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## **Semi-fundamental Abundance of the Elements**

### **Abstract and Justification**

Fusion is an important process in nature. It determines primordial helium and abundance of other elements measured throughout the universe. Fusion powers the sun and is important in aging of the stars and other high density, high temperature processes. The author developed a mass/information model of the neutron and proton in reference [1]. The model was extended to a binding energy model for atoms in reference [2] and the fundamental information from the model was applicable to fusion in general. Specifically the author developed fusion equations that followed the same theme found in the early references, i.e. description of nature based on information and probabilities. Of particular interest was a model that was largely independent of measured parameters and based on fundamentals of atomic binding energy model.

Reference [3] presented a temperature history for expansion that was similar to other cosmologies but energy in the proton mass model [1] did not support “near infinite” temperatures at “time zero”. A short time later (on the order of hundredths of seconds) the author’s proposal does produce temperatures in the range of  $1e9K$ , except the energy is related to neutron decay. Plasma exists until the temperature drops enough to allow electrons to form orbits around protons. Eventually acoustic and gravitational forces become dominant and accumulation of mass into clusters, galaxies and clusters begins. The concentration process later allows stars to “light up” with fusion when they become dense and hot. This is known in the literature as re-ionization. Stars burn up their hydrogen and follow a well documented aging cycle that depends on the kinetics of progressive fusion reactions. Literature cites measurements regarding the abundance of the heavy elements [11] that are produced by these reactions.

Fusion and heavy element formation are well studied but the author believes that the approach presented herein is almost fundamental. The main empirical factor is the amount of material subjected to the high temperature conditions.

### **Fusion kinetics**

Fusion is based on a proton and electron with kinetic energy from its environment colliding with an existing proton or atom. Since the proton and the existing atom are positively charged they repel one another creating a barrier for fusion. Reference [2]

discusses fusion, binding energy fundamentals and accurately models data from reference [8]. To match the barrier energy (BE), the proton and its associated electron must gain energy from the temperature of the environment but they also must be at high density. This prompts the properties exchange and nil release (0.511+0.111-0.622) that characterizes the re-conversion process to neutrons for about half of the protons. The barrier energy (BE) is simply the retained energy for the protons (from reference 2 about BE (mev)=0.1/4\*protons). If electrons are available and the energy related to the temperature is lower only some atoms will achieve the barrier and fuse. This makes the probability of reaction (P reaction) the important parameter. Boltzmann's approach to equilibrium kinetics characterizes the process even though it may involve several reaction paths. Following Boltzmann this probability can be characterized by the expression  $P_{\text{reaction}} = \exp(-BE/\text{mev})$ . Specifically energy of the environment is the mev of the electron that it gets from temperature. Reactions that have lower barrier energies are more frequent and this is the basis for the cosmic abundance of the elements. The proposal is as follows:

Probability of fusing/sec=Probability barrier \* density ratio \*reaction rate

Number of protons fused= probability of fusing/sec \* number of protons\* delta time

P reaction: The probability P of achieving the barrier is:

$P = \exp(-.0139/.00308) = 1.09e-2$  for a temperature of  $2.39e7$  K (the sun).

Where -0.0139 mev is the barrier energy for the reaction that converts hydrogen to helium/deuterium (see paragraph entitled "abundance of the elements" later in this document that gives the barrier energy for other elements).

Boltzmann's constant= $8.62e-11$  mev/K

Kinetic energy  $= \frac{3}{2} B * T = 1.5 * 8.62e-11 * 2.39e7 = .00308$  mev.

Probability related to density:

At higher density (density ratio) more atoms are present to undergo the reaction.

The density ratio is formed from (density/max density). (Maximum density is the initial density associated with time zero r of  $1.2e-14$  meters and is  $2.3e14$  kg/m<sup>3</sup>. See reference 3.)

Probability of reaction/second:

The number of reactions is also dependent on how fast each reaction can occur (number of reactions per second). The reaction speeds up if an electron can cross a barrier radius and reach the nucleus more quickly. The radius is the degenerate radius  $5.29e-11/\text{degeneracy}$  where degeneracy is  $(\text{density}/2699)^{.33}$ . The value 2699 is the density for protons just separated by the electron orbit  $5.29e-11$  meters. The velocity is  $v = C * (1 - (0.511/(0.511 + ke))^2)^{.5}$ .

This makes the probability of a reaction in rate/sec:

Rate=velocity/degenerate radius/exp(56.74)

The value exp(56.74) is the delay "entropy" that characterizes decay of the neutron.

Reference 15 details decay time for all the mesons and baryons. The Particle Data Group gives the decay for the neutron as 0.00112/sec. The calculation below converts the decay time to the author's N and gives the time modifier  $1/\exp(56.74)$  (one could also treat this as another probability  $P = 1/\exp(56.74)$  but based

on the study of meson decay time it is the decay time for the neutron ( $2e-22 * \exp(56.7)$  shown below).

Particle D:	Decay	mass	Ke	Velocity	gamr	R	time sec	N=LN(892/2.04e
Group	dec sec	mev	mev	v/C		meters	time=R*2*PI()/	(300000000*
1/sec								
	0.00112	892.9	939.5654	10.15	0.15	0.99	<b>R=1.97e-13/(939*20.3/0.989)^0.5</b>	
							<b>1.42E-15</b>	2.04E-22
								56.74

## Initial radius

In the authors model, “inflation” is the process of duplication of one neutron by  $\exp(180)$  times [3]. It is proposed that “time zero” is when a gravitational radius of  $1.2e-14$  meters from the R table is duplicated and fills a sphere of radius  $1.2e-14 * \exp(60) = 1.37e12$  meters.

## Initial kinetic energy of the electron

Reference [1] indicated that the electron comes from frequency transitions of the “electron quad”. As with the other frequency transitions there is a difference kinetic energy resulting from the requirement that the energy entering the transition has to equal the exiting energy. In this case the difference energy specifies the initial kinetic energy of the electron. ( $0.622 + 27.2E-6 - 2.47E-5 - 0.511 = 0.1114$  mev).

10.1361	0.5110	0.2958	2.7217E-05	<b>0.5110</b>	0.1114
0.1972	2.466E-05	10.3333	0.6224	<b>Electron mass</b>	

As the neutron decays and releases the electron, reference [1] indicates that the electron has the maximum initial kinetic energy above. The protons and electrons fill the initial gravitational radius and the kinetic energy of the electron heats the protons. Ultimately, the  $0.1114$  mev energy decreases to the kinetic energy of an electron in its base state of about  $13.6e-6$  mev but by that time most of the initial energy has been released as radiation.

As density changes the energy  $0.1114$  mev is either relaxed or degenerate [5] as its radius changes from  $5.29e-11$  meters. The equation is:

Energy of  $0.1114$  as a function of radius:  
 $= 0.1114 * (\text{density} / \text{density0})^{.33}$   
 $= 0.1114 * (\text{radius0} / \text{radius})$   
 For the expanding radius:  
 $= 0.1114 * (1.2e-14 \text{ meters} / \text{expanding radius})$

## Initial temperature of the compressed state

At time zero, no fusion has occurred and the proton mass table shows a small release related to the neutrino quad and nothing else. There appears to be no other significant energy release that might raise the initial temperature since neutrinos escape without adding energy to their surroundings. Neutrons start decaying at this point and they find themselves in a highly compressed state known as a degenerate state. The temperature of this highly compressed state is related to the electrons degeneracy as shown in the table below. The maximum degeneracy is achieved when it is compressed with its proton to  $1.2e-14$  meters.

$$\begin{aligned} E0*r/r & \\ 27.2e-6*(5.29e-11/1.2e-14) & \\ 0.120 \text{ mev} & \end{aligned}$$

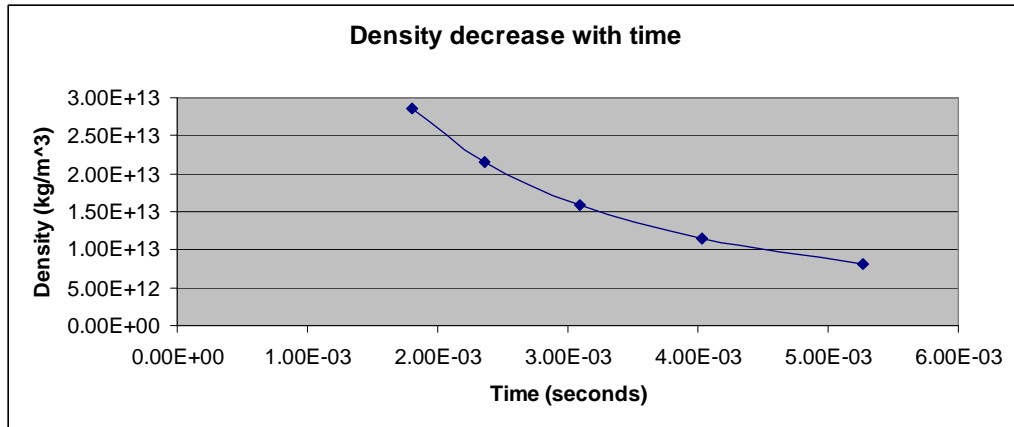
The proton (mass=938.27 mev) receives energy from the electron (m=.511 mev) and energy is shared between the electron and proton. The kinetic energy they experience can be translated to a temperature with the following equation:

$$\begin{aligned} E0*r/r & \quad \text{simple ai308} \\ 27.2e-6*(5.29e-11/1.2e-14) & \\ 0.120 \text{ mev} & \\ T=.11/1.38e-13/938.3) & \\ 8.50E+08 \text{ degrees K} & \end{aligned}$$

The details are shown in the appendix entitled "Temperature fundamentals".

The neutron decay process has a mean decay time of 855 seconds (0.5 at 855 on the plot below).

The above calculations will be shown with the author's expansion and temperature history. The initial density is defined by  $1.67e-27$  kg divided by the volume associated with the gravitational radius of  $1.2e-14$  meters ( $2.3e14$  kg/m<sup>3</sup>). Expansion changes the volume according to the expansion model as shown in the following plot:



## Primordial helium fusion

Early work by Andrei Sakharov indicated that high initial temperatures would fuse elements from primordial nucleons and “freeze” their abundances at the observed levels during expansion and cooling. Current literature cites measurements indicating that 25% of primordial matter consists of helium. This process is cited in the literature [5][12][13] as evidence of a “big bang”. Radiation pressure prevents gravitational accumulation until radiation is attenuated by expansion, a condition known as equality. Current literature cites measurements indicating that 25% of primordial matter consists of Helium 4. Fusion of neutrons, protons and electrons occurs as the temperature achieves a kinetic energy of 0.1114 mev (about 5e8K). The difference between the neutron and proton mass in mev is 1.293 and the fraction reacted would be about 0.23 based on a very simplified Boltzmann type equilibrium.

Fraction decay approximately =  $\exp(-.1114/1.293)/4=0.23$

## **Solar Example**

In the following example, the proposed fusion model is applied to the sun. The core density in the sun is about 1.5e5 kg/m<sup>3</sup> [4][12] and the temperature is about 2.4e7 K.

For the sun, the calculations follow:

Solar example				
Temp	deg K	2.39E+07		
Density	kg/m <sup>3</sup>	1.22E+05	Dmax kg/m <sup>3</sup>	2.307E+14
KE temp	1.5*B*T	3.084E-03	mev	B=8.62e-11
degeneracy		3.56E+00	$(1.22e5/2699)^{.333}$	
Degenerate radius (D		1.485E-11	$(5.29e-11/3.56)^{.5}$ m	
v/C		0.109	$(1-(0.511/(0.511+3.08e-3))^2)^{0.5}$	
Barrier	mev	-0.0139	mev	
Example calculation for above conditions				
rate		Pbarrier	Pd=(dens/max)	Preaction rate R/sec
Probability/sec		$\exp(-.0139/.003)$	$(1.2e5/2.3e14)$	$v/r/\exp(56.74)$
56.74	2.92E-18	1.09E-02	5.29E-10	5.038E-07
sun N		1.20E+57		
fract burning		0.15		
burn rate N/sec*mev/		5.24E+38		
power mev/sec		3.50E+39		
burn time (Byrs)		10.9		

In the example above Probability/sec=Pbarrier\*Pdensity\*Preaction/sec=1.09e-2\*5.29e-10\*5.04e-7=2.9e-18/sec.

### Late stage fusion in stars

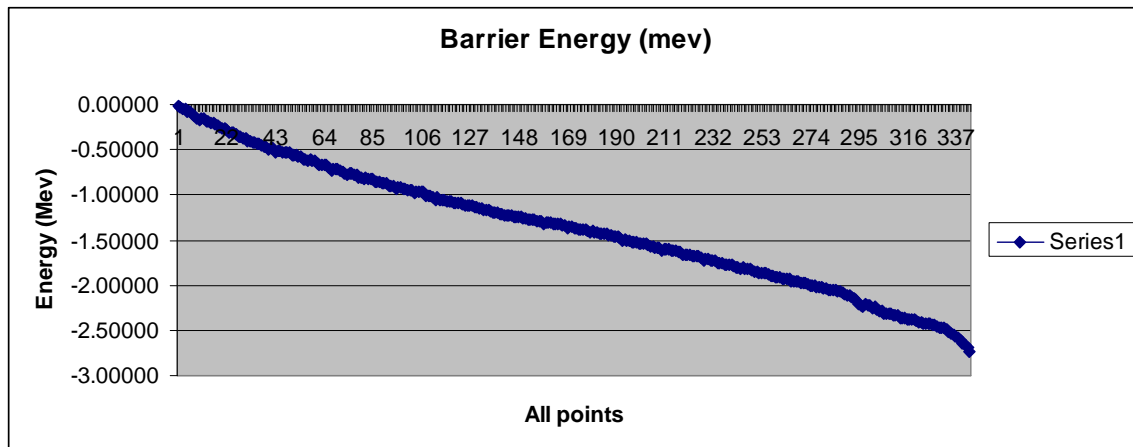
Following depletion of the hydrogen core of stars, helium, carbon, neon, oxygen, silicon and iron are currently believed to burn in successive stage of the star's life cycle [4][12][13]. Based on the proposed fusion kinetics model, very heavy elements can also be produced.

Mass accumulation results in a first generation of stars that light the skies at about 500 million years. As stars over a threshold mass age, processes are put into motion that burn hydrogen to helium, helium to carbon, carbon to neon, neon to neon, oxygen to silicon and silicon to iron. There are several sets of data regarding the temperature and density during the life cycle of stars (burns). One set [7] is given below:

Table 10-1. The major stages in the evolution

Burning Stage	Temperature (keV)	Density (kg/m <sup>3</sup> )	Time-sec
Hydrogen	5	5*10 <sup>6</sup>	7*10 <sup>6</sup> yr
Helium	20	7*10 <sup>8</sup>	5*10 <sup>5</sup> yr
Carbon	80	2*10 <sup>11</sup>	600 yr
Neon	150	4*10 <sup>12</sup>	1 yr
Oxygen	200	10 <sup>13</sup>	6 months
Silicon	350	3*10 <sup>13</sup>	1 day
Collapse	600	3*10 <sup>15</sup>	seconds
Bounce	3000	10 <sup>17</sup>	milliseconds
Explosive	100-600	varies	0.1-10 seconds

The author's binding energy model [2] results in barrier energy (BE) values for all of the elements [8]. The results are reproduced below:



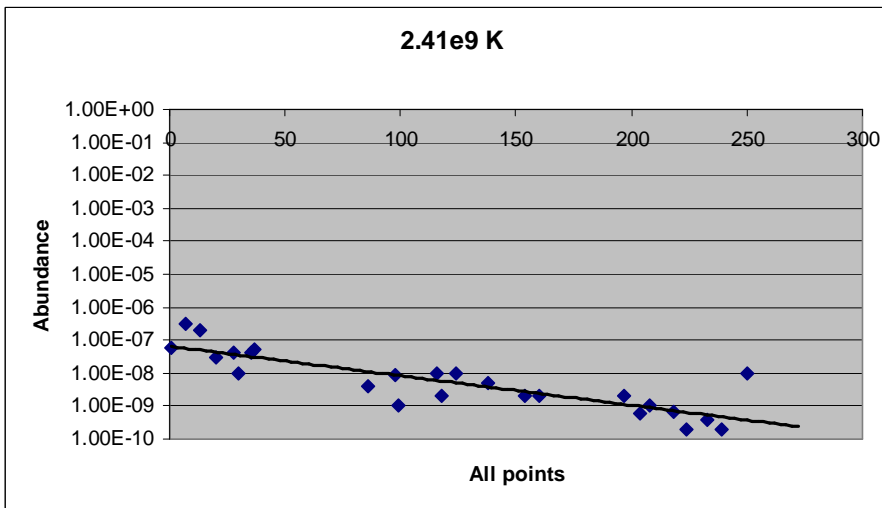
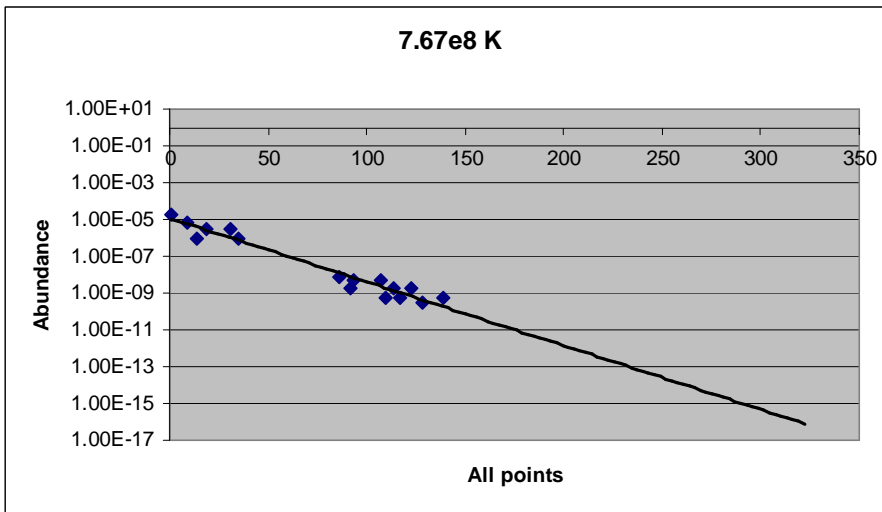
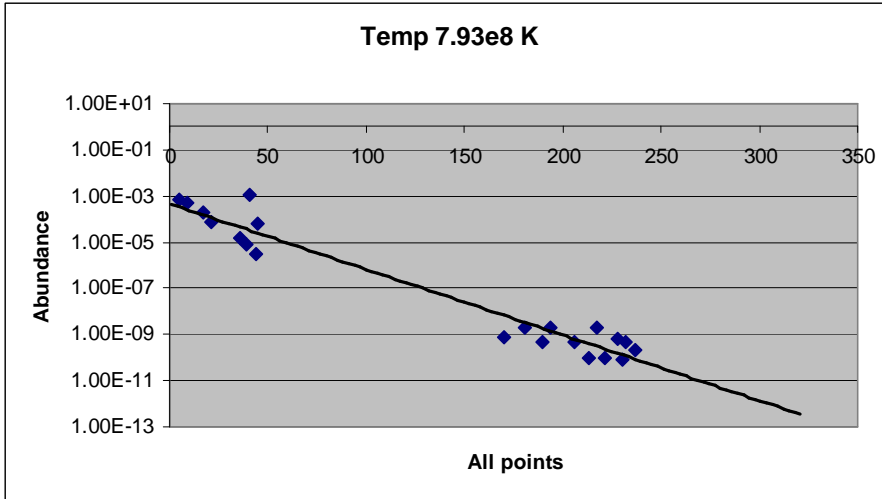
(Reminder: All points means all atomic numbers for elements and their isotopes)

Once this information is known, the temperature of the burn follows from the slope of the abundance data for a given burn. This is because the denominator in the following equation (mev) is determined by temperature alone.

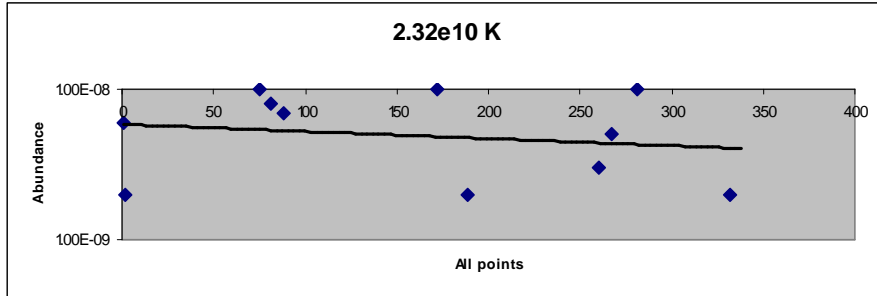
$$\text{Probability of fusing} = \exp(\text{BE}/\text{mev}).$$

$$\text{Mev} = 1.5 * 8.62e-11 * \text{Temp}$$

Abundance data was grouped by burn (this is somewhat trial and error but a pattern soon emerges) and plotted on a semi-log plot for all points (atomic numbers including isotopes). A statistical fit was determined (the line below) and the slope determined the temperature.







### ***Fusion kinetics during star evolution***

The example for solar fusion presented above was used for the remainder of the elements. It was crucial to know the Barrier Energy (BE) for each of the burns. They are presented below [2]:

	Barrier Energy
	ref 2 be
	mev
H2	-0.01392
He	-0.0541
C12	-0.1704
Ne	-0.2547
O	-0.2045
Si	-0.3589
Fe	-0.6757

Next, using the above barrier energies and the statistically determined temperatures, the burn times were calculated. Densities were determined by the following equation and density level (the value  $4e-16$ ) is the only unknown that determines the fit to the observed burn times.

$$\text{Density (kg/m}^3\text{)} = 4e-16 \cdot (T)^3$$

The fusion model gives the probability of reactions/second. Burn times are calculated from  $1/(\text{probability of reactions/sec})$ . This produces seconds for the burn that is converted to years. The following results were produced:

CF\$279*\$CD\$290 Comparison of reference (7) burn times with proposed fusion kinetics							
	protons-->H	He-->Carbo	C-->Ne	Ne-->Oxy	O-->Si	Si-->	Bounce (Fe)-->
Chap10 density	5.000E+06	7.000E+08	2.000E+11	4.00E+12	1.00E+13	3.00E+13	5.00E+15
Density (kg/m <sup>3</sup> )	1.219E+06	1.314E+08	6.930E+09	1.80E+11	1.99E+11	5.59E+12	4.997E+15
Chap10 kev	5	20	80	150	200	350	3000
Chap10 temp (t (mev)	3.87E+07	1.55E+08	6.19E+08	1.16E+09	1.55E+09	2.71E+09	2.32E+10
Temp K	0.0019	0.0089	0.0335	0.0991	0.1025	0.3115	3
V/C elect	1.45E+07	6.90E+07	2.59E+08	7.67E+08	7.93E+08	2.41E+09	2.32E+10
degeneracy	0.977	0.999	1.000	1.000	1.000	1.000	1.000
Barrier (mev)	7.7	36.5	136.9	405.8	419.4	1274.8	12277.6
degen R	-0.01392	-0.05400	-0.15430	-0.25500	-0.20500	-0.35800	-0.66400
react rate (P/se	6.90E-12	1.45E-12	3.86E-13	1.30E-13	1.26E-13	4.15E-14	4.31E-15
burn time (year	9.69E-06	4.71E-05	1.77E-04	5.25E-04	5.42E-04	1.65E-03	1.59E-02
	7.00E+09	5.00E+05	600.0	1.0	0.5	0.0027	3.17E-08

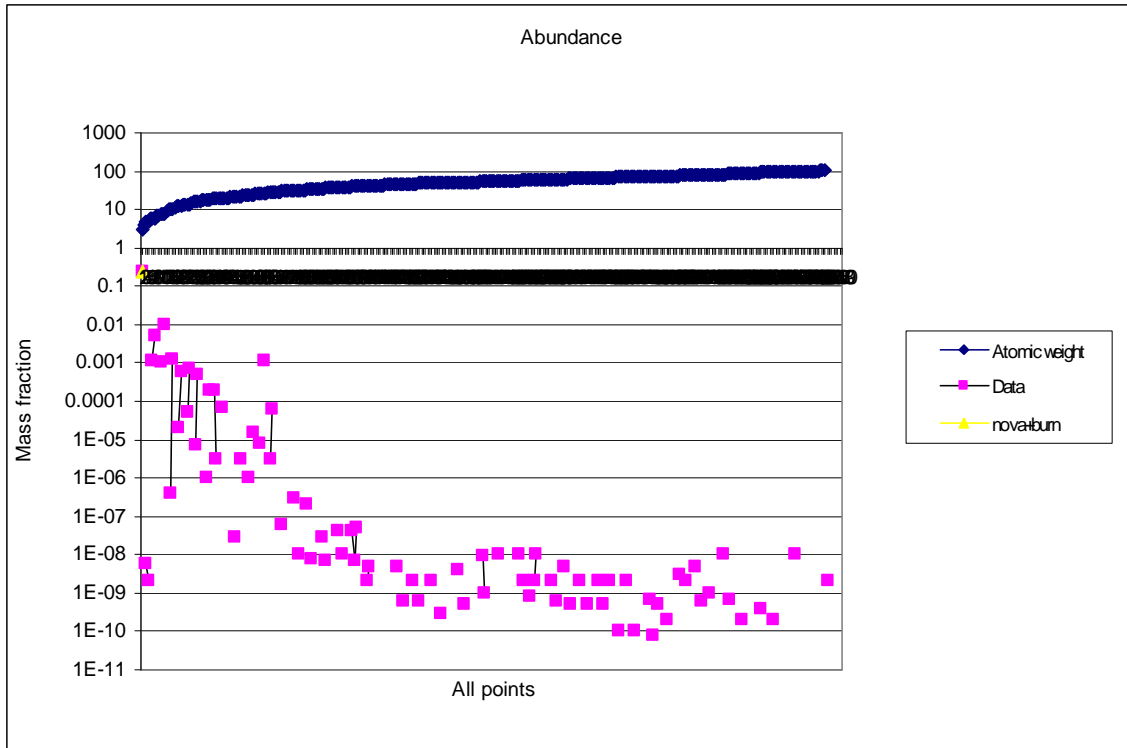
  

3.05E-17	He-->Carbon 6.31E-14 burn rate by fusion equation (P/sec) 5.02E+05 predicted burn (yrs) 5.00E+05 Chap10 burn (yrs)
1.04E+09	
7.00E+09	C-->Ne 5.28E-11 burn rate by fusion equation (P/sec) 600.56 predicted burn (yrs) 600.00 Chap10 burn (yrs)
	Ne-->Oxygen 3.13E-08 burn rate by fusion equation (P/sec) 1.01 predicted burn (yrs) 1 Chap10 burn (yrs)
	O-->Si 6.34E-08 burn rate by fusion equation (P/sec) 0.50 predicted burn (yrs) 0.5 Chap10 burn (yrs)
	Si--> 1.27E-05 burn rate by fusion (P/sec) 0.0025 predicted burn (yrs) 0.003 Chap10 burn (yrs)
reference (7) <a href="http://www.lbl.gov/abc/wallchart/chapters/10/0.html">http://www.lbl.gov/abc/wallchart/chapters/10/0.html</a>	Fe--> 2.756E-01 burn rate by fusion (P/ 1.1505E-07 predicted burn (yrs) 3.17E-08 Chap10 burn (yrs)

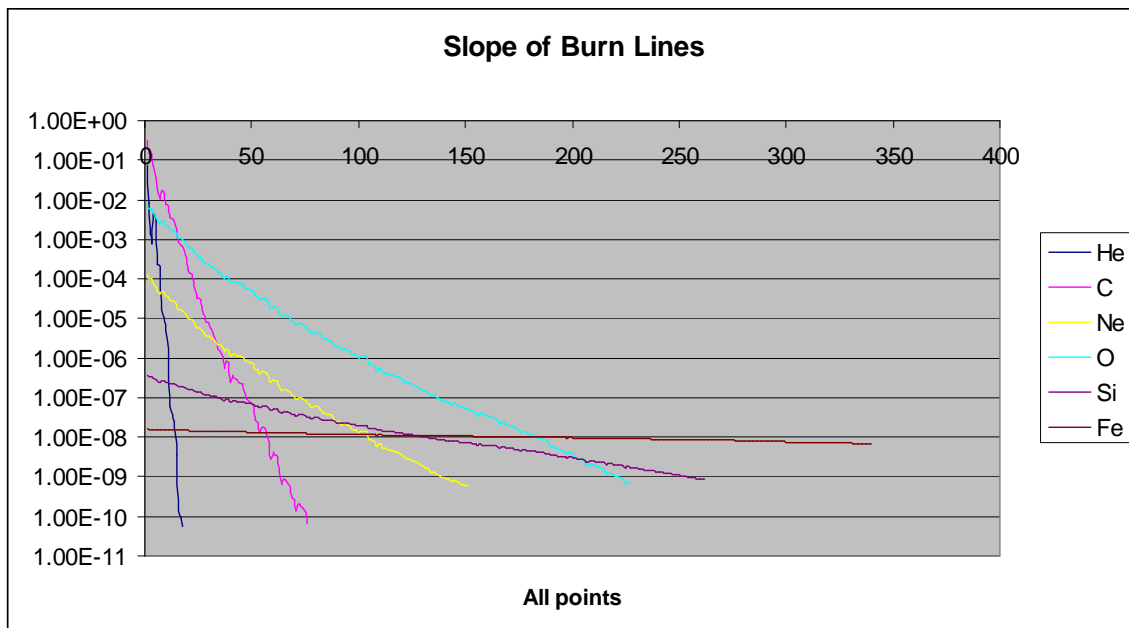
The temperatures and densities found by this analysis (in boxes above) compare favorably [5][7][11] but importantly, the temperatures and densities are used below to calculate abundance.

### Abundance of the elements

Data exists regarding the abundance of elements in the universe [11]. The raw data is presented below on the vertical axis of a semi-log plot. The x axis is all points (the atomic numbers and their isotopes).



Next, the burn lines were produced from the barrier energies of each of the elements [2], the temperatures determined above, the above densities and fusion kinetics.



The calculated burn lines can now be compared with abundance data.

Equation for abundance calculations:

$$\text{Abundance fraction} = P_{\text{barrier}} * P_{\text{reaction}} * P_{\text{density}} * \text{fraction} * \text{time}$$

Where:  $P_{\text{barrier}} = \exp(-E_b/kT)$  barrier energy is from reference 2

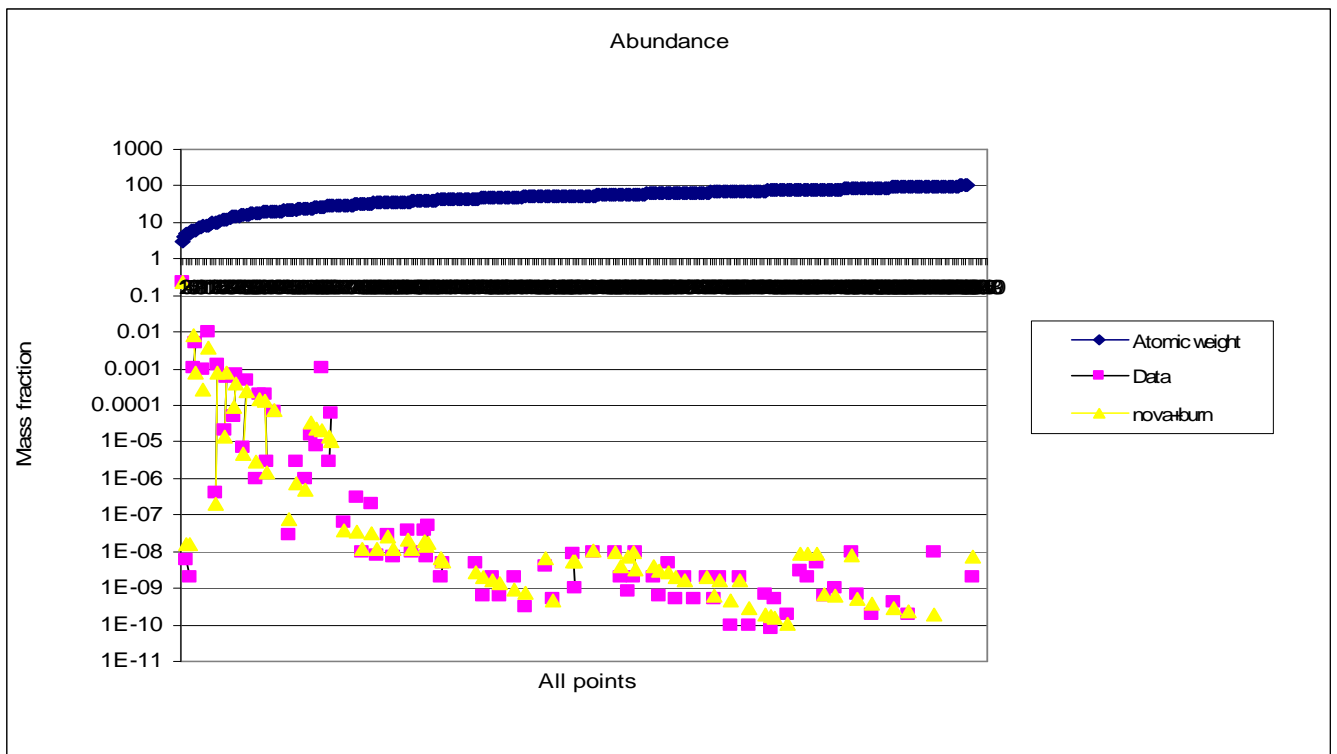
$$P_{\text{reaction}} = (v/C) * C / r_{\text{degenerate}} / \exp(56.74)$$

$$P_{\text{density}} = \text{density} / 2.3e14$$

Fraction (maximum available in core for that burn)

Time (burn time)

The ratios for each data set below compared with its temperature line are on the order of 1 or 2 standard deviations.



As indicated above, the model is semi-empirical since the vertical position of the line is also dependent on the fraction of the burning element that is subjected to high temperature. This will remain empirical until the history and types of supernovae are completely determined.

	sum abundance fraction
He burn	1.20E+04
C burn	6.00E+02
Ne burn	2.00E-04
O burn	2.00E-02
Si burn	1.00E-06

## Elements produced by each burn:

	3	5	17	21	25	9
	He-->Carb	C-->Ne	Ne-->Oxyg	O-->Si	Si-->	Fe-->
B		Ne	Ag	Ar	Au	As
C		Na	Cd	Ca	Ba	Br
N		Mg	Cl	Ce	Bi	Ga
		O	In	Co	Cu	No
		Al	K	Cr	Dy	Os
			Mo	Eu	Er	Pb
			Na	Fe	Ge	Pr
			Nb	Gd	Hg	Pt
			O	Ho	I	Xe
			P	Hf	Ir	Cm
			Pd	Lu	Kr	
			Rh	Mn	La	
			Ru	Ni	Nd	
			Sb	Pm	Rb	
			Ti	Re	Rn	
			V	S	Sc	
			Y	Si	Se	
				Tb	Sm	
				Ta	Sn	
				Tm	Sr	
				W	Te	
				Yb	Th	
					U	
					Zn	
					Zr	

## Summary

1. A fusion model was developed that relies on barrier energies from reference [2] to characterize fusion rates of stars, temperatures, densities and element abundances. The model determines which elements are formed from each of the burning phases and determines temperature from the slope of the abundance line. The model is dependent on observed burn times of supernovae and abundance data. It uses many fundamentals but, in the end, is semi-fundamental since the fraction of mass exposed to high temperature is highly complex.
2. Helium was produced in the beginning (Sakarov) but the source of initial temperature was the decay of initial neutrons. Abundance calculations show that an additional 2% of helium is produced during the hydrogen burn.

3. Supernovae are the source of heavy elements. This means that the sum of the measured abundances for elements heavier than helium is the fraction that was at some point in the core of stars.
4. The abundance of elements that burn in supernovae is the sum of the amount produced minus the burned amount. For example, Carbon was produced in copious amounts during the Helium burn but was consumed as C burned to produce Ne, Na, etc. This explains why the fraction involved in the C burn is greater than unity.
5. There is an interesting anomaly in the extremely high temperature, short lived Fe burn. Li and Be are produced in the quantities of about  $1e-9$ . These elements were all burned to higher atomic numbers early but apparently some protons became entrained in the Fe burn and produced small quantities of these elements.

## **Appendix**

### **Temperature fundamentals**

(A short review of known relationships is presented below as a reference.)

The relationship between the external kinetic energy of a particle and temperature is  $T = ke/B/1.5$ , where B is a conversion constant (Boltzmann's constant =  $8.62e-11$  meV/N/K). Interaction between particles is based on the electron resisting changes in its orbit when one particle hits another. When the electron settles into its orbit and the overall volume continues to expand, space develops between the gas molecules and hydrogen becomes diatomic. The interaction between particles becomes statistical and the ideal gas law applies. Pressure is now determined by the equation  $P = 287.05 \text{ nt/kg/K} \cdot \text{density} \cdot \text{temperature}$ , where density is  $\text{kg/m}^3$  and temperature is degrees K. The constant of proportionality (287.05) is called R and is a published value that changes slightly with temperature. Of course temperature can change independently and this changes the energy.

When the electron is forced to position itself at less than  $5.29e-11$  meters, the electron kinetic energy is degenerate. For high temperatures the electron kinetic energy from the electron quad (0.1114 meV) is scaled down by  $(\text{dens}/\text{dens}_0)^{.33}$ . If the hydrogen proton and electron were ideal and disassociated, the gas constant would be  $8.32 \text{ J/mol/K} \cdot 1000 \text{ mols/kg} = 8300 \text{ J/K/kg} = 8.6e-11 = \text{Boltzmann's constant}$ . The constant volume specific heat ideally would be  $5/3 \cdot 8300$  and the constant pressure specific heat should be about  $3/2 \cdot 8300 \text{ (J/K/kg)}$ . It is known that the equation of state for hydrogen plasma is quite complicated and that extra heat is retained [10]. The specific heat [10] gives a value of  $2.3e-13$  for a much lower temperature of 6000K and it is known that specific heat increases with temperature).

The degenerate condition ends when electrons are just packed so their orbits are in contact. For this condition the density is  $2697 \text{ kg/m}^3$ . Our atmosphere is at a much

lower density ( $1.27 \text{ kg/m}^3$ ) and the electrons are separated. We observe a temperature that is consistent with the kinetic energy divided by mass and  $c_p$  (specific heat).

The temperature associated with this kinetic energy state is about  $8.6e8 \text{ K}$ .

simple.xls cell ae175			
Compression Thermodyna		8300.00	8300.00
		0.11143395	→
		base	Expanded
degeneracy	(dens/dens0)^	4407.01807	4407.01807
Radius	compressed	1.2006E-14	1.2006E-14
Volume V	M <sup>3</sup>	7.249E-42	7.249E-42
dens-	kg/M <sup>3</sup>	2.307E+14	2.307E+14
KE electron	(r E0*r/r=E0(d/d	1.114E-01	1.114E-01
pressure=R	rh R=287.05	1.652E+27	1.651E+27
P*V	mev	7.473E-02	7.470E-02
Ke at max burn			9.24E-04
Ke =1.5BT		1.115E-01	1.115E-01
Temperature=	ke elect/cp/9	8.625E+08	8.620E+08
cp (mev/mev/K)		1.38E-13	1.38E-13

## References:

1. Barbee, Gene H., *A Top-Down Approach to Fundamental Interactions*, FQXi essay, June 2012 and vixra:1307.0082.
2. Barbee, Gene H., *A Fundamental Model of Atomic Binding Energy*, vixra:1307.0102, accompanying Microsoft® spreadsheet atom.xls.
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