Do Photons Exist in Spacetime?

Armin Nikkhah Shirazi[*]

University of Michigan
Ann Arbor, MI 48109

June 6th, 2013

Abstract

Under our current worldview, it is taken to be so obvious an assumption that objects characterized by $v = c$ exist in spacetime that it is not even bothered to explicitly mention it. This paper will present 4 simple arguments based on the special theory of relativity which at least suggest that this obvious assumption should be put into question. These arguments cannot be considered conclusive, but when considered together they support the case that this question should be seriously investigated.

Keywords: Foundations of special relativity, existence of photons

1 Introduction

A measure of how deeply an assumption about a fundamental aspect of nature is ingrained in our worldview is how ludicrous it seems to question it. Many of our fundamental assumptions in physics enjoy overwhelming direct experimental and observational evidence, and in those instances, if a claim contrary to these assumptions is not taken seriously, this can be justified by that evidence. In other instances, we may hold a strong conviction about an aspect of nature not because of the support of direct evidence, but because of how well it fits in our overall worldview: As with a piece in a puzzle set, if the assumption were to be changed, it could no longer fit so nicely in our overall conceptualization of nature, and we would be forced to change other assumptions, many (but perhaps not all) of which do enjoy considerable experimental and observational support. In those rare instances where indirectly supported fundamental assumptions about nature are finally directly tested and found to be false, they force us not only to change the individual assumptions but bring into question the tenability of the prevailing worldview.

This paper will present four simple arguments based on the special theory of relativity which are largely independent of each other, yet point all to the same conclusion: That objects characterized by $v = c$ do not exist in spacetime. It is emphasized that the arguments are surprisingly simple but, as far as this author knows, they had not been previously put together and interpreted so as to question an assumption that is at present surely deeply held by almost everyone, and this may well be because such an implication does not at all fit in our contemporary worldview.

After a few preliminary considerations, this paper will present the four arguments. These arguments cannot be considered conclusive because the claim that photons do not exist in spacetime is an extraordinary one and requires correspondingly strong evidence to be believed, whereas each of the arguments presented here may individually be vulnerable to counterarguments. Nevertheless, if one wishes to maintain a conservative stance then it seems odd that several parts of special relativity can independently be interpreted to support of this idea. On the other hand, if the conclusion to which these arguments point is accepted then it seems almost certain that a substantial modification of our present worldview is inevitable, at least insofar as it requires spacetime to be the repository of everything that exists.

---

[*] armin@umich.edu

2 Some Preliminary Considerations

There are precedents of instances in which a universally held assumption supported by strong indirect evidence was shown to be false. This is perhaps nowhere better illustrated than in the famous Michelson-Morley Experiment, which was designed to check the speed of material objects through a hypothesized all-pervasive substance, the ether. Fewer than two decades before the experiment in 1887, Maxwell had completed his *Treatise on Electricity and Magnetism*, and out of his work arose the realization that light was an electromagnetic wave. Because all waves were up to then understood to be propagating disturbances through some kind of media, it was only natural to assume that there existed a medium for light. Hence, there was a strong prevailing conviction that the ether exists, even though there was no direct experimental support for this assumption.

The experiment, carried out in what is now called a Michelson Interferometer, measured the speed of light waves split in two perpendicular directions that would be reflected back onto a single locus. If the speed along the perpendicular directions were different, then the interference between the light waves would produce a detectable fringe shift, and this would allow one to estimate the magnitude of the difference in light speed[1]. The result of this experiment, which as far as this author knows, was anticipated by no one, was the failure to detect any meaningful difference in the speed of light along the different directions. This result was subsequently confirmed numerous times with increasing precision and accuracy. As this result appeared to be incompatible with the ether hypothesis, it forced physicists to reconsider the prevailing worldview of the time. In 1905, the introduction of the special theory of relativity resolved this problem by taking the invariance of the speed of light as a fundamental postulate and thereby dispensing with the necessity for an ether altogether. Special relativity turned out to become one of our best-tested and most- confirmed theories, so not surprisingly, the physics of the 20th century, which builds in large part on it, looks significantly different from the physics of the 19th century. Empirically, this is in no small part because the merely indirectly supported assumption that light is a disturbance in an ether turned out to be false.

Before presenting the arguments for why one might consider the seemingly bizarre possibility that photons, which are described by $v = c$, do not exist in spacetime, let us address up front an immediate objection that might be brought up against this idea. In the above historical discussion, as in standard discourse on radiation, light is treated entirely as something that exists in space. One might argue that surely if something exists in space, it must also exist in spacetime. Put slightly more formally, since velocity is defined as a change in position over time, $\mathbf{v} = d\mathbf{x}/dt$, and position in space naturally seems to imply existence in spacetime, the idea that photons might not exist in spacetime would seem like a non- starter. However, this objection assumes light has a definite trajectory. In relativity light is often conceptualized this way, traveling along null geodesics which trace out definite paths in space. However, from quantum theory we know that this is merely a convenient fiction. Photons do not possess definite trajectories, and hence one cannot use this to dismiss the arguments that follow in the next section. All that one is permitted to claim based on the most direct evidence is that there are two spacetime events associated with photons: Emission events and absorption events. These describe when photons appear to come into existence and go out of existence, respectively, and *in spacetime* these events are evidently connected by no "distance"[1] at all. Furthermore, the argument that $\mathbf{v} = d\mathbf{x}/dt$ implies $\mathbf{x}$, $\mathbf{x}$ implies existence in space, and existence in space implies existence in spacetime cannot be applied to photons because there is no rest frame-and hence no frame in which $\mathbf{x}$ can be defined independently of $\mathbf{v}$-associated with photons.

3 The Four Arguments

This section will give 4 simple independent theoretical arguments which lead to the conclusion that objects described by $v = c$ do not exist in spacetime. We will use the term *photon* throughout as a convenient stand-in for such objects, even though the arguments apply of course to any massless objects. The arguments will be presented in the order from the least compelling to the most (at least in this author's view) and be given a label to facilitate discussion.

---

[1] the quotation marks are meant to acknowledge the challenge of characterizing a metric interval which can be negative-definite as a distance

1. **The Ontological Argument: The duration of existence in spacetime of a photon in its proper frame is zero** This is a direct implication of the zero proper time associated with photons: Since in their proper frame evidently there is no passage of time, the spacetime events that correspond to their emission and their absorption, and hence to their coming into and going out of existence, are one and the same. Under our present worldview, the fact that photons "observe" their own duration of existence in spacetime to be exactly zero is nothing more than a strange fact about special relativity, but since it does not seem to result in any contradictions, it is simply accepted without explanation.

   Unfortunately, the ontological argument may not seem very compelling to physicists because 1) it has a strong philosophical flavor and 2) because it assumes that it makes sense to speak of proper frames for such objects. It is not uncommon to find the view that since no spacetime observer can transform to such frames, this implies that it makes no sense to speak of such frames. However, this situation can also be interpreted from an inverse point of view: One can consider that it does make sense to speak of such frames, and in particular, that one can precisely say about such frames that no spacetime observer can transform to them. Then, if photons in fact do not exist in spacetime, this fact about photons would be consistent with a zero duration of existence in their proper frames, and explain why spacetime observers cannot transform to them. The invariance of the speed of light has been previously derived from essentially this argument[2].

2. **The Linear Dependence Argument: In a photon frame, spacetime has redundant dimensionality**

   In the standard graphical representation of two frames moving relative to each other and coinciding at the origin, one frame is taken to be the reference rest frame and its associated space and time axes are represented along orthogonal axes. As a consequence of the Lorentz transformations, the angular distance between axes associated with the moving frame are reduced by an angle 2α, where $\alpha = \tan^{-1} v/c$. For $v = c$, $\alpha = \pi/4$ and the two axes must be represented as being parallel in the moving frame (in the lightlike direction)[3]. By the principle of relativity, we are equally well allowed to assert that according to the moving frame, unit vectors along the space and time axes associated with the reference rest frame are observed to be parallel to each other. But it is a consequence of elementary linear algebra that a set of vectors which are parallel form a linearly dependent set, and that a linearly dependent set of vectors contains more elements than necessary to span a vector space, hence in a frame in which the unit vectors are parallel, spacetime has redundant dimensionality.

   It might be objected that one could turn this argument around to claim that whatever repository photons exist in also has redundant dimensionality in the reference rest frame, thereby obtaining possibly a logical contradiction. But notice that there are some asymmetries inherent in the situation. The first is produced by the fact that spacetime objects can exist during time periods that extend from before a photon comes into existence until after it goes out of existence[2]. The second and more important asymmetry arises from the fact that whereas the argument applied to the reference rest frame leads to a claim applicable to an individual photon (or, at most, a class of photons classically associated with the same direction), the argument applied to the photon frame leads to claim applicable to *every object in spacetime.* In effect, to maintain the objection, one must consider each photon a universe of its own, which we of course do not, hence the linear dependence argument applies only one way.

   Like the ontological argument, this argument depends on the assumption that it makes sense to speak of the proper frames of objects for which $v = c$. More significantly, the fact that lightlike vectors have a zero magnitude introduces a degeneracy which allows them to be nevertheless considered orthogonal to each other (and to themselves), as the magnitude of the inner product in these cases is always zero, and this may severely reduce the persuasiveness of this argument. Nonetheless, whereas we usually understand the concepts of "orthogonal" and "parallel" to be mutually exclusive, they are evidently not in this type of situation, and if one can maintain that in such frames the orthogonal unit vectors associated with the space and time axes are (also) parallel, then one can maintain the above argument, because fact that the vectors are orthogonal in those frames as well does not change the fact that they

---

[2] If one required a complete symmetry in this respect, then one would be forced to assert that a photon somehow exists before all of spacetime "came into existence" and after "it goes out of existence", respectively which, besides being of dubious logical consistency, already concedes the conclusion of this paper.

can be expressed in terms of each other and, hence, are linearly dependent.

3. **The Four-Volume argument: Photons cannot be associated with four-volumes in the absence of matter** A four-volume element in Minkowski spacetime can be defined mathematically as $\sqrt{\mid g \mid} dx^0 dx^1 dx^2 dx^3$, where $g \equiv det[\eta_{ij}]$. It is straightforward to show that if $ds^2 = \eta_{ij} dx^i dx^j = 0$, then the product of the constituents of the interval no longer yields a four-dimensional volume. Intuitively, that is because for null intervals, time and space quantities are no longer independent, as one of the four quantities can be expressed in terms of the other three. Since photons are exclusively associated with null intervals, but their constituent time and space quantities cannot be combined to form four-volumes, photons cannot exclusively (i.e. in the absence of matter) be associated with four-volumes.
   In response to this argument, it might be pointed out that it is perfectly possible to define a four-volume element which includes a spacetime event such as the absorption or emission of a photon, as for instance the small spatial region in which such an event occurs considered over an interval of time. But in those cases, one really has chosen to define a fourvolume based on the matter present in it, not the photon, because the time interval over which the 3-volume is considered can always be expressed in terms of the proper time of the massive particle that absorbed or emitted the photon. It might then be noted that one can define without any problems a four-volume in a matter-free region that lies in between the emission and absorption events associated with the same photon. But here we encounter a variant of the objection addressed under the preliminary considerations: Since photons do not have definite trajectories, one cannot assign a position vector to them and then consider a 3-volume that contains the position vector over some period of time in order to obtain a fourvolume. To be able to assign a position vector one must introduce a particle that absorbs the photon at a definite position in space, but this re-introduces the previous confounding problem: the fourvolume is then really associated with the absorber, not the photon. Essentially, for a photon the analog of considering a 3-volume element over time to obtain a fourvolume is to consider the spacetime volume swept out by null-geodesics (since photons do not "age"), but, as mentioned, there is no fourvolume associated with null intervals.
   Perhaps it also helps to consider this argument from a global perspective: If our universe were completely devoid of matter but still contained radiation, then there would be no longer any physical object present to which a spacetime observer frame can be "anchored", so in a physical sense it could no longer be considered to be four- dimensional Minkowski Spacetime. Of course, one is still free to imagine such an object as 4-dimensional spacetime with zero matter content, but this freedom is of the same kind as the freedom to imagine spatial directions beyond the three we experience in our world: it is mathematical in nature, not physical.

4. **The limit argument: At $\mathbf{v = c}$, the Lorentz transformations may take a vector outside spacetime**. It is common to refer to the speed of light as a 'limit' but perhaps it is not realized as often that while c is indeed a speed limit, it is not a limit on the Lorentz coordinate transformations because that limit does not exist. This can be easily verified, for as v approaches c from below, $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ approaches infinity, but as v approaches c from above, γ approaches $i\infty$. This fails to satisfy the definition of a limit, which requires γ to approach the same value as v approaches c from above and from below. Hence
$$\lim_{v \to c} \left[ct' = \gamma \left(ct - \frac{vx}{c} \right), x' = \gamma \left(x - vt \right), y' = y, z' = z \right] \tag{1}$$
   does not exist. The implication of this may perhaps be more easily seen if one considers Minkowski spacetime as a set M^4 of ordered 4-tuples $\mathbf{X} = (ct, x, y, z)$ with t, x, y, z real. A vector is a point in M^4 denoted by $\mathbf{X}$ and its coordinate transformations are of course the Lorentz coordinate transforms. Now, the fact that the limit $v = c$ for the Lorentz coordinate transformations does not exist can be interpreted in one of two ways: Either 1) it makes no sense to consider the Lorentz transformations at $v = c$ because there exist no rest frames associated with objects characterized by this property (the standard interpretation) or 2) it does make sense to consider the Lorentz transformations at $v = c$ [3] but if a vector $\mathbf{X}'$ were the result of a Lorentz coordinate transformation of the vector $\mathbf{X}$ in this instance, then $\mathbf{X}'$ would not be an element of M^4. Physically, this translates to the statement that an object

---

[3] Here we only mean to say that it is not nonsense to consider the transformation at v=c, not that it is actually possible

described by the vector $\mathbf{X}'$ is associated with a rest frame that is not in Minkowski spacetime. Photons are precisely the objects that would have been described by the vector that would result from such a transformation, hence by the second interpretation, photons do not exist in spacetime. Notice that if $v = c$ had constituted a legitimate limit, then the Lorentz transformations could not be considered to take a vector outside M^4 in that regime.

The existence of photons seems to disfavor the first interpretation because naively, it would have led us to believe that there exist no objects describable by $v = c$: Had we not known about their existence already, we would have surely taken the first interpretation to be an 'explanation' for their purported absence. What disfavors the second interpretation, on the other hand, seems to be merely our present worldview, which would lead one to disbelieve the specific claim that photons do not exit in spacetime. This is not uncommon and happens particularly during intermediate stages in the acceptance of formerly unobvious mathematical implications of a theory. The Aharanov-Bohm effect and Bell's theorem are particularly clear examples of this, as many physicists were very skeptical of the physical interpretation of these implications of quantum theory prior to their experimental confirmation [4] [5].

We certainly do accept other strange implications of special relativity, such as time dilation, length contraction and the relativity of simultaneity. Is it not possible that this strange implication of special relativity has not received due attention because it is still in such an intermediate stage?

4 Conclusion

This paper presented 4 arguments which suggest that photons do not exist in spacetime, but it is readily admitted that they will, for most people, fall short of being conclusive. Even so, they may be considered reasons to consider this question as a legitimate subject of further investigation.

If it is accepted that photons do not exist in spacetime, then this naturally raises the question of where they exist. Here, the fact that photons are always associated with nullsheets (i.e. boundaries of lightcones) to the rest-frame of which no spacetime observer can transform seems to provide a hint: If we consider the interior of a lightcone a topological sub-manifold of spacetime, and apply the fact that the boundary ∂S of a topological submanifold has one length dimension fewer than the manifold, then this suggests that if the boundary of the lightcone is topological, that photons exists in a $2 + 1$ analog of spacetime. This seems consistent with the fact that null-sheets have no thickness. If one accepts this, then certain counterintuitive aspects of special relativity may suddenly seem more natural. Consider, for instance, that according to the theory, an object moving at $v = c$ must be completely contracted in its direction of motion. This may seem very strange until one realizes that such objects, being lower- dimensional, inherently lack extent along a third dimension, and that since their "direction of motion" in space corresponds locally to the direction of the expansion of the lightcone with which they are associated, the "absent" dimension must be along the direction of motion. Furthermore, if one regards such objects as inherently lower- dimensional, then they must be completely length contracted in every spacetime frame, but by the relation $\gamma^{-2} + \beta^2 = 1$ that implies that they must be characterized by $v = c$ in every spacetime observer frame.

Although photons are inherently quantum mechanical objects, a discussion of quantum theory was avoided here because the aim was to uncover some of the perhaps less obvious aspects of the foundations of special relativity. Elsewhere, this author has attempted to apply the notion that photons in particular and quantum objects in general (prior to a 'measurement') may in fact be lower-dimensional objects within the context of a framework which posits that spacetime itself emerges out of a $2 + 1$ analog which it seems reasonable to call areatime, and apply this idea within the context of quantum theory to arrive at a deeper understanding of the physical origin of the Feynman Path Integral[7]. It may well be that the integration of this realization into quantum theory is required before the conclusion to which the arguments presented in this paper point is generally accepted.

References

[1] R Ferraro *Einstein's spacetime: An Introduction to Special and General Relativity* Springer Science, New York (2007)

[2] A Nikkhah Shirazi *A Derivation of the Speed of Light Postulate* Available at http://fqxi.org/community/forum/topic/329 (2008)

[3] J Callahan *The Geometry of Spacetime* Springer Verlag, New York (2000)

[4] H Batelaan, A Tonomura *The Aharonov-Bohm Effects: Variations on a Subtle Theme* Phys. Today **62**(9), 38 (2009)

[5] J Clauser *Early History of Bell's Theorem* in Coherence and Quantum Optics VIII (E Bigelow *et.al.* eds.), 19 Springer US (2004)

[6] A Nikkhah Shirazi available at http://hdl.handle.net/2027.42/83865 (2011)

[7] A Nikkhah Shirazi *A Novel Approach to 'Making Sense' Out of The Copenhagen Interpretation* in Quantum theory: Reconsideration of Foundations 6 AIP Conf. Proc. 1508, 422 (2013)