An Interpretation of the Essence of the Inertial Force

ChiYi Chen[a*]
[a]Hangzhou Normal University, Hangzhou 310036, China

In this article we discuss the formulism of particle dynamics under the framework of classical mechanics. But we propose a new dynamical equation (21) which has significant advantages comparing with Newton's second law. The essence of the inertial force is also interpreted in a very simple and natural way. Firstly, we remove the special status of inertial reference frame from particle dynamics and the equation (21) can be directly applied in any reference frame. Secondly, we prove that the essence of inertial forces is the real force exerted on the reference object, which must be deducted in a relative counting of forces between the object under study and the reference object. Finally, the general principle of relativity is realized for the first time on the particle dynamics. In general, the new dynamical equation (21) presents a more concise and universal physical picture than Newton's second law.
PACS number(s): 45.20.-d, 45.05.+x

1 INTRODUCTION

In the framework of Newtonian classical mechanics, the fundamental dynamical equation is Newton's second law. But Newton's second law is only valid in so-called inertial reference frames. If we want to invoke the form of Newton's second law in non-inertial reference frames, there is a fictitious force—inertial force must be introduced additionally. And the magnitude of the inertial force is essentially defined by the relative acceleration between the non-inertial reference frame and a certain inertial reference frame. Therefore, the classical particle dynamics is totally established on the basis of inertial reference frame. However, as we well know, we are never able to find a true inertial reference frame in the practical application. This situation is surely not satisfying.

In fact, every reference frame must be established based on a concrete and real reference object. Otherwise, there would be no reference value in measuring any object's motion in the natural world. As for the relationship between the reference object and the reference frame, in practical cases if we assign the reference object first, the reference frame can be naturally established by identifying the reference object as its origin point. Otherwise, the reference frame is assigned first, then in principle any real object which is fixed in the reference frame can be identified as its reference object.

2 PHYSICAL MOTIVATIONS AND NEW PARTICLE DYNAMICAL EQUATION

2.1 what problems exist in Newton's second law?

In theory, Newton's second law is valid only for inertial reference frames. Even we introduce the inertial force to make a compulsory application of Newton's second law in non-inertial reference frames, the calculation of inertial forces still depends on finding out an inertial reference frame firstly. In practice, we are not able to find out even one real object which can exactly correspond to the inertial reference frame. Besides, the essence of inertial forces is not clear. There are widespread controversies about it. However, the natural philosophy basis for classical mechanics is definite and clear. So these problems illustrate that Newton's second law still has somewhere for improvement.

2.2 why these problems exist in Newton's second law?

The reason is that Newton's second law is not causal symmetric in its form. The Newton's second law can be generally expressed as

$$\mathbf{F}|_p = m_p \mathbf{a}|_{p-O}. \tag{1}$$

The left hand side of this equation is given by the total force exerted on the particle p, and which only depends on p. But the right hand side of this equation not only depends on the particle p but also depends on the reference object O which corresponds to the origin point of the reference frame. If we regard the force as the cause, and regard the acceleration as the effect (namely result), the causality of Newton's second law is not symmetric and consistent in form. This is the just point to result that Newton's second law is theoretically valid only in inertial reference frames, but none of them can be found in practice.

2.3 what is the causal consistent equation for particle dynamics?

In the framework of classical mechanics, how can we write down a particle dynamical equation which is causal symmetric and consistent in form? In fact, such an equation exists and now it is firstly given as an axiom,

$$\mathbf{F}|_p - \frac{m_p}{m_O}\mathbf{F}|_O = m_p \mathbf{a}|_{p-O}. \tag{2}$$

Here the definition of the force and the acceleration is just the same as that in Newtonian mechanics. $\mathbf{F}|_p$ and $\mathbf{F}|_O$ denote the total forces from the whole universe exerted on the particle p and the reference object O respectively. m_p and m_O denote the mass of the particle p and the reference object O respectively. Since the reference object is definitely fixed in the reference frame, therefore, the acceleration ($\mathbf{a}|_{p-O}$) of the particle p relative to the reference object, equals to that relative to the origin point of the reference frame. The equation (2) is just the new dynamical equation which will be discussed as an emphasis in this article.

3 THE ADVANTAGES AND PHYSICAL EFFECTS OF THE NEW DYNAMICAL EQUATION

3.1 the realization of general principle of relativity and the essence of inertial force

The equation (2) can be directly applied in all reference frames, namely the general principle of relativity, and naturally interprets the essence of inertial force. $\mathbf{F}|_p$ and $\mathbf{F}|_O$ in the equation (2) can be calculated respectively according to existing knowledge of all types of interactions. In principle any real object or some part of object can be identified as a reference particle. Therefore, the equation (2) can be applied in all reference frames. Moreover, according to the equation (2), the inertial force is essentially the real force exerted on the reference object, which must be deducted in a relative counting of forces. To illustrate this point, we assume that there are two relatively accelerated reference frames. Their reference objects are denoted by O and O'. For above two reference frames, it is the difference in their forces being exerted that results in a relative acceleration between these two reference frames. In Newtonian mechanics, such a relative acceleration is depicted as a so-called inertial force. But now the relative acceleration between reference frames is naturally interpreted by the difference in their forces being exerted,

$$\frac{m_p}{m_{O'}}\mathbf{F}|_{O'} - \frac{m_p}{m_O}\mathbf{F}|_O = m_p\mathbf{a}|_{p-O} - m_p\mathbf{a}|_{p-O'} = m_p\mathbf{a}|_{O'-O} = \mathbf{f}|_{inertial}. \tag{3}$$

The equation (3) self-consistently goes back to the relative dynamical equation given by new dynamical law (2) when it is applied on reference objects O and O'. Above analysis illustrates that the new dynamical law (2), compared with Newton's second law, possesses more complete and symmetric theoretical structure. Therefore, it is in possession of real universality.

3.2 The correctness of the new dynamical equation

It can be proved that all successful applications of classical mechanics can also be naturally accounted for by the new dynamical equation (2). In general, we assume that there are two arbitrary particles denoted by 1 and 2(such as the Moon and Venus), exist in the solar system. The total forces from the whole universe exerted on the particle i can be written down as (Here we assume that non-gravitational forces are ignorable)

$$\mathbf{F}|_i = (\mathbf{f}_i)_{NonGrav} + (\mathbf{f}_i)_{SolarGrav} + (\mathbf{f}_i)_{OutSolarGrav} \approx (\mathbf{f}_i)_{SolarGrav} + (\mathbf{f}_i)_{OutSolarGrav}. \tag{4}$$

Here $(\mathbf{f}_i)_{SolarGrav}$ means the gravitational forces exerted on the particle i from the matter in solar system, and $(\mathbf{f}_i)_{OutSolarGrav}$ means the gravitational forces exerted on the particle i from the matter outside the solar system. According to the equations (2) and (4), the relative dynamics between 1 and 2 can be expanded as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \simeq [(\mathbf{f_1})_{SolarGrav} - \frac{m_1}{m_2}(\mathbf{f_2})_{SolarGrav}] + [(\mathbf{f_1})_{OutSolarGrav} - \frac{m_1}{m_2}(\mathbf{f_2})_{OutSolarGrav}] = m_1\mathbf{a}|_{1-2}. \tag{5}$$

Owing to both 1 and 2 being bound in the same solar system which is just one of innumerable gravitational bound systems in our universe, the gravitational forces exerted from the matter outside the solar system satisfy,

$$\frac{(\mathbf{f_1})_{OutSolarGrav}}{(\mathbf{f_2})_{OutSolarGrav}} \approx \frac{m_1}{m_2}. \tag{6}$$

So the relative dynamics (5) between particles 1 and 2 can be approximately rewritten as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \approx [(\mathbf{f_1})_{SolarGrav} - \frac{m_1}{m_2}(\mathbf{f_2})_{SolarGrav}] = m_1\mathbf{a}|_{1-2}. \tag{7}$$

Specifically, if we select the center of sun as the particle 2 in above equation, at that time the reference object can be the whole sun, or can be any real object at the center of sun. For example, if we choose the whole sun as the reference object, the second term in above equation would be drove down since $m_1 \ll m_2$. Otherwise, we may choose a real object at the center of sun as the reference object 2, which has a comparative mass as the object 1. Because for the reference object 2, the distribution of the matter in the solar system has a high approximated spherical symmetry, the forces exerted on the reference object 2 from the solar system may usually be approximated to be zero, namely $(\mathbf{f_2})_{SolarGrav} \approx 0$. Therefore, if we don't desire a high precision, the dynamics of particle 1 can always be expressed as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \approx (\mathbf{f_1})_{SolarGrav} = m_1\mathbf{a}|_{1-2}. \tag{8}$$

That is the true formula which we always conform with, when the classical mechanics is practically applied on objects in the solar system. Obviously, Newton's second law is substantially an approximation of new dynamical law (2) when it is applied in gravitational bound systems(It undergoes about two steps of approximations). The approximation such as (8) is valid for usual sun-centered reference frame and earth-centered reference frame. Even for the most common ground-based reference frame, it substantially comes down to the earth-centered reference frame. Therefore, all successful applications of classical mechanics can be naturally accounted for again by the new dynamical law (2). Besides, why the gravitational forces exerted from the outside of the solar system do not have to be taken into account? The direct reason is not the magnitude of these forces being so small as to be ignored, but in fact the forces are deducted in the relative counting of forces.

Now we consider the case that the equation (2) is applied in ground-based reference frame. We assume that A is the particle under study which is moving on the ground and B denotes the reference object which is at rest relative to the earth's surface. According to the equation (2), the relative dynamics between A and B can be directly expressed as

$$\mathbf{F}|_A - \frac{m_A}{m_B}\mathbf{F}|_B = m_A\mathbf{a}|_{A-B}. \tag{9}$$

In theory, the total forces exerted on the A and B can be expanded as

$$\begin{aligned}\mathbf{F}|_A &= (\mathbf{f}_A)_{OutEarthGrav}(\propto m_A) + (\mathbf{f}_A)_{EarthGrav}(\propto m_A) + (\mathbf{f}_A)_{NonGrav},\\ \mathbf{F}|_B &= (\mathbf{f}_B)_{OutEarthGrav}(\propto m_B) + (\mathbf{f}_B)_{EarthGrav}(\propto m_B) + (\mathbf{f}_B)_{NonGrav}.\end{aligned} \tag{10}$$

The non-gravitational forces exerted on B can be equivalently calculated from the case that B is at rest relative to a smooth and horizontal surface. Therefore, the non-gravitational force must numerically equal to the support force from the ground,

$$(\mathbf{f}_B)_{NonGrav} = -m_B\mathbf{g}. \tag{11}$$

Substituting it into the equation (9), at the same time we adopt an approximation that the gravitational forces from the earth and outside of the earth can be counterbalanced, we finally obtain,

$$\begin{aligned}\mathbf{F}|_A - \frac{m_A}{m_B}\mathbf{F}|_B &= (\mathbf{f}_A)_{NonGrav} - \frac{m_A}{m_B}((\mathbf{f}_B)_{NonGrav}) = (\mathbf{f}_A)_{NonGrav} - \frac{m_A}{m_B}(-m_B\mathbf{g})\\ &= (\mathbf{f}_A)_{NonGrav} + m_A\mathbf{g} = m_A\mathbf{a}|_{A-B}.\end{aligned} \tag{12}$$

This is the actual application expression of Newton's second law in ground-based reference frames. There is one point should also be emphasized. The change from $(\mathbf{f}_A)_{EarthGrav}$ in (10) to $m_A\mathbf{g}$ in (12), is usually interpreted by a fictitious inertial force owing to the rotation of the earth. But now we can see that it actually should be attributed to the real force from the ground exerted on the reference object B. Broadly speaking, the surface of the earth is rigid in spatial distance relative to the center of the earth, so the measurement of kinematical effect in ground-based reference frames is equivalent to that relative to the center of earth. Accordingly, for the part of counted force, the gravitational forces exerted from the outside of the earth system should not be considered.

Furthermore, if we assume the reference object B is an arbitrary object moving over the ground(including the case of acceleration), at that time $(\mathbf{f}_B)_{NonGrav} \neq -m_B\mathbf{g}$, so the expression for an available particle dynamics (12) should be changed as,

$$(\mathbf{f}_A)_{NonGrav} - \frac{m_A}{m_B}(\mathbf{f}_B)_{NonGrav} = m_A\mathbf{a}|_{A-B} = [(\mathbf{f}_A)_{NonGrav} + m_A\mathbf{g}] - \frac{m_A}{m_B}[(\mathbf{f}_B)_{NonGrav} + m_B\mathbf{g}]. \tag{13}$$

Therefore, in all senses, the practical Newton's second law can be regarded as an approximation of the new dynamical equation (2) under some special conditions.

3.3 the universality of the new dynamical equation

The equation (2) is directly applicable in the relative dynamics between galaxy clusters. For instance, we assume that there are only two particles (such as 1 and 2) exist in the whole universe, and there is only gravitational interaction exists between them. In the framework of Newtonian mechanics, none of real particles can be approximated as a inertial reference frame, so Newton's second law can not be directly applied in this situation. Even someone resorts to the center of mass for this system, but essentially the center of mass method is finally attributed to an assumption that the center of mass is rest relative to a certain inertial reference frame. But according to (2), the relative dynamics between above two particles can be directly written down as

$$\mathbf{F}|_1 - \frac{m_1}{m_2}\mathbf{F}|_2 \ = \ (\frac{Gm_1m_2}{r^3}\mathbf{r}_{1\to 2}) - \frac{m_1}{m_2}(\frac{Gm_1m_2}{r^3}\mathbf{r}_{2\to 1}) = m_1\mathbf{a}|_{1-2} = m_1\frac{d^2\mathbf{r}_{2\to 1}}{dt^2}. \tag{14}$$

4 THE PHYSICAL LOGIC OF THE NEW DYNAMICAL EQUATION

4.1 the new dynamical equation is also a logical result from Newton's second law

In all concepts of reference frame, besides the property of space-time scales, there is only the reference object is physical. Therefore under the framework of classical mechanics, since the space-time scales have been assumed to be constant for all reference frames, any object's dynamical problem in any reference frame can be substantially regarded as a two-body problem. In fact, even in the framework of Newtonian mechanics, the new dynamical equation (2) can also be logically derived. Firstly, we assume there really exists an inertial reference frame which is denoted by Ω. According to Newton's second law, an arbitrary object p satisfies the particle dynamics as

$$\mathbf{F}|_p = m_p\mathbf{a}|_{p-\Omega}. \tag{15}$$

Similarly, to an arbitrary reference object O, which must also obey the same natural law,

$$\mathbf{F}|_O = m_O\mathbf{a}|_{O-\Omega}. \tag{16}$$

Performing a simple algebraic manipulation, we obtain

$$\mathbf{F}|_p - \frac{m_p}{m_O}\mathbf{F}|_O = m_p\mathbf{a}|_{p-\Omega} - \frac{m_p}{m_O}(m_O\mathbf{a}|_{O-\Omega}) = m_p[\mathbf{a}|_{p-\Omega} - \mathbf{a}|_{O-\Omega}] = m_p\mathbf{a}|_{p-O}. \tag{17}$$

Therefore, the new dynamical equation (2) which is given as the axiom at the beginning of this article is recovered. In this sense, the new dynamical equation must be right under the framework of classical mechanics if we assume the existence of an inertial reference frame.

But according to above discussions, the new dynamical equation has definite advantages comparing with Newton's second law. Especially, we remove the necessity of the existence of inertial reference frames and inertial forces from the formulism of the theory. Therefore, if we furthermore want to logically remove the special status of inertial reference frames, in principle the equation (2) should not be derived on the basis of Newton's second law.

4.2 an interpretation for the new dynamical equation at the level of natural philosophy

The new dynamical equation (2) embodies a form symmetry on the causal relationship. The right hand side of this equation is a relative acceleration, meanwhile the left hand side of this equation is a relative counting of forces. We suppose that every natural principle which is described as a pure relative law, must exist an absolute basis. In this spirit, the most simple natural philosophy basis which is logically required to construct the new dynamical equation (2) is nothing but the existence of an absolute cosmic spatial background. Now we derive the new dynamical equation (2) from the requirement of causal consistent principle.

First question, what is the exact information that all experiments of classical mechanics tell us? In fact, the experiments of classical mechanics didn't directly guarantee the validness of Newton's second law since every term of force appeared in Newton's second law must be defined to count force totally. Newton's second law is essentially an empirical law from ground based experiments which depicts the quantitative relation between the new additionally exerted force and the resulting relative acceleration. Therefore, the classical mechanics experiments did not actually tell us that Newton's second law is correct at all situations. Inversely, we find that Newton's second law can not be exactly applied in any situation. Now we know the reason is that there exists a causal inconsistency problem in Newton's second law.

Second question, how to solve the problem of causal inconsistency? We suppose that the particle dynamics is certainly to be a theory with causal principle. We regard the force as the cause, and regard the acceleration as the effect(result). Then, to the total force exerted on the particle p from the whole universe, how should we express the corresponding effect? The total force exerted on a single particle should be objective, and not change with the alternation of observers. Therefore, the corresponding effect should be objective, and be not relevant with any reference frame. Therefore, a completely objective acceleration can only be expressed as the acceleration in the spatial background of the whole universe(called cosmic spatial background),

$$\mathbf{F}|_p = m_p \frac{d^2}{dt^2}(X, Y, Z)|_p. \tag{18}$$

Particularly, here the objective position of the particle p in the cosmic spatial background is denoted by $(X, Y, Z)|_p$. In fact, the cosmic spatial background can be understand intuitively. As the name implies, the spatial background is just what still exist in a space region after all objects inside it were moved away. And the cosmic spatial background is just what still exist in the whole universe after all concrete objects in the universe were moved away. If we believe that the motion state for any object is objective, not only the state of relative motion as we have known is objective and definitely determinable, but the state of any object's motion itself should also be objective. That is to say there exists absolute state of motion for every object. Therefore, we hold that there exists an absolute and eternally unchangeable spatial background for the whole universe. The motion of all objects in the universe must be located in this common absolute three-dimensional space background, because only on this basis then the existence of objective dynamical laws is possible. Strictly speaking, the existence of an absolute comic spatial background is an assumption at the natural philosophy level. The key point which must be distinguished here is that the existence is proposed for absolute background, instead of absolute space-time. Here *absolute* means that the cosmic spatial background has no dynamical evolution and is irrelative to the existence or motion of any object. The reason is that both Newton's view of absolute spacetime and Einstein's spacetime theory of relativity are mainly concerned with the rules on the change of spacetime scales. But the spacetime scales are radically defined by the intervals of proper events occurred in objects, and so belong to the property of concrete objects. However, the cosmic spatial background do not contain any concrete object. Therefore, the existence of absolute cosmic spatial background will not be conflicted with the core ideas of modern physics, and we must distinguish the spatial background of the universe from the spatial intervals in Einstein's theory of relativity. Definitely, the cosmic spatial background can be included into Newton's view of absolute spacetime. But in contrast, the former do not require that the spacetime intervals are absolute, which is the key point only for the latter.

In summary, the cosmic spatial background really exists for our universe although it is not concrete like general objects. For example, we have known that the dimensions of cosmic spatial background should be of three. But there is still a problem that the objective position in cosmic spatial background can not be directly measured. What we can really measure is the difference between any two objective positions, which substantially constructs a mathematical vector,

$$\mathbf{x}|_{p-O} = (X, Y, Z)|_p - (X, Y, Z)|_O. \tag{19}$$

Third question, how to construct a particle dynamical equation which is really applicable for any observer? All objects in the universe, including objects under study (p) and reference objects (O), should be of equal status in the most fundamental law of dynamics. For example, the dynamics of any real reference object should also satisfy $\mathbf{F}|_O = m_O \frac{d^2}{dt^2}(X,Y,Z)|_O$. The introduction of reference frames is to make relative measurements on kinematical quantities. As a causal correspondence, the forces should also be relatively counted in nature.

$$m_O\mathbf{F}|_p - m_p\mathbf{F}|_O = m_p m_O \frac{d^2}{dt^2}((X,Y,Z)|_p - (X,Y,Z)|_O) = m_p m_O \frac{d^2\mathbf{x}|_{p-O}}{dt^2}. \tag{20}$$

Here the reference object O naturally corresponds to the origin point of a reference frame. Finally, we obtain

$$\mathbf{f}|_{p-O} = m_p\mathbf{a}|_{p-O} = \mathbf{F}|_p - \frac{m_p}{m_O}\mathbf{F}|_O. \tag{21}$$

In this way, the new dynamical equation (2) is almost logically derived without resorting to the concept of inertial reference frame and inertial force. This equation is also discussed in detail in another paper[1]. The equation (21) is proposed to replace Newton's second law under the framework of classical mechanics since the new dynamical law (21) presents a more natural and concise physical picture based on reinterpreting all existing successful experiments for classical mechanics.

The general principle of relativity is essentially a practical requirement. On one hand, we are never able to know about the actual state of motion of our terrestrial reference frame where our observers exist. On the other hand, we are always able to determine the rotation of any reference frame relative to cosmic spatial background by resorting to the galaxies far enough away since the cosmic spatial background is objective and motionless. Therefore, what the practical observation really requires is that dynamical laws must keep form invariant to all irrotational reference frames. Furthermore, in particle dynamics, the rotation phenomena can always be attributed to the relative motion between different particles. But for single particle, there is no concept of rotation. In other words, once any reference object is regarded as a particle, no problem of rotation exists for reference particle. Just as the problem of variable mass system under the framework of Newtonian mechanics, the variable mass phenomena should be attributed to the relative motion between different particles in the system of particles, so the fundamental particle dynamics is still $\mathbf{F}|_p = m_p\mathbf{a}|_{p-O}$. But for $\mathbf{F}|_p = \frac{d(\mathbf{p}|_{p-O})}{dt}$, it actually can be generalized from the former equation. Therefore, the problem of reference frames' rotation is actually a mathematical problem. Ultimately, the problem of the rotation of reference frames can be separated from the problem of dynamical relativity.

5 THE PHYSICAL SIGNIFICANCE OF THE NEW DYNAMICAL EQUATION

First, the new dynamical equation may improve the precision on the physical application. In principle, we should always make an approximation on the inertial reference frame before Newton's second law can be applied. Actually, such an approximation is made in theory, rather than in practical measurement. Therefore, if the forces exerted on the reference origin can not be ignored, the error would be significant. By contrast, if we adopt the equation (21) as the new particle dynamical law, it at least has entirely solved the problem of the approximation on inertial reference frames.

Second, the new dynamical equation (21) illustrates that the general principle of relativity for particle dynamics can be realized in a very simple and natural approach which is obviously different with Einstein's view[3]. Because the concept of inertial force still exists in Einstein's theory of relativity, and Einstein's (strong) equivalence principle further claims that the inertial force is equivalent to the gravitational force. However, we can see from the equation (21), which obviously is more natural and simple in physical picture, the essence of the inertial force is the real force exerted on the reference object. Therefore, the so-called inertial force can actually be all kind of interactions such as the gravitational interaction, electromagnetic interaction and so on.

Third, the new approach to realize the general principle of relativity must bring about modifications[2] on physical pictures under non-classical frameworks. The existence of an absolute cosmic spatial background is actually not conflicted with the principle of invariant light speed and the idea of gravity being geometrized. The reason is that both Einstein's special relativity and geometric theory of gravity are mainly concerned with the laws on the change of space-time scales which all belong to the property of concrete objects, instead of the change on the cosmic spatial background[3–5]. Besides, the idea that forces should be relatively counted in any reference frame embodies the principle of causal consistency, so it is a very fundamental requirement and should also be retained.

6 CONCLUSION

In summary, Newton's second law is essentially an empirical law from ground based experiments which depicts the quantitative relation between the new additionally exerted force and the resulting relative acceleration and thereby a further reformulation according to the general principle of relativity is required. The new dynamical law (21) presents a more concise physical picture based on reinterpreting all existing successful experiments for classical mechanics. Consequently, we propose that the Newton's second law should be replaced by the new dynamical law (21) in the actual mechanics analysis.

Acknowledgments: The author would like to thank all his friends who have provided their constructive criticisms and comments. This work has been supported from the Nature Science Foundation of Zhejiang Province under the grant number Y6110778. This research was supported in part by the Project of Knowledge Innovation Program (PKIP) of Chinese Academy of Sciences, Grant No. KJCX2.YW.W10. This report is dedicated to my grandfather Mr. Chen ChengDong.

---

* Electronic address: `iamchiyi@gmail.com;chenchiyi@hznu.edu.cn`

[1] ChiYi Chen, The Realization of General Principle of Relativity in Particle Dynamics, hep-th/0312225 V9, 26 Nov 2012.
[2] ChiYi Chen, On the Dynamical Relativity Principle and Cosmological MetricarXiv:hep-th/0411047v8.
[3] Bernard F. Schutz, A first course in general relativity, Cambridge Univers press 1985.
[4] Liu Liao, General Relativity (High Education Press, Shanghai, China, 1987), pp 26-30; 188-190.
[5] Steven. Weinberg "Gravitational and Cosmology: Principles and Applications of the General Theory of Relativity"(John Wiley & Sons, Inc, New York, 1972), Part 3.