The Creator's Equation

Jin He and Xiaoli Yang

http://www.galaxyanatomy.com/
Wuhan FutureSpace Scientific Corporation Limited,
Wuhan, Hubei 430074, China
E-mail: mathnob@yahoo.com

Abstract Is the sum of rational structures also a rational structure? It is called the Creator's big question for humans. Numerical calculation suggests that it is approximately rational for the fitted parameter values of barred spiral galaxies. However, we need mathematical justification. The authors are very old and are not experts in mathematics. Please help us humans to resolve the question.

keywords: Rational Structure; Calculus; Partial Differential Equation

PACS: 02.30.Jr

1 Rational Structure

Rational structure in two dimension

$$\rho(x,y) \tag{1}$$

means that not only there exists an orthogonal net of curves in the plane

$$x = x(\lambda,\mu),\ y = y(\lambda,\mu) \tag{2}$$

but also, for each curve, the matter density on one side of the curve is in constant ratio to the density on the other side of the curve. Such a curve is called a proportion curve or a Darwin curve. Such a distribution of matter is called a rational structure.

Because the ratio of density $\rho(x,y)$ is proportional to the derivative to the logarithm of the density

$$f(x,y) = \ln\rho(x,y) \tag{3}$$

we, from now on, are only concerned with the logarithmic density $f(x,y)$. We know that, given the two partial derivatives

$$\frac{\partial f}{\partial x},\quad \frac{\partial f}{\partial y} \tag{4}$$

the structure $f(x,y)$ is determined provided that the Green's theorem is satisfied

$$\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x} \tag{5}$$

Now we are interested in rational structure and ignore the partial derivatives (4). Instead, we calculate the directional derivatives along the tangent direction to the above curves (2)

$$\frac{\partial f}{\partial l_\lambda},\quad \frac{\partial f}{\partial l_\mu} \tag{6}$$

where l_λ is the linear length on the (x, y) plane and is along the row curves whose parameter is λ while l_μ is the linear length along the column curves whose parameter is μ. Given the two partial derivatives (6), however, the structure $f(x, y)$ may not be determined. A similar Green's theorem must be satisfied

$$\frac{\partial}{\partial \mu}\left(P\,\frac{\partial f}{\partial l_\lambda}\right) - \frac{\partial}{\partial \lambda}\left(Q\,\frac{\partial f}{\partial l_\mu}\right) = 0 \tag{7}$$

where

$$P(\lambda, \mu) = \sqrt{x'^2_\lambda + y'^2_\lambda}, \quad Q(\lambda, \mu) = \sqrt{x'^2_\mu + y'^2_\mu} \tag{8}$$

are the lengths or magnitudes of the vectors (x'_λ, y'_λ) and (x'_μ, y'_μ) on the (x, y) plane, respectively. Note that we have used the simple notation $x'_\lambda = \frac{\partial x}{\partial \lambda}$. From now on, we always use the similar simple notation. To simplify the expression of our equations, we introduce one more notation

$$u(\lambda, \mu) = \frac{\partial f}{\partial l_\lambda}, \quad v(\lambda, \mu) = \frac{\partial f}{\partial l_\mu} \tag{9}$$

As you might know, the above notation is very useful. However, our articles on rational structure which employ the notations were rejected hundreds of times by the editors of over fifty scientific journals. We are afraid that the editors could not understand the notation at all.

The condition of rational structure is that u depends only on λ and v depends only on μ

$$u = u(\lambda), \;\; v = v(\mu) \tag{10}$$

Now we prove the condition. Assume you walk along a row curve. The logarithmic ratio of the density on your left side to the immediate density on your right side is approximately the directional derivative of $f(x, y)$ along the column direction. That is, the logarithmic ratio is approximately the directional derivative $v(\lambda, \mu)$. Because $v(\lambda, \mu)$ is constant along the row curve (rational), $v(\lambda, \mu)$ is independent of λ: $v = v(\mu)$. Similarly, we can prove that $u(\lambda, \mu) = u(\lambda)$.

2 Rational Structure Equation

In the case of rational structure, the directional derivatives, $u = \frac{\partial f}{\partial l_\lambda}$ and $v = \frac{\partial f}{\partial l_\mu}$, are the functions of the single variables λ and μ, respectively (see the formula (10)). Therefore, the Green's theorem (7) turns out to be much simpler that is called the rational structure equation [1,2]

$$u(\lambda) P'_\mu = v(\mu) Q'_\lambda \tag{11}$$

To transform the equation and find its geometric meaning, we calculate

$$\begin{aligned} P'_\mu &= (x'_\lambda x''_{\lambda\mu} + y'_\lambda y''_{\lambda\mu})/P, \\ Q'_\lambda &= (x'_\mu x''_{\lambda\mu} + y'_\mu y''_{\lambda\mu})/Q \end{aligned} \tag{12}$$

That is,

$$\begin{aligned} P'_\mu &= \hat{\mathbf{x}}'_\lambda \cdot \mathbf{x}''_{\lambda\mu}, \\ Q'_\lambda &= \hat{\mathbf{x}}'_\mu \cdot \mathbf{x}''_{\lambda\mu} \end{aligned} \tag{13}$$

where boldface letters are the notations of vectors

$$\mathbf{x} = (x, y), \quad \mathbf{x}'_\lambda = (x'_\lambda, y'_\lambda), \quad \mathbf{x}''_{\lambda\mu} = (x''_{\lambda\mu}, y''_{\lambda\mu}), \quad \text{etc.} \tag{14}$$

The hats above letters mean that the corresponding vectors are unit ones. The dot symbol is the inner product of vectors. The geometric meaning of the formula (13) is that P'_μ is the projection of the vector $\mathbf{x}''_{\lambda\mu}$ in the direction of the vector $\mathbf{x}'_\lambda$ and Q'_λ is the projection of the same vector in the direction of the vector $\mathbf{x}'_\mu$. They are also the geometric meaning of our final rational structure equation (16) (see also the paper [3]).

Finally, our rational structure equation becomes

$$u(\lambda)\hat{\mathbf{x}}'_\lambda \cdot \hat{\mathbf{x}}''_{\lambda\mu} = v(\mu)\hat{\mathbf{x}}'_\mu \cdot \hat{\mathbf{x}}''_{\lambda\mu} \tag{15}$$

However, the solution $f(x, y)$ of the equation may not be rational structure because the net of curves may not be orthogonal. The solution of the following equation system must be rational

$$\begin{cases} u(\lambda)\hat{\mathbf{x}}'_\lambda \cdot \hat{\mathbf{x}}''_{\lambda\mu} = v(\mu)\hat{\mathbf{x}}'_\mu \cdot \hat{\mathbf{x}}''_{\lambda\mu}, \\ \mathbf{x}'_\lambda \cdot \mathbf{x}'_\mu = 0 \end{cases} \tag{16}$$

where the second equation is the orthogonal condition.

3 The Creator's Equation

Now we want to change the rational structure equation into the Creator's equation. It is true that the Creator's equation is a different form of the rational structure equation but the new form is more elegant and should be easier for its solution.

We know that $f(x, y)$ is the function of Cartesian coordinates x, y. However, the equation system (2) suggests that it is also the composite function of the parameters λ, μ. We use the same symbol f to denote the composite function. For example, we denote its partial derivatives by f'_λ, f'_μ. It is straightforward to show that the partial derivatives are

$$\begin{aligned} f'_\lambda &= u(\lambda)P(\lambda, \mu), \\ f'_\mu &= v(\mu)Q(\lambda, \mu) \end{aligned} \tag{17}$$

That is,

$$\begin{aligned} u(\lambda) &= f'_\lambda / P(\lambda, \mu), \\ v(\mu) &= f'_\mu / Q(\lambda, \mu) \end{aligned} \tag{18}$$

Therefore,

$$\begin{aligned} (u(\lambda))'_\mu &= (f'_\lambda / P(\lambda, \mu))'_\mu = 0, \\ (v(\mu))'_\lambda &= \left(f'_\mu / Q(\lambda, \mu)\right)'_\lambda = 0 \end{aligned} \tag{19}$$

That is,

$$\begin{aligned} f''_{\lambda\mu} &= \tfrac{f'_\lambda}{P} P'_\mu, \\ f''_{\mu\lambda} &= \tfrac{f'_\mu}{Q} Q'_\lambda \end{aligned} \tag{20}$$

Finally we have our Creator's equation

$$\frac{f'_\lambda}{P} P'_\mu = \frac{f'_\mu}{Q} Q'_\lambda \tag{21}$$

Noticing the formulas (18), we find out that the Creator's equation is nothing but the different form of the rational structure equation (11).

Using the derivative rule of composite functions for f, we have the second form of the Creator's equation

$$\frac{(f'_x x'_\lambda + f'_y y'_\lambda)(x'_\lambda x''_{\lambda\mu} + y'_\lambda y''_{\lambda\mu})}{x'^2_\lambda + y'^2_\lambda} = \frac{(f'_x x'_\mu + f'_y y'_\mu)(x'_\mu x''_{\lambda\mu} + y'_\mu y''_{\lambda\mu})}{x'^2_\mu + y'^2_\mu} \tag{22}$$

Using the same vector notation in the last Section, we have the third form of the Creator's equation

$$\nabla f \cdot \hat{\mathbf{x}}'_\lambda \, \hat{\mathbf{x}}'_\lambda \cdot \mathbf{x}''_{\lambda\mu} = \nabla f \cdot \hat{\mathbf{x}}'_\mu \, \hat{\mathbf{x}}'_\mu \cdot \mathbf{x}''_{\lambda\mu} \tag{23}$$

The geometric meaning of the Creator's equation is that the two vectors

$$\begin{aligned} \mathbf{A} &= \begin{pmatrix} \hat{\mathbf{x}}'_\lambda \cdot \mathbf{x}''_{\lambda\mu} \\ \hat{\mathbf{x}}'_\mu \cdot \mathbf{x}''_{\lambda\mu} \end{pmatrix} \\ \mathbf{B} &= \begin{pmatrix} \nabla f \cdot \hat{\mathbf{x}}'_\mu \\ \nabla f \cdot \hat{\mathbf{x}}'_\lambda \end{pmatrix} \end{aligned} \tag{24}$$

are parallel to each other. Therefore, we have the fourth form of the Creator's equation

$$\mathbf{A} \sim \mathbf{B} \tag{25}$$

The geometric meaning of the formula is shown in the Figure 1. It is that the directions of the two vectors $\mathbf{x}''_{\lambda\mu}$ and $\nabla f = (f'_x, f'_y)$ are symmetric with respect to the bisector line between the two vectors $\mathbf{x}'_\lambda$ and $\mathbf{x}'_\mu$.

If we assume $P(\lambda, \mu) = Q(\lambda, \mu)$,

$$\sqrt{x'^2_\lambda + y'^2_\lambda} = \sqrt{x'^2_\mu + y'^2_\mu} \tag{26}$$

then we can take off hats from the letters in our formulas, and the Creator's equation (25) takes the fifth form

$$\begin{pmatrix} x'_\lambda & y'_\lambda \\ x'_\mu & y'_\mu \end{pmatrix} \begin{pmatrix} x''_{\lambda\mu} \\ y''_{\lambda\mu} \end{pmatrix} \sim \begin{pmatrix} x'_\mu & y'_\mu \\ x'_\lambda & y'_\lambda \end{pmatrix} \begin{pmatrix} f'_x \\ f'_y \end{pmatrix} \tag{27}$$

Equivalently, the Creator's equation is,

$$\begin{pmatrix} x''_{\lambda\mu} \\ y''_{\lambda\mu} \end{pmatrix} \sim \begin{pmatrix} (x'_\mu y'_\mu - x'_\lambda y'_\lambda) f'_x + (y'^2_\mu - y'^2_\lambda) f'_y \\ (x'^2_\lambda - x'^2_\mu) f'_x - (x'_\mu y'_\mu - x'_\lambda y'_\lambda) f'_y \end{pmatrix} \tag{28}$$

We present the Creator's equation in the sixth form which might have some different meaning

$$\begin{pmatrix} x'_\lambda & y'_\lambda \\ x'_\mu & y'_\mu \end{pmatrix} \begin{pmatrix} x''_{\lambda\mu} \\ y''_{\lambda\mu} \end{pmatrix} \sim \begin{pmatrix} f'_\mu \\ f'_\lambda \end{pmatrix} \tag{29}$$

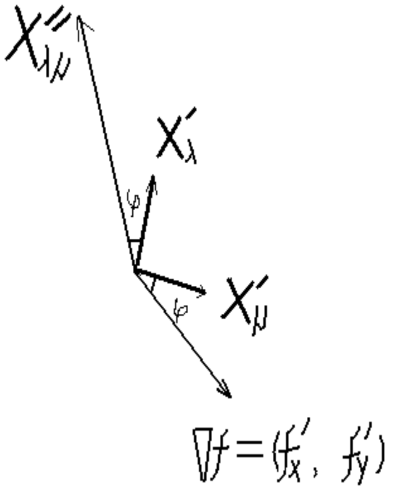

Figure 1: The directions of the two vectors $\mathbf{x}''_{\lambda\mu}$ and $\nabla f = (f'_x, f'_y)$ are symmetric with respect to the bisector line between the two vectors $\mathbf{x}'_\lambda$ and $\mathbf{x}'_\mu$.

Equivalently, it is,

$$\begin{pmatrix} x''_{\lambda\mu} \\ y''_{\lambda\mu} \end{pmatrix} \sim \begin{pmatrix} y'_\mu f'_\mu - y'_\lambda f'_\lambda \\ -x'_\mu f'_\mu + x'_\lambda f'_\lambda \end{pmatrix} \tag{30}$$

If we further require that the two functions (2) satisfy Cauchy-Riemann equations,

$$\begin{aligned} x'_\lambda &= y'_\mu, \\ x'_\mu &= -y'_\lambda \end{aligned} \tag{31}$$

then the fifth-form of the Creator's equation (28) reduces to

$$\begin{pmatrix} x''_{\lambda\mu} \\ x''_{\lambda\lambda} \end{pmatrix} \sim \begin{pmatrix} 2x'_\mu x'_\mu f'_x + (x'^2_\lambda - x'^2_\mu) f'_y \\ (x'^2_\lambda - x'^2_\mu) f'_x - 2x'_\lambda x'_\mu f'_y \end{pmatrix} \tag{32}$$

and the sixth-form of the Creator's equation (30) reduces to

$$\begin{pmatrix} x''_{\lambda\mu} \\ x''_{\lambda\lambda} \end{pmatrix} \sim \begin{pmatrix} x'_\lambda f'_\mu + x'_\mu f'_\lambda \\ -x'_\mu f'_\mu + x'_\lambda f'_\lambda \end{pmatrix} \tag{33}$$

4 Exponential Disk

Exponential disk is rational structure. Its logarithmic density is

$$f(x, y) = d_1 r \tag{34}$$

where d_1 is a constant and $r = \sqrt{x^2 + y^2}$. Its orthogonal net of curves is

$$x = e^{d_2\lambda}\cos(\mu),\ y = e^{d_2\lambda}\sin(\mu) \tag{35}$$

where d_2 is another constant.

5 Heaven Breasts Structure

Heaven breasts structure is rational [4,5]. Its logarithmic density is

$$f(x, y) = (b_2/3)\ \left((r^2 - b_1^2)^2 + 4b_1^2x^2\right)^{3/4} \tag{36}$$

where b_1, b_2 are constants. Its orthogonal net of curves is

$$x = b_1 \sinh(\lambda)\sin(\mu),\ \ y = b_1 \cosh(\lambda)\cos(\mu) \tag{37}$$

6 Galaxy Application

If we ignore those strongly interacting galaxies or some extremely small galaxies, there exist only two types of galaxies: spiral galaxies and elliptical galaxies. There are two kinds of spiral galaxies. A spiral galaxy with a bar is called a barred spiral, and a spiral galaxy without a bar is called an ordinary spiral. Astronomers found out that the stellar density distribution of ordinary spiral galaxies is basically an axi-symmetric disk described by the exponential disk (see the Section 4)

$$\rho_0(x, y) = d_0 \exp(d_1 r) \tag{38}$$

where d_0 is a constant. Its logarithmic density is

$$f_0(x, y) = d_1 r \tag{39}$$

Therefore, ordinary spiral galaxies are rational structure.

Astronomers have found out that the main structure of barred spiral galaxies is also the exponential disk. Therefore, we subtract the fitted exponential disk from a barred spiral galaxy image. What is left over? Jin He discovered that the left-over resembles human breasts [4,5]

$$\rho_i(x, y) = b_{i0} \exp\left((b_{i2}/3)\ \left((r^2 - b_{i1}^2)^2 + 4b_{i1}^2x^2\right)^{3/4}\right) \tag{40}$$

where b_{i0}, b_{i1}, b_{i2} are constants. Its logarithmic density is

$$f_i(x, y) = (b_{i2}/3)\ \left((r^2 - b_{i1}^2)^2 + 4b_{i1}^2x^2\right)^{3/4} \tag{41}$$

Jin He calls it Heaven Breasts structure (see the above Section 5). Barred spiral galaxies, however, generally have more than a pair of breasts. The bar of barred spiral galaxies is composed of two or three pairs of breasts which are usually aligned. The addition of the two or three pairs of breasts to the major structure of exponential disk becomes a bar-shaped pattern which crosses galaxy center.

Therefore, our model of barred spiral galaxy structure is

$$\rho(x, y) = \rho_0(x, y) + \rho_1(x, y) + \rho_2(x, y) \tag{42}$$

The logarithmic density of barred spiral galaxies is

$$f(x, y) = \ln \rho(x, y) = \ln(\rho_0(x, y) + \rho_1(x, y) + \rho_2(x, y)) \tag{43}$$

It is straightforward to show that

$$\nabla f = (\rho_0 \nabla f_0 + \rho_1 \nabla f_1 + \rho_2 \nabla f_2)/(\rho_0 + \rho_1 + \rho_2) \tag{44}$$

That is, the gradient of sum is the sum of weighted gradients. It is some kind of averaging which is usually denoted by a bar above the corresponding letter. We follow the notation. Here is an example

$$f'_x = (\rho_0 f'_{0x} + \rho_1 f'_{1x} + \rho_2 f'_{2x})/(\rho_0 + \rho_1 + \rho_2) = \overline{f'_{ix}} \tag{45}$$

Here comes the big question for humans.

7 The Creator's Question for Humans

Is the sum of rational structures also a rational structure (see the formula (42) in the above Section)? It is called the Creator's big question for humans. Numerical calculation suggests that it is approximately rational for the fitted galaxy values of the parameters $d_0, d_1, b_{i0}, b_{i1}, \cdots$ (see the paper [5]). However, we need mathematical justification. The authors are very old and are not experts in mathematics. Please help us humans to resolve the question.

The Creator's Question: If we substitute the expression (44) into the right-hand side of the equation (28) or (32), does the solution (2) exist?

In case you need them, we present some useful formulas in the Appendix.

References

[1] He J. (2005) PhD thesis, Department of Physics and Astronomy, The University of Alabama. Publication Number: AAT 3210452; ISBN: 0542620561

[2] He J. (2010) Electronic Journal of Theoretical Physics 24, 361

[3] He J. (2013) viXra:1304.0144, http://vixra.org/abs/1304.0144

[4] He J. (2012) viXra:1209.0072, http://vixra.org/abs/1209.0072

[5] He J. (2012) viXra:1211.0020, http://vixra.org/abs/1211.0020

8 Appendix

In the following formulas, $\alpha, \beta, \gamma, \delta$ may be the Cartesian coordinates x or y, or may be the parameters λ or μ.

$$f'_\alpha = \overline{f'_{i\alpha}}, \tag{46}$$

$$f''_{\alpha\beta} = \overline{f'_{i\alpha} f'_{i\beta}} + \overline{f''_{i\alpha\beta}} - \overline{f'_{i\alpha}}\;\overline{f'_{i\beta}}, \tag{47}$$

$$\begin{aligned} f'''_{\alpha\beta\gamma} \quad &= \overline{f'_{i\alpha} f'_{i\beta} f'_{i\gamma}} + \overline{f''_{i\alpha\beta} f'_{i\gamma}} + \overline{f''_{i\beta\gamma} f'_{i\alpha}} + \overline{f''_{i\alpha\gamma} f'_{i\beta}} + \overline{f'''_{i\alpha\beta\gamma}} \\ &\quad - \overline{f'_{i\alpha}}\;\overline{f'_{i\beta}}\;\overline{f'_{i\gamma}} - \overline{f''_{i\alpha\beta}}\;\overline{f'_{i\gamma}} - \overline{f''_{i\beta\gamma}}\;\overline{f'_{i\alpha}} - \overline{f''_{i\alpha\gamma}}\;\overline{f'_{i\beta}}, \end{aligned} \tag{48}$$

$$\begin{aligned} & f''''_{\alpha\beta\gamma\delta} \\ &= \overline{f'_{i\alpha} f'_{i\beta} f'_{i\gamma} f'_{i\delta}} \\ &+ \overline{f''_{i\alpha\beta} f'_{i\gamma} f'_{i\delta}} + \overline{f''_{i\alpha\gamma} f'_{i\beta} f'_{i\delta}} + \overline{f''_{i\alpha\delta} f'_{i\gamma} f'_{i\beta}} + \overline{f''_{i\beta\gamma} f'_{i\alpha} f'_{i\delta}} + \overline{f''_{i\beta\delta} f'_{i\gamma} f'_{i\alpha}} + \overline{f''_{i\gamma\delta} f'_{i\alpha} f'_{i\beta}} \\ &+ \overline{f''_{i\alpha\beta} f''_{i\gamma\delta}} + \overline{f''_{i\alpha\gamma} f''_{i\beta\delta}} + \overline{f''_{i\alpha\delta} f''_{i\beta\gamma}} \\ &+ \overline{f'''_{i\alpha\beta\gamma} f'_{i\delta}} + \overline{f'''_{i\alpha\beta\delta} f'_{i\gamma}} + \overline{f'''_{i\alpha\gamma\delta} f'_{i\beta}} + \overline{f'''_{i\beta\gamma\delta} f'_{i\alpha}} + \overline{f''''_{i\alpha\beta\gamma\delta}} \\ &- \overline{f'_{i\alpha}}\;\overline{f'_{i\beta}}\;\overline{f'_{i\gamma}}\;\overline{f'_{i\delta}} \\ &- \overline{f''_{i\alpha\beta}}\;\overline{f'_{i\gamma}}\;\overline{f'_{i\delta}} - \overline{f''_{i\alpha\gamma}}\;\overline{f'_{i\beta}}\;\overline{f'_{i\delta}} - \overline{f''_{i\alpha\delta}}\;\overline{f'_{i\gamma}}\;\overline{f'_{i\beta}} - \overline{f''_{i\beta\gamma}}\;\overline{f'_{i\alpha}}\;\overline{f'_{i\delta}} - \overline{f''_{i\beta\delta}}\;\overline{f'_{i\gamma}}\;\overline{f'_{i\alpha}} - \overline{f''_{i\gamma\delta}}\;\overline{f'_{i\alpha}}\;\overline{f'_{i\beta}} \\ &- \overline{f''_{i\alpha\beta}}\;\overline{f''_{i\gamma\delta}} - \overline{f''_{i\alpha\gamma}}\;\overline{f''_{i\beta\delta}} - \overline{f''_{i\alpha\delta}}\;\overline{f''_{i\beta\gamma}} \\ &- \overline{f'''_{i\alpha\beta\gamma}}\;\overline{f'_{i\delta}} - \overline{f'''_{i\alpha\beta\delta}}\;\overline{f'_{i\gamma}} - \overline{f'''_{i\alpha\gamma\delta}}\;\overline{f'_{i\beta}} - \overline{f'''_{i\beta\gamma\delta}}\;\overline{f'_{i\alpha}} \end{aligned} \tag{49}$$