An Interpretation of the Essence of the Inertial Force

ChiYi Chen[a*]
[a] *Hangzhou Normal University, Hangzhou 310036, China*

In this article we discuss the formulism of particle dynamics under the framework of classical mechanics. But we propose a new dynamical equation (5) which has significant advantages than Newton's second law. The essence of the inertial force is also interpreted in a very simple and natural way. Firstly, we remove the concept of inertial reference frame from particle dynamics and the equation (5) can be directly applied in any reference frame. Secondly, we prove that the essence of inertial forces is the real force exerted on the reference object, which must be deducted in a relative counting of forces between the object under study and the reference object. Finally, the general principle of relativity is first time realized on the particle dynamics. In one word, the new form of dynamical law (5) is more general and presents a more concise physical picture than Newton's second law.
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1 INTRODUCTION

In the framework of Newtonian classical mechanics, the fundamental dynamics equation is Newton's second law. But Newton's second law is only valid in so-called inertial reference frame. If we want to invoke the form of Newton's second law in non-inertial reference frame, there is a fictitious force—inertial force must be introduced additionally. And the magnitude of inertial force is essentially defined by the relative acceleration between the non-inertial reference frame and a certain inertial reference frame. Therefore, the whole classical particle dynamics is established on the basis of inertial reference frame. However, as we well know, we are never able to find a true inertial reference frame in the practical application. This situation is surely not satisfying.

In fact, every reference frame must be established based on a concrete and real reference object. Otherwise, there would be no reference value in measuring any object's motion in the natural world. As for the relationship between the reference object and the reference frame, in practical cases if we assign the reference object first, the reference frame can be established naturally identifying the reference object as its origin point. Otherwise, the reference frame is assigned first, then in principle any real object which is fixed in the reference frame can be identified as its reference object.

1 THE MOTIVATIONS AND PHYSICS LOGIC

1.1 what problems exist in Newton's second law?

In Theory, Newton's second law is valid only for inertial reference frames. Even we introduce the inertial force to make a compulsory application in non-inertial reference frames, the calculation of inertial forces still depends on finding out an inertial reference frame firstly. In practice, we are not able to find out even one reference object which can exactly correspond to the inertial reference frame. Besides, the essence of inertial forces is not clear. There are widespread controversies about it. However, the natural philosophy basis for classical mechanics is definite and clear. So these problems illustrate that Newton's second law still has somewhere for improvement.

1.2 why these problems exist in Newton's second law?

The reason is that Newton's second law is not causal symmetric in its form. The Newton's second law is

$$\mathbf{F}|_p = m_p \mathbf{a}|_{p-O}. \tag{1}$$

The left hand side of this equation is given by the total force exerted on the particle p, and only depends on p. But the right hand side of this equation not only depends on the particle p but also depends on the reference object O which corresponds to the origin point of reference frame. If we regard the force as the cause, and regard the acceleration as the effect, the causality of Newton's second law is not symmetric and consistent in form. This is the just point to result that Newton's second law is theoretically valid only in inertial reference frames, but none of them can be found in practice.

1.3 how to solve the problem of causal consistency?

To the total force exerted on the particle p from the whole universe, how should we express the corresponding effect? The total force of a single particle should be objective, and not change with the alternation of observers. Therefore, the corresponding effect should be objective, and not relevant with any reference frame. In fact the objective effect can only correspond to the acceleration in the cosmic spatial background,

$$\mathbf{F}|_p = m_p \frac{d^2}{dt^2}(X, Y, Z)|_p. \tag{2}$$

Particularly, the objective position of the particle p in the cosmic spatial background here is denoted by $(X, Y, Z)|_p$. As the name implies, the spatial background is just what left in a space region after all objects inside it were moved away. And the cosmic spatial background is just what left in the whole universe after all concrete objects in the universe were moved away. We believe that there exists an absolute and eternally unchangeable background for cosmic space. the motion of all objects in the universe must be located in this common absolute three-dimensional space backgroun, because only on this basis then the existence of objective dynamical laws is possible. The key point which must be distinguished here is that the existence is proposed for absolute background, instead of absolute space-time. Here *absolute* means that the cosmic spatial background has no dynamical evolution and is irrelative to the existence or motion of any object. The reason is that both Newton's view of absolute spacetime and Einstein's spacetime theory of relativity are mainly concerned with the rules on the change of spacetime scales. But the spacetime scales are radically defined by the intervals of proper events occurred in objects, and so belong to the property of concrete objects. However, the cosmic spatial background do not contain any concrete object. Therefore, the existence of absolute cosmic spatial background will not be conflicted with modern physics, and we must distinguish the spatial background of the universe from the spatial intervals in Einstein's theory of relativity. Definitely, the cosmic spatial background can be included into Newton's view of absolute spacetime. But in contrast, the former do not require that the spacetime intervals are absolute, which is the key point only for the latter. In summary, the cosmic spatial background really exists for our universe although it is not concrete like general objects. For example, we have known that the dimensions of cosmic spatial background should be of three. But there is still a problem that the objective position in cosmic spatial background can not be directly measured. What we can really measure is the difference between any two objective positions, which substantially constructs a mathematical vector,

$$\mathbf{x}|_{p-O} = (X, Y, Z)|_p - (X, Y, Z)|_O. \tag{3}$$

1.4 how to construct a particle dynamical equation which is really applicable for any observer?

All objects in the universe, including objects under study (p) and their reference objects (O), should be of equal status in the most fundamental law of dynamics. For example, the dynamics of any real reference object should also satisfy $\mathbf{F}|_O = m_O \frac{d^2}{dt^2}(X, Y, Z)|_O$. The introduction of reference frames is to make relative measurements on kinematical quantities. As a causal correspondence, the forces should be relatively counted in nature.

$$m_O \mathbf{F}|_p - m_p \mathbf{F}|_O = m_p m_O \frac{d^2}{dt^2}((X, Y, Z)|_p - (X, Y, Z)|_O) = m_p m_O \frac{d^2 \mathbf{x}|_{p-O}}{dt^2}. \tag{4}$$

Here the real object identified as the origin point of reference frame is denoted by O. Finally, we obtain

$$\mathbf{f}|_{p-O} = m_p \mathbf{a}|_{p-O} = \mathbf{F}|_p - \frac{m_p}{m_O}\mathbf{F}|_O. \tag{5}$$

This is just the main idea proposed by a more detailed article (hep-th/0312225), in which[1] it was expressed by the equation (9). This equation (5) is proposed to replace Newton's second law under the framework of classical mechanics since the new dynamical law (5) presents a more concise physical picture based on reinterpreting all existing successful experiments for classical mechanics.

The general principle of relativity is essentially a practical requirement. On one hand, we are never able to know about the actual state of motion of our terrestrial reference frame where our observers exist. On the other hand, we are always able to determine the rotation of any reference frame relative to cosmic spatial background by resorting to the galaxies far enough away since the cosmic spatial background is objective and motionless. Therefore, what the practical observation really requires is that dynamical laws must keep form invariant to all irrotational reference frames. Furthermore, in particle dynamics, the rotation phenomena can always be attributed to the relative motion between different particles. But for single particle, there is no concept of rotation. Once any reference object is regarded as a particle, no problem of rotation exists for reference particle. Just as the problem of variable mass system under the framework of Newtonian mechanics, the variable mass phenomena should be attributed to the relative motion between different particles in the system of particles. Thus the fundamental particle dynamics is still $\mathbf{F}|_p = m_p\mathbf{a}|_{p-O}$, while $\mathbf{F}|_p = \frac{d(\mathbf{p}|_{p-O})}{dt}$ can be generalized from the former equation. Therefore, the problem of reference frames' rotation is actually a mathematical problem. Ultimately, the problem of the rotation of reference frames can be separated from the problem of dynamical relativity.

In all concepts of reference frame, besides the property of space-time scales, there is only the reference object is physical. Therefore, under the framework of classical mechanics where the space-time scales remain the same for all reference frames, then any object's dynamical problem in any reference frame can be substantially regarded as a two-body problem. In fact, according to the formal similarity between equations (1) and (2), it is easy to see that above more general dynamical equation (5) can actually be also derived by making a substraction between the dynamical equations of the object under study and the reference object under the framework of Newton's second law. But in principle the equation (5) should not be derived on the basis of Newton's second law, moreover, it must be pointed out that the grounding equation (2) is only valid for the cosmic spatial background, instead of any concrete reference frame, thereby the special status for inertial reference frames is logically removed.

2 THE ADVANTAGES AND PHYSICS EFFECTS

2.1 the realization of general principle of relativity and the essence of inertial force

The equation (5) can be directly applied in all reference frames(namely the general principle of relativity), and naturally interpret the essence of inertial force. $\mathbf{F}|_p$ and $\mathbf{F}|_O$ in the equation (5) can be calculated respectively according to existing knowledge of all types of interactions. Any real object or some part of object can be identified as a reference particle. Therefore, the equation (5) can be applied in all reference frames. Moreover, according to the equation (5), the inertial force is essentially the force exerted on the reference origin that must be deducted in the relative counting of forces. We assume that there are two relative accelerated reference frames. Their reference objects denoted by O and O'. For above two reference frames, it is the difference in their forces being exerted that results in a relative acceleration between these two reference frames. In Newtonian mechanics, such a relative acceleration is depicted as a so-called inertial force. But now the relative acceleration between reference frames is naturally interpreted by the difference in their forces being exerted,

$$\frac{m_p}{m_{O'}}\mathbf{F}|_{O'} - \frac{m_p}{m_O}\mathbf{F}|_O = m_p\mathbf{a}|_{p-O} - m_p\mathbf{a}|_{p-O'} = m_p\mathbf{a}|_{O'-O} = \mathbf{f}|_{inertial}. \tag{6}$$

The equation (6) self-consistently goes back to the relative dynamical equation given by new dynamical law (5) when it is applied on objects O and O'. Above analysis illustrates that the new dynamical law (5), compared with Newton's second law, possesses more complete and symmetric theoretical structure. Therefore, it is in possession of real universality.

2.2 The correctness of the new dynamical equation

It can be proved that all successful applications of classical mechanics can be naturally accounted for again by the new dynamical law (5). We assume there are two arbitrary particles(such as the Moon and Venus) exist in the solar system which are denoted by 1 and 2. The total forces from the whole universe exerted on the particle i can be written down as (Here we assume that non-gravitational forces are ignorable)

$$\mathbf{F}|_i = (\mathbf{f}_i)_{NonGrav} + (\mathbf{f}_i)_{SolarGrav} + (\mathbf{f}_i)_{OutSolarGrav} \approx (\mathbf{f}_i)_{SolarGrav} + (\mathbf{f}_i)_{OutSolarGrav}. \tag{7}$$

Here $(\mathbf{f}_i)_{SolarGrav}$ means the gravitational forces exerted on the particle i from the matter in solar system, and $(\mathbf{f}_i)_{OutSolarGrav}$ means the gravitational forces exerted on the particle i from the matter outside the solar system. According to the equation (5), the relative dynamics between 1 and 2 is given by

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \approx [(\mathbf{f_1})_{SolarGrav} - \frac{m_1}{m_2}(\mathbf{f_2})_{SolarGrav}] + [(\mathbf{f_1})_{OutSolarGrav} - \frac{m_1}{m_2}(\mathbf{f_2})_{OutSolarGrav}] = m_1\mathbf{a}|_{1-2}. \tag{8}$$

Owing to the solar system being a gravitational bound system in the universe, the gravitational forces exerted from the matter outside the solar system satisfy,

$$\frac{(\mathbf{f_1})_{OutSolarGrav}}{(\mathbf{f_2})_{OutSolarGrav}} \approx \frac{m_1}{m_2}. \tag{9}$$

So the relative dynamics (8) between particles 1 and 2 can be approximately rewritten as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \approx [(\mathbf{f_1})_{SolarGrav} - \frac{m_1}{m_2}(\mathbf{f_2})_{SolarGrav}] = m_1\mathbf{a}|_{1-2}. \tag{10}$$

Specifically, if we select the center of sun as the particle 2 in above equation, the forces exerted on it from the solar system may usually be approximated to be zero, namely $(\mathbf{f_2})_{SolarGrav} \approx 0$. Then the dynamics of particle 1 can be expressed as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \approx (\mathbf{f_1})_{SolarGrav} = m_1\mathbf{a}|_{1-2}. \tag{11}$$

That is the true formula always conformed with when the classical mechanics was practically applied on objects in the solar system. Obviously, Newton's second law is actually the approximation of new dynamical law (5) when it is applied in gravitational bound systems(we should make two steps of approximations). The approximation such as (11) is valid for usual sun-centered reference frame and earth-centered reference frame. Even for the most common ground-based reference frame, it substantially comes down to the earth-centered reference frame. Therefore, all successful applications of classical mechanics can be naturally accounted for again by the new dynamical law (5). Besides, why the gravitational force exerted from the outside of the solar system does not have to be taken into account? The direct reason is not the magnitude of forces being so small as to be ignored, but in fact the forces are deducted in the relative counting of forces.

Now we consider the case that the equation (5) is applied in ground-based reference frame. We assume that O denotes the center of the earth, A is the particle under study which is moving on the ground and B denotes the reference object which is at rest relative to the earth's surface. According to the equation (5), the relative dynamics between A and B can be given by

$$\mathbf{F}|_A - \frac{m_A}{m_B}\mathbf{F}|_B = m_A\mathbf{a}|_{A-B}. \tag{12}$$

In theory, the total forces exerted on the A and B can be written down as

$$\begin{aligned}\mathbf{F}|_A &= (\mathbf{f}_A)_{OutEarthGrav}(\propto m_A) + (\mathbf{f}_A)_{EarthGrav}(\propto m_A) + (\mathbf{f}_A)_{NonGrav},\\ \mathbf{F}|_B &= (\mathbf{f}_B)_{OutEarthGrav}(\propto m_B) + (\mathbf{f}_B)_{EarthGrav}(\propto m_B) + (\mathbf{f}_B)_{NonGrav}.\end{aligned} \tag{13}$$

The non-gravitational forces exerted on B can be equivalently calculated by the case when B is at rest relative to a smooth and horizontal surface. Hence the non-gravitational force equals to the support force from the ground, namely

$$(\mathbf{f}_B)_{NonGrav} = -m_B\mathbf{g}. \tag{14}$$

Substituting it in the equation (12), at the meantime we adopt an approximation that the gravitational forces from the earth and outside of the earth can be counterbalanced, we finally obtain,

$$\begin{aligned}\mathbf{F}|_A - \frac{m_A}{m_B}\mathbf{F}|_B &= (\mathbf{f}_A)_{NonGrav} - \frac{m_A}{m_B}((\mathbf{f}_B)_{NonGrav}) = (\mathbf{f}_A)_{NonGrav} - \frac{m_A}{m_B}(-m_B\mathbf{g})\\ &= (\mathbf{f}_A)_{NonGrav} + m_A\mathbf{g} = m_A\mathbf{a}|_{A-B}.\end{aligned} \tag{15}$$

This is the actual application of Newton's second law in ground-based reference frames. There is one point should also be emphasized. The change from $(\mathbf{f}_A)_{EarthGrav}$ in (13) to $m_A\mathbf{g}$ in (15), is usually interpreted by a fictitious inertial force owing to the rotation of the earth. But now we can see that it should be attributed to the real force from the ground exerted on the reference object B. But broadly speaking, the surface of the earth is rigid relative to the center of the earth, so the kinematical effect in ground-based reference frames is equivalently measured relative to the center of earth. Accordingly, for the counted force part, the gravitational forces exerted from the outside of the earth system should not be considered. Furthermore, if we assume the reference object B is an arbitrary object moving over the ground, namely $(\mathbf{f}_B)_{NonGrav} \neq -m_B\mathbf{g}$, the expression for an available particle dynamics (15) should be changed to be,

$$(\mathbf{f}_A)_{NonGrav} - \frac{m_A}{m_B}(\mathbf{f}_B)_{NonGrav} = m_A\mathbf{a}|_{A-B} = [(\mathbf{f}_A)_{NonGrav} + m_A\mathbf{g}] - \frac{m_A}{m_B}[(\mathbf{f}_B)_{NonGrav} + m_B\mathbf{g}]. \tag{16}$$

Therefore, in all senses, the practical Newton's second law can be regarded as an approximation of the new form of dynamical law (5) under some special circumstances.

2.3 the universality of the new dynamical equation

The equation (5) is directly applicable in the relative dynamics between galaxy clusters. For instance,we assume that there are only two particles (such as 1 and 2) exist in the whole universe, and there is only gravitational interaction exists between them. In the framework of Newtonian mechanics, none of real particles can be approximated as a inertial reference frame, so Newton's second law can not be directly applied in this situation. Even someone resorts to the center of mass for this system, but essentially the center of mass method is finally attributed to the cosmic spatial background. But according to (5), the relative dynamics between above two particles can be directly written down as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 = (\frac{Gm_1m_2}{r^3}\mathbf{r}_{1\to2}) - \frac{m_1}{m_2}(\frac{Gm_1m_2}{r^3}\mathbf{r}_{2\to1}) = m_1\mathbf{a}|_{1-2} = m_1\frac{d^2\mathbf{r}_{2\to1}}{dt^2}. \tag{17}$$

2.4 The advantage of the new dynamical equation

It may improve the precision on the physical application. In principle, we should always make an approximation on the inertial reference frame before Newton's second law can be applied. Actually, such an approximation is made in theory, rather than in practical measurement. Therefore, if the forces exerted on the reference origin can not be ignored, the error would be significant. By contrast, if we adopt the equation (5) as the new particle dynamical law, it at least has entirely solved the problem of the approximation on inertial reference frames.

2.5 physics effects

It must bring about modifications[2] on physical pictures under non-classical frameworks. The existence of cosmic spatial background is actually not conflicted with the principle of invariant light speed and the idea that gravity can be geometrized. The reason is that both Einstein's special relativity and geometric theory of gravity are mainly concerned with the laws on the change of space-time scales which all belong to the property of concrete objects, instead of the change on the cosmic spatial background[3–5]. Besides, the idea that forces should be relatively counted in any reference frame embodies the principle of causal consistency, so it is a very fundamental requirement.

3 CONCLUSION

In summary, Newton's second law is essentially an empirical law from ground based experiments which depicts the quantitative relation between the new additionally exerted force and the resulting relative acceleration and thereby a further reformulation according to the general principle of relativity is required. The new dynamical law (5) presents a more concise physical picture based on reinterpreting all existing successful experiments for classical mechanics. Consequently, we propose that the Newton's second law should be replaced by the new dynamical law (5) in the actual mechanics analysis.

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* Electronic address: `iamchiyi@gmail.com;chenchiyi@hznu.edu.cn`

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