metric the method gives Kerr metric [3], from Reissner-Nordström metric gives the Kerr-Newman metric [4]. The development of method: [5].

So the precedent is there: the theory, what not allways working, not quite explained, but still is accepted in a top journal. Therefore I propose to publish following new idea. I use spacetimes, where the norm of a timelike vector is negative. I use comma as index to show partial derivative. I use semicolon as index to show covariant derivative (the one with Christoffel symbols).

II. THE IDEA

By coordinate transformation one can allways turn the spacetime so, that g_{00} is independent from x^0. Secondly, let the spacetime satiesfy the energy conditions (i.e. be physically reasonable) throughout the space and time, hereby the Einstein equations let include the Dark Energy as part of Geometry law (i.e. sit on left hand side); namely $G^\mu_\nu + \kappa\Lambda\delta^\mu_\nu = \kappa\, T^\mu_\nu$, the $\kappa = 8\pi\, G/c^4$. Is sufficiently, but not necessary, that $T^1_0 = T^2_0 = T^3_0 \equiv 0$. Then cirtainly the following quantity

$$NP = -\int T^0_0\,\sqrt{-g}\,dV\,, \tag{1}$$

where $dV = dx^1\,dx^2\,dx^3$ (in spherical coordinates $dV = \frac{1}{c}\,dr\,d\theta\,d\phi$), is independent from x^0. The integral can have finite or infinite ranges. It is the System Integral of Motion. The *integrals of motion* are important. Let us say, that discovery of new one means the Nobel Prize (NP). Using them the "unsolvable" problems solve.

Even if several above conditions are violated, nevertheless the NP may occur as constant. You shall assume, that it leads to the natural solution, till the opposite is proved.

III. EXAMPLES

Marshall's corrected dust ball collapse [6] spherical-symmetric metric

$$ds^2 = -d\tau^2 + \Big(\frac{\partial W(\tau,R)}{\partial R}\Big)^2\,dR^2 + W^2(\tau,R)(d\theta^2 + \sin^2\theta\,d\theta^2)\,, \tag{2}$$

where

$$W(\tau,R) := R\,\Big(1 - \frac{3}{2}\,\tau\exp\big(\frac{3}{2}(1-R)\big)\Big)^{2/3}\,, \tag{3}$$

the centre has $R = 0$, the surface has $R = 1$, the gravitational mass is $M = 1/2$. After transformation $\tau = f(u)$ the spacetime violates $NP = const$, if is not $f(u) = C_1\,u + C_2$. In last case the $NP = const$ holds. Moreover, if $C_1 = 1$ the $NP = M = 1/2$.

I propose to study plane-symmetric dust collapse

$$ds^2 = -dt^2 + A^2(t,z)\,dz^2 + W^2(t,z)(dx^2 + dy^2) \tag{4}$$

and the axis-symmetric dust collapse

$$ds^2 = -dt^2 + \Big(\frac{\partial W(t,r)}{\partial r}\Big)^2\,dr^2 + W(t,r)^2\,d\phi^2 + B^2(t,r)\,dh^2\,. \tag{5}$$

One can try to find exact solutions, but I left the problem. All these two forms have matter tensor $T^\mu_\nu = \mathrm{diag}(-\rho,0,0,0)$, where the ρ is function of two coordinates. From matter equation

$$T^\mu_{\nu\,;\mu} = 0 \tag{6}$$

follows, that

$$\rho = \rho_0\sqrt{\frac{g_0}{g}}\,, \tag{7}$$

where index 0 means values at initial moment $t = t_0$. Thus, the $\rho\,\sqrt{g}$ is truly time-independent: $NP = const$.

In case of flat metric of Universe

$$ds^2 = -dt^2 + R^2(t)(dx^2 + dy^2 + dz^2) \tag{8}$$

the choices $R(t) = \sqrt{t}$ and $R(t) = t$ satiesfy the weak and null energy conditions (if Dark Energy is matter).

From Synge argument (reading the T^μ_ν from $g_{\mu\,\nu}$) holds $T^\mu_\nu = \mathrm{diag}(-F(t), A(t), A(t), A(t))$. The Einstein theory only allows knowing the natural $F(t)$ get $A(t)$ and $R(t)$. I believe the $NP = const$ selects from all possible functions $F(t)$ the historical, the natural one.

Looking at left hand of Einstein equations, the energy conditions are more hard to satiesfy, if the Dark Energy is Geometry. For any causal (i.e. timelike of null) vector must be

$$T_{\mu\nu}U^\mu U^\nu = G_{\mu\nu}U^\mu U^\nu + \kappa\Lambda\, U_\nu U^\nu \geq 0\,. \qquad (9)$$

Indeed, occurs negative addition with Λ. Shall we write minus sign before the Λ? No, putting the minus is against the observations. But choices $R(t) = \sqrt{t}$ and $R(t) = t$ violate energy conditions after certain point in time. So no wonder, that $NP = const$ is violated: these are not the true choices and truely, the Dark Energy is Geometry.

Consider Friedmann Universe with $NP = const$, Dark Energy is Geometry and free choice of closed $k = +1$, open $k = -1$ and flat $k = 0$ Geometry. Then the $(R_{,t})^2 \geq 0$ never turns to zero. In any variation of measured parameters. So the Universe is not cyclic: it expands and never will be shrinking.

I took Marshall's dust ball collapse and made coordinate transformation $\tau = f(t, R)$. The $NP = const$ holds, if $f(t, R) = t\, f_1(R) + f_2(R)$. This means, that g_{tt} is time-independent and the energy conditions are not violated in all space and time. Note, that making the $g_{tt} = -1$ in all spacetime doesn't give gravitational mass of stable liquid ball $NP \neq M$, however in case of Marshall's dust-ball collapse the $NP = M$.

I have derived (compare with [7]) the solution of stable liquid ball of constant density ρ is (the Dark Energy is ignored)

$$ds^2 = -A(r)\, dt^2 + \frac{dr^2}{B(r)} + r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2)\,, \qquad (10)$$

joint on surface to vacuum

$$B(r) = 1 - \frac{8}{3}\pi\,\rho\, r^2\,,$$

$$A(r) = \frac{5}{2} - 6\,R^2\,\pi\,\rho -$$

$$\frac{1}{2}\sqrt{(3 - 8\,R^2\,\pi\,\rho)\,(3 - 8\,\pi\,\rho\, r^2)} - \frac{2}{3}\pi\,\rho\, r^2\,,$$

$$p(r) = \rho\left(-1 + \sqrt{\frac{B(R)}{A(r)}}\right),$$

with R as star radius and $p(r)$ is pressure, the perfect liquid has $T^\mu_\nu = \mathrm{diag}(-\rho, p, p, p)$. The *natural units* set gravitational constant and light-speed to 1. Turns out, that down after $R = 2.25\, M$ there are singularities of $p(r)$. Thus, solution holds, if $R > 2.25\, M$.

IV. FIRST DERIVATION

I was tought, that divergence is the intensity of springs (sources). So I define the flux of a tensor as integral over its devergents. Demanding, that flux shall be invariant, I get covariant divergence, where invariant 4-volume element is $d\Omega := \sqrt{-g}\, dx^0\, dV$. Suppose the time axis changes $\Delta t = C_1\, \Delta u$, where units are $[t] = [sec]$, $[u] = [betasec]$, $[C_1] = [sec/betasec]$. The time-etalon is redefined and renamed. So the flux Φ as leaked mass per time changes:

$$\Phi_u := \frac{\Delta m}{\Delta u} = C_1\, \frac{\Delta m}{\Delta t} = C_1\, \Phi_t\,. \qquad (11)$$

Moreover, calculations show, that

$$NP_u = C_1\, NP_t\,. \qquad (12)$$

So, demand of invariance is demand of unchanging time-axis, in particular $C_1 = 1$. Thus transformation formula has

$$\frac{\partial x^\nu}{\partial u} = (1, 0, 0, 0)\,. \qquad (13)$$

Therefore, following flux is invariant:

$$\Phi = \int F^\mu_{\nu\,;\mu}\, \sqrt{-g}\, dx^0\, dV\,, \qquad (14)$$

where $\nu = 0$.

The invariant flux can be defined alternatively. It's the projections of the tensor on surface vectors. The invariant scalar products are

$$F^\nu_0\, d\sigma^{(1)}_\nu = \sqrt{-g}\, F^0_0\, dV\,, \qquad (15)$$

$$F^\nu_0\, d\sigma^{(2)}_\nu = \sqrt{-g}\, dx^0\, F^1_0\, dx^2\, dx^3\,. \qquad (16)$$

It is because the contravariant component $i = 0, 1$ in F^i_0 transform like dx^i inside the formula of invariant [7] 4-volume element $d\Omega = \sqrt{-g}\, dx^0\, dx^1\, dx^2\, dx^3 = inv.$

Thus, we demand the rightness of both flux definitions, so the Flux Divergence Theorem:

$$\oint F^\nu_0\, d\sigma_\nu = \int F^\mu_{\nu\,;\mu}\Big|_{\nu=0} d\Omega\,. \qquad (17)$$

The insertion of very first metric and F has satiesfied the theorem. Indeed, take $F^\mu_\nu = A^\mu A_\nu$, where $A^\mu = (t, r, 0, 0)/(1+r)^4$, $ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$, where $g_{tt} = -1 - 1/(1+r)$, $g_{rr} = (2+r)/(3+r)$. A container with volume Ω: upper plane has $t = t2$, lower plane has $t = t1$, sides have $r = R$. The spacetime violates the weak energy condition, but g_{tt} is time-independent.

Then Eq.(17) is satisfied only if on upper plane $d\sigma_\mu = (d\sigma, 0, 0, 0)$, where $d\sigma := \sqrt{-g}\, dr\, d\theta\, d\phi$, on lower plane $d\sigma_\mu = (-d\sigma, 0, 0, 0)$, on sides $d\sigma_\mu = (0, d\psi, 0, 0)$, where $d\psi := \sqrt{-g}\, dt\, d\theta\, d\phi$.

If it is $F^\mu_\nu := \delta^\mu_\nu$, then there is counterexample: dust ball collapse.

V. SECOND DERIVATION

What is physical meaning of this NP? If metric is diagonal, the

$$dM = -T^0_0 \sqrt{-g_{11} g_{22} g_{33}}\, dV \tag{18}$$

is energy (not density) measured by local observer. The

$$M = \sum \sqrt{-g_{00}}\, dM = const\,. \tag{19}$$

is quantity, what conserves during evolution; like the integral of motion of test particle in Schwarzschild spacetime [8]

$$E \sqrt{-g_{00}} = const\,, \tag{20}$$

where E is total energy of test particle, measured locally by stationary observer. As you see, the dM must be valid energy of system. Thus, at least all energy conditions must be satiesfied.

VI. WHO WANTS BUCKET OF LIQUID?

A liquid-ball. Distant observer slowly takes the water amount dm using a bucket on very long cord and destributes it throughout the Universe. He losts on his efforts dE_0 energy to pull the bucket. The local stationary observers at the surface measure the energy of the thin layer dR (outside the ball is Schwarzschild metric)

$$dm = T^0_0 \sqrt{\frac{g}{g_{tt}}}\, dV = 4\pi\, \rho\, \frac{R^2\, dR}{\sqrt{1-2M/R}} = \frac{dM}{\sqrt{1-2M/R}}\,. \tag{21}$$

There is energy conservation

$$dm - dE_0 = dm \sqrt{1-\frac{2M}{R}} = dM\,. \tag{22}$$

Thus, integral gives

$$\Delta m - \Delta E_0 = M\,, \tag{23}$$

where amount of gotten liquid is

$$\Delta m = \int \frac{dM}{\sqrt{1-2M/R}}\,. \tag{24}$$

Liquid ball solution Eq.(10) of constant density provides

$$M = \frac{4}{3}\, \pi\, \rho\, R^3\,. \tag{25}$$

I calculated the NP, if on ball solution is made $g_{uu} = -1$ by coordinate transformation $t = u\, f(r)$. Then holds exactly:

$$\Delta m = NP\,, \tag{26}$$

where Δm and NP are calculated in different coordinates.

Conclusion: in stationary case the $\Delta E_0 = NP - M$ is work needed to be done to get needed substance of amount NP. The ΔE_0 is hight of potential barrier needed to overcome.

But also in unstationary, Marshall's dust collapse spacetime (2). In distant past $\tau \to -\infty$ the dust would fill all spacetime $r := W(\tau, 1) \to \infty$. Thus, the dust is distributed around the world even without use of bucket. Therefore, the obstacle to distribute the dust is zero $NP - M = 0$.

VII. MORE ON APPLICATION TO UNIVERSE

In General Relativity from geometrical reasons (s.c. Synge argument) the "metric" of isotropic, homogeneous Universe is

$$ds^2 = -c^2\, dt^2 + R^2(t) \left(\frac{dr^2}{1-k\, r^2} + r^2 (d\theta^2 + \sin^2\!\theta\, d\phi^2) \right). \tag{27}$$

Inserting it into Einstein Equations, I get true form of matter tensor:

$$T^\mu_\nu = \mathrm{diag}(-F(t), A(t), A(t), A(t))\,. \tag{28}$$

The physical meaning of $F(t)$ is energy density, because stationary observer $u^\mu = (1,0,0,0)$ measures it $\delta = F(t)$, see Eq.(50). The physical meaning of $A(t)$ is not fixed: we don't know the matter tensors of dark matter and perhaps electromagnetic fields, but sum of all factors produces the Eq.(28). Applying $NP = const$ we get

$$F(t) = F_0 \left(\frac{R_0}{R(t)} \right)^3. \tag{29}$$

From Eqs.(29),(6) and established $R_{,t} \neq 0$ the $A(t) \equiv 0$. You try to reject my paper, on the believe, that $A(t)$ is pressure and, thus, can't remain zero? Then the zero is less strange, than the negative pressure of the believed Universe dynamics. Hereby in fixation of $A(t) \equiv 0$ I see the *fine tuning*, we talk about.

I use modificated Einstein Equations, where "dark energy" is expressed through geometric term on the left, see Wikipedia:

$$G^\mu_\nu + \kappa \Lambda \delta^\mu_\nu = \kappa\, T^\mu_\nu\,, \tag{30}$$

where G^μ_ν is s.c. Einstein's tensor. The Geometry Lagrangian is then $L = \hat{R} + \Lambda$, where $\hat{R}$ is s.c. scalar curvature. Thus now the dark energy is part of Geometry law. It's not materia, which on the right sides of Eqs. (30) and (33). Thus,

$$2R\, R_{,tt} + (R_{,t})^2 + kc^2 = R^2\, c^2 \kappa\, \Lambda\,, \tag{31}$$

$$-3R\,(R_{,t})^2/c^2 - 3k\, R + R^3\, \kappa\, \Lambda = -\kappa\, F_0\, (R_0)^3\,. \tag{32}$$

Taking derivative from second equation (32) gives first one (31), so if second is satiesfied also holds first one.

Let us assume, that dark energy is matter, i.e. holds

$$G^{\mu}_{\nu} = \kappa T^{\mu}_{\nu} , \qquad (33)$$

where $F_0 = \rho_0 c^2 + \Lambda$, here ρ_0 includes *dark matter* and ordinary, baryonic matter. Then

$$2R R_{,tt} + (R_{,t})^2 + kc^2 = 0 , \qquad (34)$$

$$-3R (R_{,t})^2/c^2 - 3k R = -\kappa F_0 (R_0)^3 . \qquad (35)$$

Taking derivative I get first equation (34). Structure of first equation shows, that must be $k = -1$ (because is established $R_{,tt} > 0$). This is not case of the popular flat Universe model, also it is not the Biblical model.

A. Comparison with experiments

From Wikipedia (also Russian one) the set of parameters, I use, is:

$$\Lambda \approx 5.98 \,\mathrm{x}\, 10^{-10}\,\mathrm{J/m^3} , \quad F_0 \approx \frac{27.2}{72.8}\Lambda , \quad F_0 \approx \frac{27.2}{72.8}\Lambda + \Lambda , \qquad (36)$$

$$H_0 \approx 69.32 \,\frac{\mathrm{km/s}}{\mathrm{Mpc}} , \qquad (37)$$

where the higher F_0 is for "dark energy is matter" variant. Then this variant, that dark energy is materia, is excluded: any possible curves are close to line (1) dismatching the experimental points (see Fig.1). So, the dark energy is Geometry.

In such case can we distinguish betwean different k? Possibly. The line (2) is not perfect: entire 4 points are out of "standart" range. But curve (3) with $R_0 = 27.1\,\mathrm{Gpc}$ lefts only one point out. Note, that allready 1.5x 27.1 Gpc is "bad" match: 3 points out. This line is closer to line (2). Taking higher the R_0 we get even more closer to line (2). If you take $k = 0$ then solutions are at line (2), hereby my eye can not distinguish lines of different R_0. If $k = -1$ solution is higher then line (2), so the higher mismatch than the line (2).

But the set Eq.(37) with $R_0 = 27.1\,\mathrm{Gpc}$, $k = 1$ violates Einstein's equation (32) at present time $t = t_0 = 0$. But if to replace Λ with 1.3 percent higher value, then Eq.(32) is satiesfied and the solution is between (2) and (3) lines, so still having better match, than line (2). Hereby case 5x 27.1 Gpc closely resembles line (2), being partly higher than it.

Conclusion: the Biblical model more beautifully matches the data.

B. Biblical model

Let us draw a circle in the closed Friedmann universe [10]. The idea is to assume squared interval $ds^2 = -c^2 dt^2 + R^2 (d\xi^2 + \sin^2\xi\,(d\theta^2 + \sin^2\theta\, d\phi^2))$. Scale factor R expands with time t. We get the perimeter of the circle $L = 2\pi R_0 \sin\xi = 30\,cubit$ and diameter $2a = 2R_0\xi = 10\,cubit$, where metric spatial coordinate $\xi = \pi/6$ and scale factor $R_0 = 30/\pi\, cubit$. The 30 and 10 are in 1 Kings 7:23.

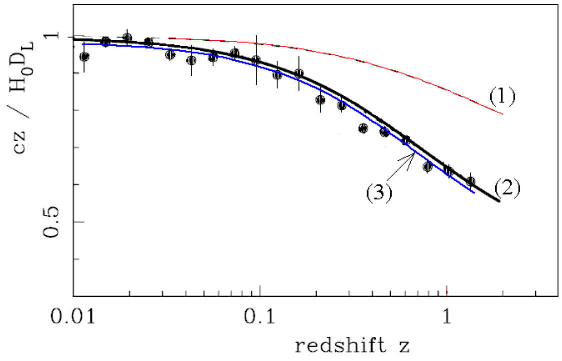

FIG. 1: Picture from internet. My numerical blue line (3) more perfectly matches the experimental points. Analytic solution (1) is from assumptions, that dark energy is matter and $k = 0$. It matches the $\Omega_m = 1$ line of Ref.[9]. Line (2) is the thick line $\Omega_m = 0.3$, $k = 0$ from Ref.[9].

The molten sea was observed in Solomon's temple. The diameter of the observable universe is $2a = 28.3\,\mathrm{Gpc}$ [11]. Thus, using the equation for the diameter $2a = 2R_0\xi$ again, at the present time, the $R = R_0 \approx 27.1\,\mathrm{Gpc}$.

The topology scale d of hypothetical non-trivial topology in [11, 12] is not the geometrical R. These specialists distinguish geometry and topology. Note, that we have also "topological censorship theorem".

Because the most severe constraint cannot reach 28 Gpc, (read [12]), the line of termination will be always on the very edge of the Biblical Model. In case 27.1 Gpc is wrong, I will take the next possible number between 27.1 Gpc and 28 Gpc (but the current Biblical model would be disproved). Why? Take into account that God is not forcing us into a Biblical World. So, as I can accept, He left the Biblical Model in balancing between proof and disproof.

C. Singularity

Let's find the moment of virtual inflation. In Biblical model the edge of observable Universe has $\xi = \pi/6$. This corresponds to parameter $t \approx -14.0995$ billion years, scale factor $R(t) \approx 53\,\mathrm{Mpc}$ and maximum cosmological redshift $z_m \approx 512$. The singularity has $t \approx -14.101$ (it is almost the $1/H_0$).

Taking not the Biblical model: $R_0 = 20 \,\mathrm{x}\, 27.1\,\mathrm{Gpc}$ we get $\xi \approx 0.02611$, singularity has $t = -13.8992$, the relict "light" has $t = -13.8982$, the $R(t) \approx 799\,\mathrm{Mpc}$ and $z_m \approx 677$.

Indeed, the Wikipedia tells, that the redshift of relict photons is in the order of thousand, however found Ia supernovas have $z < 2$ some quasars have $z \approx 6$. Of course, not all relict light is hitting telescope right now, so the sphere of relict light's ξ origin is really the "observable Universe".

In these two models ($R_0 = 27.1\,\mathrm{Gpc}$ and $R_0 = 20\,\mathrm{x}\,27.1\,\mathrm{Gpc}$) the relict light is at million years after singularity, however in Wikipedia it is allmost half million years. So there is new result in present paper: the scientific world does not know our integral of motion $NP = const.$

VIII. OTHER INTEGRALS OF MOTION

Let's start with $F^{i\,k\,l}$, which

$$F^{i\,k\,l} = -F^{i\,l\,k}\,. \tag{38}$$

Then

$$\frac{\partial^2 F^{i\,k\,l}}{\partial x^k \partial x^l} \equiv F^{i\,k\,l}_{,k\,,l} = 0\,. \tag{39}$$

Integrating

$$\int F^{i\,k\,l}_{,k\,,l} dV \equiv \int \Big(\frac{\partial F^{i\,0\,l}_{,l}}{\partial x^0} + \frac{\partial F^{i\,1\,l}_{,l}}{\partial x^1} + \frac{\partial F^{i\,2\,l}_{,l}}{\partial x^2} + \frac{\partial F^{i\,3\,l}_{,l}}{\partial x^3}\Big) dV\,, \tag{40}$$

where $dV := dx^1\,dx^2\,dx^3$. Such kind of calculations is well shown in [13], page 876. If holds

$$\int \frac{\partial F^{i\,1\,l}_{,l}}{\partial x^1} dV + \int \frac{\partial F^{i\,2\,l}_{,l}}{\partial x^2} dV + \int \frac{\partial F^{i\,3\,l}_{,l}}{\partial x^3} dV = 0\,, \tag{41}$$

then

$$\int \frac{\partial F^{i\,0\,l}_{,l}}{\partial x^0} dV = 0\,. \tag{42}$$

If the ranges of integration are time-independent (or do not influence), then

$$\frac{\partial}{\partial x^0} \int F^{i\,0\,l}_{,l} dV = 0\,. \tag{43}$$

Only then I have "System Integral of Motion"

$$P^i = \int F^{i\,0\,l}_{,l} dV = const. \tag{44}$$

It can be, but not necessary, the energy-momentum vector.

From Landau's book [8] (see the *energy-momentum pseudotensor* paragraph)

$$F^{i\,k\,l} := \frac{c^4}{16\pi G}\,[(-g)\,(g^{i\,k}\,g^{l\,m} - g^{i\,l}\,g^{k\,m})]_{,m}\,. \tag{45}$$

Landau formula (applied to my liquid ball) gave negative $P^0 \approx -0.002897$, which absolute value is not gravitational mass $M \approx 0.015451$. Negative energy can not be, such energy is against the energy conditions. But in Landau's book the P^0 equals the energy (in $c = 1$ units system). But it turned out negative and with wrong absolute value (further read Sections X and IX).

Eq.(5) in Ref.[14], the "Einstein complex" gives

$$F^{k\,l}_i := \frac{c^4}{16\pi G}\,\frac{g_{i\,s}}{\sqrt{-g}}\,[(-g)\,(g^{k\,s}\,g^{l\,m} - g^{l\,s}\,g^{k\,m})]_{,m}\,. \tag{46}$$

Using it in above formulas simply picture index i being written below (i.e. as covariant one).

Eq.(11) in Ref.[14], the Møller formula gives

$$F^{k\,l}_i := \frac{c^4}{8\pi G}\,\sqrt{-g}\,g^{k\,m}\,g^{l\,j}\,(g_{i\,j\,,m} - g_{i\,m\,,j})\,. \tag{47}$$

Generally, variants to think out is infinitely many. More examples. Because Ricci tensor is symmetric like $g^{i\,k}$

$$F^{i\,k\,l} := \psi(x^0, x^1, x^2, x^3)\,[R^{ik}R^{lm} - R^{il}R^{km}]_{,m} \tag{48}$$

with arbitrary, tunable function ψ of spacetime coordinates (or simply $\psi \equiv 1$ if you choose). Important could become things for matter tensor $\mathbf{T}$, for example

$$F^{i\,k\,l} := [T^{ik}T^{lm} - T^{il}T^{km}]_{,m} \tag{49}$$

Landau's integral of motion (from Eq.(45)), if instead of g^{ik} there is combination of tensors $a\,G^{ik} + b\,R^{ik} + d\,g^{ik} + const^{ik}$ and instead of $(-g)$ a tunable function $w(t,x) := W(t)\,f(x)$. Tuning the W, I still have not found restriction for Universe scale factor $R(t)$ (considered simplest, $-dt^2 + R^2(t)(dx^2 + dy^2 + dz^2)$ metric). The wanted equation would with combination of $NP = const$ theoretically determine the experimental constants $k, H_0, R_0, \rho_0, \Lambda$. At least, one could prove, that $NP = const$ is law in nature of Universe. The curvature tensor product $F^{i\,s\,k\,l} = W(t)\,f(x)\,R^{iskl}$ with antisymmetric pair $k\,l$ was also checked with no result. Conclusion: the initial point is any antisymmetric thing F^{ik}. I found in components, that this theory says, what $f(t) := F^{1\,0}_{,1} + F^{2\,0}_{,2} + F^{3\,0}_{,3}$ is time independent, if $df(t)/dt = 0$. Thus, it is identity expressions. This method does not constrain the metric functions. As also Landau case showed. The known conservation laws [8, 14], "system integrals of motion" (Einstein, Møleer and Landau) are just identity forms, may be usefull in theoretical considerations. Like the very useful identity is $g^{\nu\mu}_{;\mu} = 0$, which however does not constrain the metric functions. Indeed, we started with identity $F^{ik}_{,i\,,k} = 0$, which holds for any metric. Thus, it and its consequences does not constrain the metric functions. But, my Gauss flux theorem with arbitrary $\mathbf{F}$ works not for any metric in Synge argument. Thus, it is not an identity and may constrain the metric functions.

IX. ON ENERGY IN GENERAL RELATIVITY

The isolated system has gravitational mass as integral of motion. Like in the Birkhoff's theorem. The total energy also conserves. I recognize the energy as gravitational mass (times c^2) of an asymptotically flat system. In General Relativity the gravitational mass equals the inertial mass (theoretical proof is in [8]) and inertial mass times c^2 is energy [8], according to Special Relativity. Looking at electron we recognizing it: the space integral of energy density T_0^0 does not provide the experimental value of energy even in "Minkowski spacetime" approximation (integrating outside sphere of s.c. classical radius). The Tolman's formula uses other diagonal elements of matter tensor and provides correct energy of electron. Indeed, because electron is splitted [15], it is not a point; and observed perfect sphere [16] of, indeed, classical radius (as reported in media) has sharp boundaries, which "fur coat" of virtual particles around pointlike electron can not have. Here I can not see the place for Higgs boson.

The local energy density δ observer measures the known way: $\delta = T_{\mu\nu}\, u^\mu\, u^\nu$, for observer at rest

$$\delta = -T_0^0\,. \tag{50}$$

X. AGAIN ON GAUSS THEOREM

In Landau's book [8]

$$\oint X^{i\,k}\, dS_k = \int X^{i\,k}_{,k}\, dx^0\, dx^1\, dx^2\, dx^3\,. \tag{51}$$

Here on left is box

$$A_0 < x^0 < B_0\,, \quad A_1 < x^1 < B_1\,, \quad A_2 < x^2 < B_2\,,$$

$$A_3 < x^3 < B_3\,. \tag{52}$$

With accompanying number complects $dS_k = (dx^1\, dx^2\, dx^3, 0, 0, 0)$ at B_0, $dS_k = (-dx^1\, dx^2\, dx^3, 0, 0, 0)$ at A_0, $dS_k = (0, dx^0\, dx^2\, dx^3, 0, 0)$ at B_1, $dS_k = (0, -dx^0\, dx^2\, dx^3, 0, 0)$ at A_1, $dS_k = (0, 0, dx^0\, dx^1\, dx^3, 0)$ at B_2, $dS_k = (0, 0, -dx^0\, dx^1\, dx^3, 0)$ at A_2, $dS_k = (0, 0, 0, dx^0\, dx^1\, dx^2)$ at B_3, $dS_k = (0, 0, 0, -dx^0\, dx^1\, dx^2)$ at A_3.

It is in Cartesian coordinates t, x, y, z the closed box. But looking from our room on box of spherical coordinates $x^\mu = (t, r, \theta, \phi)$ in formula (52) it is not closed surface. The Gauss theorem, which likes to use the Landau, uses just the closed surface. So the Landau's result holds only for Cartesian coordinates (in which the metric is asymptotically Minkowskian).

Indeed, example of liquid ball, shows, that in spherical coordinates Landau formula for P^0 does not give the gravitational mass. The last we get after coordinate transformation $x = r\cos\phi\sin\theta$, $x = r\sin\phi\sin\theta$, $z = r\cos\theta$ and integrating also outside the ball. In Landau formula the curved vacuum gives nonzero, negative addition to gravitational mass. But there are "positive mass theorem" [17], which says, that mass can not be negative; I believe, even that of curved vacuum. The Møller formula [14] seems to give gravitational mass in every coordinate system, but in case of dust collapse it does not working.

[1] Newman ET. 1973. J Math Phys 6:774.
[2] Talbot CJ. 1969. Newman-Penrose Approach to Twisting Degenerate Metrics. Commun math Phys 13:45-61.
[3] Newman ET, and Janis AI. 1965. J Math Phys 6:915.
[4] Newman ET. 1965. J Math Phys 6:918.
[5] Hernando Quevedo. 1992. Complex Transformation of the Curvature Tensor. Gen Rel Grav 24:693.
[6] Trevor W Marshall, Max K Wallis. 2010. Journal of Cosmology 6:1473-84. Available from: http://journalofcosmology.com/MarshallWallis.pdf
[7] Lightman AP, Press WH, Price RH, Teukolsky SA. Problem Book in Relativity and Gravitation. Princeton, Princeton University Press, 1975.
[8] Landau LD, Lifshitz EM. The classical theory of fields. Oxford, Pergamon Press, 1975.
[9] Olive KA, Peacock JA. 2012. *Chapter 19. Big-Bang cosmology*, http://pdg.lbl.gov/2011/reviews/rpp2011-rev-bbang-cosmology.pdf, Nakamura K, et al. 2010. Review of Particle Properties. J Phys G 37:075021
[10] Hawking SW, and Ellis GFR. The Large Scale Structure of Space-Time. Cambridge University Press, 1975.
[11] Bielewicz, P., and Banday AJ. 2011. Constraints on the topology of the universe derived from the 7-yr WMAP data. Monthly Notices of the Royal Astronomical Society 412:2104-10. Available on arXiv:1012.3549v2
[12] Neil J Cornish, David N Spergel, Glenn D Starkman, and Eiichiro Komatsu. 2004. Constraining the Topology of the Universe. Physical Review Letters 92:201302. Available on arXiv:astro-ph/0310233v1
[13] Tolman RC. 1930. On the use of the energy-momentum principle in General Relativity. The Phys Rev 35:875-95.
[14] Florides PS. 2009. On the Tolman and Møller mass-energy formulae in general relativity. J Phys: Conf Ser. 189:012014.
[15] Zeeya Merali. Not-quite-so elementary, my dear electron. Nature News. 2012 April 18; Ralph Claessen. 2012. Solid-state physics: Electrons do the split. Nature 485:46-7; 2012. Nature 485:82-5.
[16] Edwin Cartlidge. Rounding the electron. Nature News. 2011 May 25; 2011. Nature 473:493-6.
[17] Horowitz GT, and Perry MJ. 1982. Gravitational mass cannot become negative. Phys Rev Lett 48:371, Schoen, R., and Yau ST. 1982. Proof that the Bondi mass is positive. Phys Rev Lett 48:369, Ludvigsen, M., and Vickers JAG. 1981. The positivity of the Bondi mass. J Phys A 14:L389.