

A formula based on twin primes that generates chains of primes in arithmetic progression

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Abstract. I was studying recurrences of the form $P(n) = P(n - 1) + 2^k - 2$, when incidentally I found a chain of 5 primes in arithmetic progression that satisfy this recurrence (8329, 8839, 9349, 9859, 10369). But, interesting, instead of find easily other chains of primes based on this recurrence, I obtained easily such chains (up to AP-6) defining in other way, based on twin primes, the relation between those 5 primes.

The formula is: $p + 2 + 30*m + p*30*n$, where m and n are integers, m non-negative and n positive, and p is the smaller prime from a pair of twin primes; giving a constant value to m and consecutive values to n we obtain for the following pairs of twin primes:

(11,13): we have $p = 11$.

We take $m = 1$. The formula becomes $43 + 330*n$. We obtain for n from 14 to 17 a chain of four primes in arithmetic progression (4663, 4993, 5323, 5653) and for n from 35 to 38 another such chain (11593, 11923, 12253, 12583).

We take $m = 2$. The formula becomes $73 + 330*n$. We obtain for n from 5 to 8 a chain of four primes in arithmetic progression (1723, 2053, 2383, 2713).

We take $m = 3$. The formula becomes $103 + 330*n$. We obtain for n from 3 to 6 a chain of four primes in arithmetic progression (1093, 1423, 1753, 2083).

We take $m = 4$. The formula becomes $133 + 330*n$. We obtain for n from 3 to 6 a chain of four primes in arithmetic progression (1123, 1453, 1783, 2113) and for n from 43 to 48 a chain of six primes in arithmetic progression (14323, 14653, 14983, 15313, 15643, 15973).

We take $m = 5$. The formula becomes $163 + 330*n$. We obtain for n from 36 to 39 a chain of four primes in arithmetic progression (12043, 12373, 12703, 13033).

(17,19): we have $p = 17$.

We take $m = 0$. The formula becomes $19 + 510 \cdot n$. We obtain for n from 14 to 18 a chain of five primes in arithmetic progression (7159, 7669, 8179, 8689, 9199).

We take $m = 1$. The formula becomes $49 + 510 \cdot n$. We obtain for n from 23 to 26 a chain of four primes in arithmetic progression (11779, 12289, 12799, 13309) and for n from 30 to 34 a chain of five primes in arithmetic progression (15349, 15859, 16369, 16879, 17389).

We take $m = 2$. The formula becomes $79 + 510 \cdot n$. We obtain for n from 10 to 14 a chain of five primes in arithmetic progression (5179, 5689, 6199, 6709, 7219).

We take $m = 3$. The formula becomes $109 + 510 \cdot n$. We obtain for n from 40 to 45 a chain of six primes in arithmetic progression (20509, 21019, 21529, 22039, 22549, 23059).

We take $m = 5$. The formula becomes $169 + 510 \cdot n$. We obtain for n from 16 to 20 a chain of five primes in arithmetic progression (8329, 8839, 9349, 9859, 10369).

(29,31): we have $p = 29$.

We take $m = 1$. The formula becomes $61 + 870 \cdot n$. We obtain for n from 30 to 34 a chain of five primes in arithmetic progression (26161, 27031, 27901, 28771, 29641).

We take $m = 2$. The formula becomes $91 + 870 \cdot n$. We obtain for n from 43 to 46 a chain of four primes in arithmetic progression (37501, 38371, 39241, 40111).

We take $m = 4$. The formula becomes $151 + 870 \cdot n$. We obtain for n from 42 to 45 a chain of four primes in arithmetic progression (36691, 37561, 38431, 39301).

We take $m = 5$. The formula becomes $181 + 870 \cdot n$. We obtain for n from 40 to 44 a chain of five primes in arithmetic progression (34981, 35851, 36721, 37591, 38461).

(41,43): we have $p = 41$.

We take $m = 1$. The formula becomes $73 + 1230 \cdot n$. We obtain for n from 13 to 17 a chain of five primes in arithmetic progression (16063, 17293, 18523, 19753, 20983).

We take $m = 5$. The formula becomes $193 + 1230 \cdot n$. We obtain for n from 4 to 7 a chain of four primes in arithmetic progression (5113, 6343, 7573, 8803).

(59,61): we have $p = 59$.

We take $m = 0$. The formula becomes $61 + 1770 \cdot n$. We obtain for n from 7 to 11 a chain of five primes in arithmetic progression (12451, 14221, 15991, 17761, 19531).

We take $m = 2$. The formula becomes $121 + 1770 \cdot n$. We obtain for n from 17 to 20 a chain of four primes in arithmetic progression (30211, 31981, 33751, 35521) and for n from 25 to 28 another such chain (44371, 46141, 47911, 49681).

We take $m = 4$. The formula becomes $181 + 1770 \cdot n$. We obtain for n from 28 to 32 a chain of five primes in arithmetic progression (49741, 51511, 53281, 55051, 56821).

Conclusion: we obtained two AP-6, eight AP-5, twelve AP-4 and many AP-3, considering just first five pairs of twin primes, beside of course the pairs (3,5) and (5,7), values for m up to 5 and values for n up to 50.