

# Extended mass relation for seven fundamental masses and new evidence of Large Numbers Hypothesis

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## Abstract:

A previously derived mass relation has been extended to seven equidistant fundamental masses covering an extremely large mass range from  $\sim 10^{-69}$  kg to  $\sim 10^{53}$  kg. Six of these masses are successfully identified as mass of the observable universe, Eddington mass limit of the most massive stars, mass of hypothetical quantum “Gravity Atom” whose gravitational potential is equal to electrostatic potential  $e^2/S$ , Planck mass, Hubble mass and mass dimension constant relating masses of stable particles with coupling constants of fundamental interactions. The seventh mass,  $\sim 10^{-48}$  kg remains unidentified and could be considered as a prediction of the suggested mass relation for an unknown fundamental mass, potentially a yet unobserved light particle. First triad of these masses describes macro objects, the other three masses belong to particle physics masses, and the Planck mass appears intermediate in relation to these two groups. Additionally, new evidences of Dirac’s Large Numbers Hypothesis (*LNH*) have been found in the form of series of ratios relating cosmological parameters and quantum properties of space-time. A very large number on the order of  $5 \times 10^{60}$  connects mass, density, age and size of the observable universe with Planck mass, density, time and length, respectively.

**Key words:** mass relation, fundamental masses, Dirac’s Large Numbers Hypothesis, Newtonian Constant of Gravitation, Avogadro’s Number

## 1. Introduction

Discovery of theoretical or empirical mass relations for the many various particles is a great challenge for the recent high-energy physics and astrophysics, and derivation of mass relations covering a very large range of particle masses is most desirable. Known are a few formulas connecting the masses of particles having similar properties, one such, is Hadron’s multiplets (octets and decuplets of particles having close masses).

Though imprecise, one of the first attempts to empirically derive ‘Balmer’s law’ for several particles has been attempted in [1], wherein,  $m_n \sim 137m_e n$  is the mass of the  $n$ th particle,  $m_e$  is mass of the electron, and  $n$  is an integer or half-odd. Based on SU(3) symmetry, the Gell-Mann – Okubo mass formula [2, 3] has been derived for baryon decuplet:  $m_{\Delta} - m_{\Sigma} = m_{\Sigma} - m_{\Xi} = m_{\Xi} - m_{\Omega}$ , where  $m_{\Delta}$ ,  $m_{\Sigma}$ ,  $m_{\Xi}$  and  $m_{\Omega}$  are the masses of respective hyperons. This formula

successfully predicted the mass for the then undiscovered  $\Omega^-$  hyperon. The mass relations of Georgi-Jarlskog [4] ensue from the SO(10) model and relate masses of charged leptons ( $e$ ,  $\mu$  and  $\tau$ ) and down-type quark ( $d$ ,  $s$  and  $b$ )  $m_e = m_d/3$ ,  $m_\mu = 3m_s$  and  $m_\tau = m_b$ . However, these mass relations yield results that deviate significantly as compared to experimental data. It is postulated in [5] that a quantized magnetic self-energy of magnitude  $3 m_e n^4/2\alpha$  be added to the rest mass of a lepton to get the next heavy lepton in the chain  $e$ ,  $\mu$ ,  $\tau$ , ..., with  $n=1$  for  $\mu$ ,  $n=2$  for  $\tau$ , etc. Here,  $\alpha$  is the fine structure constant,  $m_e \approx 0.511$  MeV is the rest mass of the electron and  $n$  is a new quantum number. Thus it was predicted  $M_\mu=1786.08$  MeV, and for the next lepton  $M_\delta = 10293.7$  MeV. Koide has pointed out [6] that the mass relation  $m_e + m_\mu + m_\tau = (2/3)(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2$  is consistent with the measurements of the tau lepton mass. Found in [7] is a simple mass relation  $m_i = m_e \alpha_i/\alpha(0)$ , connecting masses of stable particles ( $p$ ,  $e$ ,  $\nu_e$  and graviton) with coupling constants  $\alpha_i(0)$  of the four interactions, and  $i = 1, 2, 3, 4$ . This mass relation covers an extremely wide range of values, exceeding 40 orders of magnitude and predicts a graviton mass on the order of  $10^{-69}$  kg.

Found in [8] is the derived mass relation:

$$M_n = m_e(\pi N\sqrt{3})^n \alpha^{(2n-1)} \quad (1)$$

where  $N \sim 6.02 \times 10^{23}$  is a large pure number and  $n = 1, 2, 3, 4$ .

This mass formula produces four equidistant masses covering a large range of 61 orders of magnitude. Mass  $M_1 \sim 2.18 \times 10^{-8}$  kg is apparent Planck mass,  $M_2 \sim 3.80 \times 10^{12}$  kg, the apparent mass of a hypothetical quantum "Gravity Atom" whose gravitational potential is equal to electrostatic potential  $e^2/S$ ,  $M_3 \sim 6.62 \times 10^{32}$  kg has not been identified and  $M_4 \sim 1.16 \times 10^{53}$  kg is the assumed proper mass of the observable universe. Now, in the present paper, we extend mass relation (1) to produce seven equidistant fundamental masses covering extremely large mass range of 122 powers of magnitude.

It was noticed in [9] that the ratio of the age of the universe  $H^{-1}$ , the inverse of the Hubble parameter, and the atomic unit of time,  $\tau = e^2/m_e c^3 \cong 10^{-23}$  s, is a large number  $N_D \sim 4.64 \times 10^{40}$ , where  $e$  is electron charge and  $c$  is speed of light in vacuum. Additionally, the ratio of mass of the observable universe  $M_u$  and nucleon mass is of the order of  $N_D^2$ , and the ratio of electrostatic  $e^2/r^2$  and gravitational forces  $Gm_e m_p/r^2$  between proton and electron in a hydrogen atom is  $2.27 \times 10^{39}$ , where  $G$  is the Newtonian constant of gravitation and  $m_e$  and  $m_p$  are electron and proton masses respectively. These "coincidences" hint at a possible connection between macro and microphysical world known as Dirac Large Numbers Hypothesis (LNH). Many other interesting ratios have been found approximately relating some cosmological parameters and microscopic properties of matter. For example, Narlikar [10] shows that the ratio of the observable universe radius,  $cH^{-1}$ , and the classical electron radius,  $e^2/m_e c^2$ , is exactly equal to  $N_D$ . Additionally, the ratio of the electron mass and Hubble mass parameter  $\hbar H/c^2$  is  $3.39 \times 10^{38}$  [11]. Here  $\hbar = h/2\pi$  is the reduced Planck constant and  $H$  is the Hubble constant.

Jordan [12] noted that the mass ratio for a typical star and an electron is of the order of  $10^{60}$ . Further, the ratio of observable universe mass and Planck mass is on the order of  $10^{61}$  [13]. Peacock [14] points out that the ratio of Hubble distance and Planck length is on the order of  $10^{60}$ . Finally, the ratio of Planck density  $\rho_{pl}$  and recent critical density of the universe  $\rho_c$  is found to be on the order of  $10^{121}$  [15]. These ratios between astrophysical parameters and microscopic properties of matter result mostly in large numbers that roughly agree with order of magnitude accuracy. In [16] has been derived a series of ratios relating cosmological parameters (mass  $M$ , density  $\rho_c$ , age  $H^{-1}$  and size  $cH^{-1}$  of the observable universe) and Planck (mass  $m_{pl}$ , density  $\rho_{pl}$ , time  $t_{pl}$  and length  $l_{pl}$ ) respectively, resulting in a very large number  $N_V$ , wherein  $m_{pl}$  is defined as the mass whose reduced Compton wavelength and Schwarzschild radius  $r_s$  are equal,  $l_{pl}$  is identical with  $r_s$ , and  $\rho_{pl}$  is defined as the density of a sphere having mass  $m_{pl}$  and radius  $l_{pl}$ .

$$\left(\frac{M}{m_H}\right)^{\frac{1}{2}} = \frac{M}{m_{pl}} = \frac{m_{pl}}{m_H} = \frac{cH^{-1}}{l_{pl}} = \frac{H^{-1}}{t_{pl}} = \left(\frac{\rho_{pl}}{\rho_c}\right)^{\frac{1}{2}} = \left(\frac{c^5}{2G\hbar H^2}\right)^{\frac{1}{2}} = N_V \quad (2)$$

These ratios *exactly* connect cosmological and quantum parameters of space-time and appear to be a precise formulation and proof of *LHN*. In this paper, we have found new evidences in support of *LNH* connecting cosmological parameters and microscopic properties of matter.

## 2. Extended mass relation

### 2.1 Review of mass relation concerning four fundamental masses

In Section IIA of previous paper [8], Newton's law of universal gravitation is derived, based on postulated mass/energy resonance waves, wherein the apparent Newtonian constant of gravitation factors as:

$$G = \frac{c^3 \lambda_\phi^2}{6\pi \hbar N^2} = \frac{\hbar c}{6\pi m_\phi^2 N^2} = \frac{\hbar c}{3(\pi \alpha m_e N)^2} \cong 6.6629 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \quad (3)$$

where  $m_e$  is electron rest mass,  $\lambda_\phi$  the resonance wavelength,  $m_\phi$  the associated particle mass, and  $N$  is a large pure number, curiously comparably with  $N_A$ , the 2006 recommended numerical value of Avogadro's number, and in terms of the fine structure constant  $\alpha$ , and  $\pi$ , is shown to be given by:

$$N = (4\pi\alpha^5)^2 \times \sqrt{\frac{8}{3}} \cong 6.022\,139\,582 \times 10^{23} \quad (4)$$

The Planck mass by convention is  $\sqrt{\hbar c/G}$  [17]. It follows, therefore, from Eq. (3) that the apparent Planck mass is given by:

$$m_{pl} = \pi\alpha m_e N\sqrt{3} = m_e(\pi N\sqrt{3})^1\alpha^1 \cong 2.178 \times 10^{-8} \text{ kg} \quad (5)$$

Additionally shown is that the resonance wavelength is equal to twice the first Bohr orbit thus leading directly to :

$$m_\phi = \pi\alpha m_e \cong 2.088 \times 10^{-32} \text{ kg} \quad (6)$$

It is known that the fine structure constant, the coupling constant of electromagnetic interaction, i. e. a measure of its strength, is determined by the formula  $\alpha = e^2/\hbar c$ . Taking into consideration this formula, we find from Eq. (3) that:

$$G = \frac{e^2}{3\pi^2\alpha^3 m_e^2 N^2} \quad (7)$$

In Section IIC of paper [8], a hypothetical quantum ‘‘Gravity Atom’’ has been proposed, comprised of an electrically neutral central mass  $M_G$  orbited by an electrically neutral particle having electron mass  $m_e$ , such that the gravitational potential  $GM_G m_e/S$  is equal to an electrostatic potential  $e^2/S$ , and  $S$ , the orbital radius, is a Bohr orbit. Thus,  $GM_G m_e = e^2$ , that in conjunction with Eq. (7) results in:

$$M_G = 3\pi^2\alpha^3 m_e N^2 = m_e(\pi N\sqrt{3})^2\alpha^3 \cong 3.801 \times 10^{12} \text{ kg}. \quad (8)$$

It is also of interest to note that this is the mass for which the Schwarzschild radius is equal to twice the classical electron radius.

Noted in [8] is that examination of Eqs. (5) and (8) reveals the masses  $m_{pl}$  and  $M_G$  are members of the series suggested by Eq. (1), that in conjunction with Eq. (6) can also be expressed as  $M_n = \pi^{(n-1)} \times \alpha^{(2n-2)} \times m_\phi (N\sqrt{3})^n$ , where  $n$  is the placement within the series. Employing Eq. (1) and beginning at  $n = 1$ , it is found that:

$$M_1 = m_e(\pi N\sqrt{3})^1\alpha^1 \cong 2.178 \times 10^{-8} \text{ kg} \quad (9)$$

$$M_2 = m_e(\pi N\sqrt{3})^2\alpha^3 \cong 3.801 \times 10^{12} \text{ kg} \quad (10)$$

$$M_3 = m_e(\pi N\sqrt{3})^3\alpha^5 \cong 6.632 \times 10^{32} \text{ kg} \quad (11)$$

$$M_4 = m_e(\pi N\sqrt{3})^4\alpha^7 \cong 1.157 \times 10^{53} \text{ kg} \quad (12)$$

Identified above is the physical significance attributed to masses  $M_1$  and  $M_2$ . Mass  $M_4$  appears to be well within the range of estimates for the observable universe proper mass  $M_u$  [16, 18, 19] and as such, it represents the upper limit of the series.

2.2. *Extended mass relation for seven fundamental masses, a new fundamental constant and the Hubble Parameter*

Upon extending the series downwards to  $n \leq 0$ , we obtain:

$$M_0 = m_e(\pi N\sqrt{3})^0 \alpha^{-1} = \frac{m_e}{\alpha} \cong 1.248 \times 10^{-28} \text{ kg} \quad (13)$$

$$M_{(-1)} = m_e(\pi N\sqrt{3})^{-1} \alpha^{-3} = \frac{m_e}{\pi N \alpha^3 \sqrt{3}} \cong 7.154 \times 10^{-49} \text{ kg} \quad (14)$$

$$M_{(-2)} = m_e(\pi N\sqrt{3})^{-2} \alpha^{-5} = \frac{m_e}{3\pi^2 N^2 \alpha^5} \cong 4.100 \times 10^{-69} \text{ kg} \quad (15)$$

It is found that the ratio of any two consecutive masses in the series (9):(15) is a constant,  $K$ , wherein:

$$M_{(n+1)}/M_n = K = \pi N \alpha^2 \sqrt{3} \cong 1.774 \times 10^{20} \quad (16)$$

Therefore:

$$M_n = \frac{m_e}{\alpha} (\pi N \alpha^2 \sqrt{3})^n \cong \frac{m_e}{\alpha} K^n = M_0 K^n \quad (17)$$

where  $n = -2, -1, 0, \dots, 4$ . We now find that:

$$\frac{2M_1}{\pi K^3} = \frac{2M_2}{\pi K^4} = \frac{2M_3}{\pi K^5} = \frac{2M_4}{\pi K^6} \cong 2.61 \times 10^{-69} \text{ kg} \quad (18)$$

The current best estimates of  $H_o$  center around about  $70 \text{ km s}^{-1} \text{ Mps}^{-1}$ . Thus, when  $m_H$ , the Hubble mass [20, 21], is defined as:

$$m_H = \frac{\hbar H}{c^2} \quad (19)$$

we find from Eq. (19) an approximate value for  $m_H$  of  $2.66 \times 10^{-69} \text{ kg}$ . This result is close to that from Eq. (18). It is close enough in fact that all symbolic members of Eq. (18) are assumed to express accurately the value of the Hubble mass concomitant with  $H$ . Therefore, regarding the 1<sup>st</sup> and 4th members:

$$\frac{2m_{pl}}{\pi K^3} = m_H \quad (20)$$

and

$$\frac{2M_u}{\pi K^6} = m_H \quad (21)$$

from which, upon elimination of  $K$ , results:

$$m_H = \frac{2m_{pl}^2}{\pi M_u} \quad (22)$$

and the final result upon substituting the right-hand member of Eq. (22) into Eq. (19) for  $m_H$  and solving for  $H$ , becomes:

$$H = \frac{2c^2 m_{pl}^2}{\pi \hbar M_u} = \frac{4c^2 m_{pl}^2}{h M_u} \quad (23)$$

If the Hubble mass is defined as  $(hH/c^2)$ , as in [8] and [22], the value for  $m_H$  would be  $\sim 1.64 \times 10^{-68}$  kg, so the left-hand members of Eqs. (20) and (21) must then be multiplied by  $2\pi$  to preserve the equalities, and Eq. (22) is still the final result.

As was proposed in [22], predicated upon the rate of cosmic expansion apparently transitioning from deceleration to acceleration at redshift  $\sim 0.5$  [23], the deceleration parameter must have passed through a zero null point at transition, as the opposing operatives of cosmic expansion reached a transient state of equilibrium. Intuitively it would seem that the Hubble parameter at that juncture  $H_{eq}$ , the tipping point between deceleration and acceleration, must be tied to the mass of the universe via means of a unique relationship that existed at that juncture, as developed through Eqs. (19), (20), (21), and (22), leading to Eq. (23). However, it does not necessarily follow that the Hubble parameter is increasing along with the accelerating rate of cosmic expansion. Some theoretical considerations suggest that the Hubble parameter has now assumed a truly constant value in time and space. Others predict that even as the expansion accelerates, the Hubble parameter will continue to decrease asymptotically, approaching a limiting value of about 62, as the influence of the cosmological constant becomes more and more dominant over the contribution of matter after several billions of years and a several fold increase in the scale factor. It is thus reasonable to propose that  $H_o$ , the present day Hubble parameter, and  $H_{eq}$  are essentially identical. Thus:

$$H = H_{eq} = \left(\frac{4c^2}{h}\right) \left(\frac{m_{pl}^2}{M_u}\right) \cong H_o \cong 68.634 \frac{\text{km}}{\text{s} \times \text{Mpc}} \quad (24)$$

A theoretical value for  $H_o$  of  $68.658 \pm 0.1 \text{ km.s}^{-1}$ , obtained via an entirely independent approach [24], is in excellent agreement with the above.

Since by convention, the square of the Planck mass is  $hc/2\pi G$ , Eq. (23) can be restated as:

$$H_{eq} = \frac{2c^3}{\pi G M_u} \cong 68.634 \frac{\text{km}}{\text{s} \times \text{Mpc}} \quad (25)$$

and from Eq. (25), we obtain:

$$M_u = \frac{(2/\pi)c^3}{GH_{eq}} \cong 1.157 \times 10^{53} \text{ kg} \quad (26)$$

the exact same result as that of Eq. (12). Additionally, from Eqs. (12) and (26) another interesting relationship results:

$$M_4 = M_u = \frac{(2/\pi)c^3}{GH_{eq}} = m_e(\pi N\sqrt{3})^4 \alpha^7 \cong 1.157 \times 10^{53} \text{ kg}$$

### 2.3. Review of three fundamental masses obtained by dimensional analysis

In previous paper [25], three fundamental masses have been derived by dimensional analysis namely:

$$m_1 = \frac{\hbar H}{c^2} = m_H \quad (27)$$

$$m_2 = \frac{k c^3}{GH} \cong M_u \quad (28)$$

$$m_3 = \left( \frac{H \hbar^3}{G^2} \right)^{\frac{1}{5}} \cong 1.43 \times 10^{-20} \text{ kg} \quad (29)$$

In form, Eq. (26) coincides closely with Eq. (28) and the two are an identity when the dimensionless parameter  $k$ , on the order of unity, is identical with  $2/\pi$ .

The papers [8], [22] do not attribute any physical significance to mass  $M_3$  ( $6.632 \times 10^{32} \text{ kg}$ ) in the original  $n_1$  through  $n_4$  series. Recently we have identified this mass with the Eddington stellar mass limit where the outward pressure of the star's radiation balances the inward gravitational force [26, 27]. Additionally, we have identified the mass  $M_0$  ( $\sim 1.248 \times 10^{-28} \text{ kg}$ ) as exactly coinciding with the mass dimension constant in a basic mass equation from paper [7] relating masses of stable particles and coupling constants of the four fundamental interactions. It is interesting that this mass is approximately a half-charged pion mass  $M_0 = m_e/\alpha \cong \frac{1}{2} m_{\pi^\pm}$ . Mass  $M_{(-1)}$  ( $\sim 7.154 \times 10^{-49} \text{ kg}$ ) is presently unidentified and

could feasibly be regarded as a prediction by the suggested model, Eq. (9), for a fundamental, albeit as yet unobserved light particle. Finally, mass  $M_{(-2)}$  ( $\sim 4.100 \times 10^{-69}$  kg) in the extended series is easily identifiable with the Hubble mass Eq. (19) as  $0.5\pi m_H$ . It is of further interest to note that the extended mass series includes seven equidistant fundamental masses covering a mass interval of 122 orders of magnitude, and that masses  $M_{(-2)}$ ,  $M_{(-1)}$ , and  $M_0$  are particle physics masses, whereas the masses  $M_2$ ,  $M_3$ ,  $M_4$  describe macro objects, and the Planck mass  $M_1$  appears intermediate in relation to these two groups. In fact, it is easily shown that the Planck mass, as given by Eq. (9), is the geometric mean of the extreme masses  $M_{(-2)}$  and  $M_4$  as given by Eqs. (15) and (12), as is the geometric mean of masses  $m_1$  and  $m_2$  from Eqs. (27) and (28) when  $k=1$ . The physical significance of mass  $m_3$  from Eq. (29), if any, has not yet been identified.

### 3. New evidences of Dirac's large numbers hypothesis

Recalling Eq. (2) and the definition of terms therein, it is found that  $N_V \cong 5.73 \times 10^{60}$  when the defined terms are evaluated according to:

$M = c^3/2GH$ ;  $m_H = \hbar H/c^2$ ;  $m_{pl} = (\hbar c/2G)^{1/2}$ ;  $l_{pl} = (2G\hbar/c^3)^{1/2}$ ;  $t_{pl} = l_{pl}/c = (2G\hbar/c^5)^{1/2}$ ;  $\rho_{pl} = 3c^5/16\pi\hbar G^2$ ;  $\rho_c = 3H^2/8\pi G$  is recent density of the universe equal to the critical one;  $H^{-1}$ , the age of the universe and  $cH^{-1}$  is the Hubble distance.

The Eq. (2) ratios appear very important because they relate cosmological parameters and the fundamental microscopic properties of matter. The Planck units imply quantization of space-time at extremely short range. Thus, the ratios represent connection between cosmological and quantum parameters of space-time and thus appear to be a precise formulation and proof of *LHN*. In addition, the very large number  $N_V$  and Dirac's large number  $N_D$ [9] seem connected by the approximate formula:

$$N_D \simeq N_V^{\frac{2}{3}} = \left( \frac{c^5}{2G\hbar H^2} \right)^{\frac{2}{3}} \cong 3.2 \times 10^{40} \quad (30)$$

We now construct a similar series to (2) involving ratios of the same parameters producing the very large, number  $N_{VF}$ , as follows:

$$\left( \frac{2 M_u}{\pi m_H} \right)^{\frac{1}{2}} = \frac{M_u}{m_{pl}} = \frac{2 m_{pl}}{\pi m_H} = \frac{2 cH^{-1}}{\pi l_{pl}} = \frac{2 H^{-1}}{\pi t_{pl}} = \frac{2}{\pi} \left( \frac{\rho_{pl}}{2\rho_c} \right)^{\frac{1}{2}} = \frac{2}{\pi} \left( \frac{c^5}{G\hbar H^2} \right)^{\frac{1}{2}} = N_{VF} \cong 5.31 \times 10^{60} \quad (31)$$

Where now:

$$H \text{ is } H_{eq} = 2c^3/\pi GM_u; \quad M_u \text{ is apparent proper mass of universe} = 2c^2/\pi GH; \quad m_H = \hbar H/c^2; \quad m_{pl} = (\hbar c/2\pi G)^{1/2}; \quad l_{pl} = (G\hbar/c^3)^{1/2}; \quad t_{pl} \text{ is } l_{pl}/c = (G\hbar/c^5)^{1/2}; \quad \rho_{pl} =$$



$3m_{pl}/4\pi l_{pl}^3 = 3c^5/4\pi\hbar G^2$ ; and  $\rho_c = 3H^2/8\pi G$ , and  $G$  is according to Eq. (3). These ratios also represent a connection between cosmological and quantum parameters of space-time and so likewise appear to be possible new evidences of *LNH*. Recalling Eq. (17), it is noteworthy that apparently:  $N_{VF} = K^3 = (\pi N\alpha^2\sqrt{3})^3 \cong 5.313 \times 10^{60}$  and that  $N_{VF}$  and Dirac's large number  $N_D$  seem connected by the approximate formula:

$$N_D \cong N_{VF}^{\frac{2}{3}} = K^2 = (\pi N\alpha^2\sqrt{3})^2 = \left(\frac{4c^5}{\pi^2 G\hbar H^2}\right)^{\frac{2}{3}} \cong 3.04 \times 10^{40} \quad (32)$$

Thus, by independent approaches it is apparent that we obtain very similar results, (30), (32) and (2), (31).

From Eq. (32), it follows that:

$$N^2 = \frac{(N_{VF})^{\frac{2}{3}}}{3\pi^2\alpha^4} = \frac{K^2}{3\pi^2\alpha^4} \quad (33)$$

that upon substitution into Eq. (3) for the square of  $N$  results in:

$$G = \frac{\alpha^2\hbar c}{m_e^2(N_{VF})^{\frac{2}{3}}} = \frac{\alpha^2\hbar c}{m_e^2 K^2} \quad (34)$$

Thus, Eqs. (33) and (34) connect  $N$  to *LNH* and therefore to  $G$  through the unique and apparently new fundamental constant  $K$ , as given by Eq. (16).

#### 4. Conclusions

Mass relation (1) obtained in [8] has been extended from  $n = -2$  to  $n = 4$ . The result is seven equidistant fundamental masses  $M_n$ , covering a mass interval of 122 orders of magnitude, have been obtained. Six of these masses are successfully identified, namely:

$M_1 \sim 2.178 \times 10^{-8}$  kg the apparent Planck mass,  $\sqrt{(\hbar c/G)}$ , that is very important in recent particle physics.  $M_2 \sim 3.801 \times 10^{12}$  the central mass of a hypothetical quantum "Gravity Atom" whose gravitational potential  $GM_G m_e/S$  is equal to electrostatic potential  $e^2/S$  and  $S$  is a Bohr orbit.

$M_3 \sim 6.632 \times 10^{32}$  kg is of the order of the Eddington mass limit of the most massive stars.

$M_4 \sim 1.157 \times 10^{53}$  kg is close to the mass of the Hubble sphere and most probably appears to be mass of the observable universe.  $M_0 \sim 1.248 \times 10^{-28}$  kg coincides with a mass dimension constant in a basic mass equation relating masses of stable particles and coupling constants of the four interactions; approximately a half charged pion mass.  $M_{(-2)} \sim 4.100 \times 10^{-69}$  kg is easily identifiable with the Hubble mass as  $0.5\pi m_H$ . The sixth mass  $M_{(-1)} \sim 7.154 \times 10^{-49}$  kg remains

yet unidentified and could be regarded as a prediction by the suggested mass relation for unknown fundamental mass, most likely a yet unobserved light particle. Apparently, masses  $M_{(-2)}$ ,  $M_{(-1)}$ , and  $M_0$  are particle physics masses, whereas the masses  $M_2, M_3, M_4$  describe macro objects, and the Planck mass  $M_1$  appears intermediate in relation to these two groups.

Finally, new evidences of *LNH* have been found in the form of series of ratios relating cosmological parameters and quantum properties of space-time. In addition, the very large number  $N_{VF} = K^3 = (\pi N \alpha^2 \sqrt{3})^3 \cong N_V = \sqrt{c^5 / 2G\hbar H^2} \cong 5.313 \times 10^{60}$  connects mass, density, age and size of the observable universe with Planck mass, density, time and length respectively, and K is apparently a unique and new fundamental constant.

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