Connection between Extenics and Refined Neutrosophic Logic

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Contents

- **Two-Valued Logic**
- **Three-Valued Logic & Extenics**
- **Four-Valued Logic & Extenics**
- **Five-Valued Logic & Extenics**
- **Seven-Valued Logic & Extenics**
- **n-Valued Logic & Extenics**
- **Neutrosophic Cube and its Extenics Part**
- **Example of Extenics in 3-Valued Neutrosophic Logic**
- **Example of Extenics in 4-Valued Neutrosophic Logic**

Abstract

The aim of this presentation is to connect Extenics with new fields of research, i.e. fuzzy logic and neutrosophic logic.

We show herein:

- How Extenics is connected to the 3-Valued Neutrosophic Logic;
- How Extenics is connected to the 4-Valued Neutrosophic Logic;
- How Extenics is connected to the n-Valued Neutrosophic Logic;

When contradiction occurs.

Two-Valued Logic

1. The Two Symbol-Valued Logic is the classical or *Boolean Logic*, which has two values: true and false (or 1 and 0).

In Chinese philosophy: *Yin and Yang* (or Femininity and Masculinity) as contraries:

2. The Two Numerical-Valued Logic is the *Fuzzy Logic*, where the truth (t) and the falsity (f) can be any numbers in [0,1] such that t + f=1.

More general, t and f can be subsets of [0,1].

Three-Valued Logic & EXtenics

1. The <u>Three Symbol-Valued Logics:</u>

a) *Łukasiewicz 's Logic*: True, False, and Possible.

b) *Kleene's Logic*: True, False, Unknown (or Undefined), or with numbers 1, 0, and ½.

Use "min" for /\, "max" for \/, and "1-" for negation.

2. The <u>Three Numerical-Valued Logic </u>is the *Neutrosophic Logic* [Smarandache, 1995], where the truth (t) and the falsity (f) and the indeterminacy (i) can be any numbers in [0, 1], then $0 \le t + I + f \le 3$.

More general: Truth (T), Falsity (F), and Indeterminacy (I) are standard or nonstandard subsets of the nonstandard interval]-0, 1+[.

When $t + f > 1$ we have conflict, hence Extenics.

Three-Valued Logic & Extenics (2)

- Chinese philosophy extended to: *Yin, Yang, and Neuter* (or Femininity, Masculinity, and Neutrality).
- Neutrosophy philosophy was born from neutrality between various philosophies. Connected with Extenics (Prof. Cai Wen, 1983), and Paradoxism (F. Smarandache, 1980).

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>).

The <neutA> and <antiA> ideas together are referred to as <nonA>.

Three-Valued Logic & Extenics (3)

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well. Such contradictions involves Extenics.

Neutrosophy is the base of all neutrosophics and it is used in engineering applications (especially for software and information fusion), medicine, military, airspace, cybernetics, physics.

Four-Valued Logic & Extenics

1. The <u>Four Symbol-Valued Logic</u>

Belnap's Logic: True (T), False (F), Unknown (U), and Contradiction (C), where T, F, U, C are symbols, not numbers.

Now we have **EXTENICS**, thanks to C = contradiction.

Below is Belnap's conjunction operator table:

$\cap$	F	U	C	T
F	F	F	F	F
U	F	U	F	U
C	F	F	C	C
T	F	U	C	T

Restricted to T,F,U and to T,F,C, Belnap connectives coincide with connectives in Kleene's logic.

Four-Valued Logic & Extenics (2)

2. The Four Numerical-Valued Logic

- *Four Numerical-Valued Neutrosophic Logic:* Indeterminacy I is refined (split) as U = Unknown, and C = contradiction.
- T, F, U, C are subsets of [0, 1];
- Generalizes Belnap's logic since one gets degree of truth, degree of falsity, degree of unknown, and degree of contradiction.
- C = T/\F involves the Extenics.

Five-Valued Logic & Extenics

1. Five Symbol-Valued Neutrosophic Logic [Smarandache, 1995]: Indeterminacy I is refined (split) as U = Unknown, C = contradiction, and G = ignorance; where the symbols represent:

T = truth;

F = falsity;

U = neither T nor F (undefined);

C = $T \wedge F$, which involves the **Extenics**;

G = $T \vee F$.

2. If T, F, U, C, G are subsets of [0, 1] then we get: *Five Numerical-Valued Neutrosophic Logic*.

Seven-Valued Logic & Extenics

1. Seven Symbol-Valued Neutrosophic Logic [Smarandache, 1995]: I is refined (split) as U, C, G, but T also is refined as T_1 = absolute thuth and T_2 = relative truth, and F is refined as F_1 = absolute falsity and F_2 = relative falsity. Where:

U = neither T_j nor F_k (undefined);

C = $T_j \wedge F_k$, which involves the **Extenics**;

G = $T_j \vee F_k$. All are symbols.

2. But if T_1, T_2, F_1, F_2, U, C, G are subsets of [0, 1] then we get: *Five Numerical-Valued Neutrosophic Logic*.

n-Valued Logic & Extenics

1. The <u>n-Symbol-Valued Logic</u> is

The Refined n-Symbol-Valued Neutrosophic Logic [Smarandache, 1995].

In general:

T can be split into types of truths: T_1, T_2, ..., T_p, and I into types of indeterminacies: I_1, I_2, ..., I_r, and F into types of falsities: F_1, F_2, ..., F_s,

where p + r + s = n.

All subcomponents T_k, I_j, F_l are symbols.

If at least one $I_j = T_k \wedge F_l$ = contradiction, we get again the Extenics.

n-Valued Logic & Extenics (2)

2. The <u>*n*-Numerical-Valued Logic</u> is

The Refined n-Numerical-Valued Neutrosophic Logic. In the same way:

T is be split into types of truths: T_1, T_2, ..., T_p,

and I into types of indeterminacies: I_1, I_2, ..., I_r,

and F into types of falsities: F_1, F_2, ..., F_s,

where p + r + s = n.

But all subcomponents T_k, I_j, F_l are not symbols, but subsets of [0,1], for all k in {1,2,…,p}, all j in {1,2,…,r}, and all l in {1,2,…,s}.

For at least one $I_j = T_k \wedge F_l$ = contradiction,

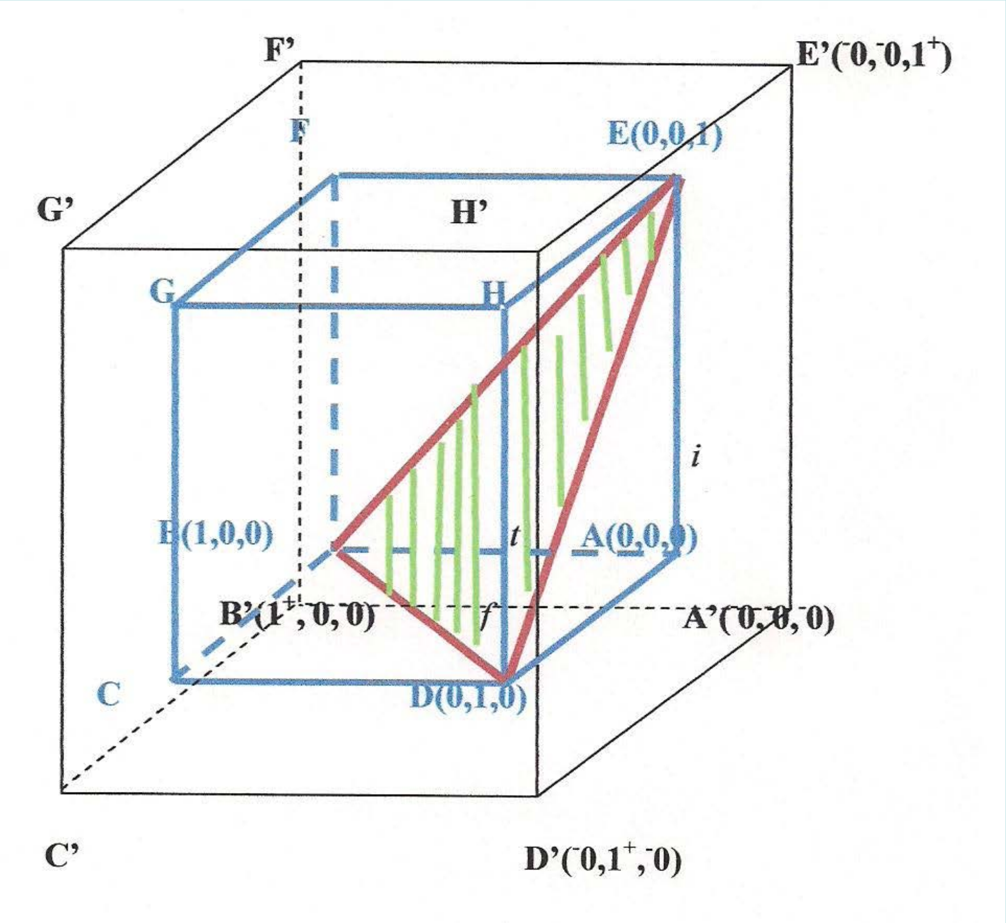
we get again the Extenics

Neutrosophic Cube and its Extenics Part

- The most important distinction between IFS and NS is showed in the below **Neutrosophic Cube** A’B’C’D’E’F’G’H’ introduced by J. Dezert in 2002.
- Because in technical applications only the classical interval is used as range for the neutrosophic parameters , we call the cube the **technical neutrosophic cube** and its extension the **neutrosophic cube** (or **absolute neutrosophic cube**), used in the fields where we need to differentiate between *absolute* and *relative* (as in philosophy) notions.

Neutrosophic Cube and its Extenics Part (2)

Extenics-->



Neutrosophic Cube and its Extenics Part (3)

Let's consider a 3D-Cartesian system of coordinates, where t is the truth axis with value range in $]^{-}0,1^{+}[$, i is the false axis with value range in $]^{-}0,1^{+}[$, and similarly f is the indeterminate axis with value range in $]^{-}0,1^{+}[$.

We now divide the technical neutrosophic cube ABCDEFGH into three disjoint regions:

1) The equilateral triangle BDE, whose sides are equal to $\sqrt{(2)}$ which represents the geometrical locus of the points whose sum of the coordinates is 1.

If a point Q is situated on the sides of the triangle BDE or inside of it, then $t_Q+i_Q+f_Q=1$ as in Atanassov-intuitionistic fuzzy set (A-IFS).

Neutrosophic Cube and its Extenics Part (4)

- 2) The pyramid EABD {situated in the right side of the triangle EBD, including its faces triangle ABD(base), triangle EBA, and triangle EDA (lateral faces), but excluding its face: triangle BDE } is the locus of the points whose sum of coordinates is less than 1 (Incomplete Logic).
- 3) In the left side of triangle BDE in the cube there is the solid EFGCDEBD (excluding triangle BDE) which is the locus of points whose sum of their coordinates is greater than 1 as in the paraconsistent logic. This is the **Extenics** part.

Neutrosophic Cube and its Extenics Part (4)

- It is possible to get the **sum of coordinates strictly less than 1** (in Incomplete information), or **strictly greater than 1 (in contradictory** Extenics**)**. For example:
- We have a source which is capable to find only the degree of membership of an element; but it is unable to find the degree of non-membership;
- Another source which is capable to find only the degree of non-membership of an element;
- Or a source which only computes the indeterminacy.
- Thus, when we put the results together of these sources, it is possible that their sum is not 1, but smaller (Incomplete) or greater (Extenics).

Example of Extenics in 3-Valued Neutrosophic Logic

- About a proposition P, the first source of information provides the truth-value T=0.8.
- Second source of information provides the false-value F=0.7.
- Third source of information provides the indeterminacy-value I=0.2.
- Hence $NL_3(P) = (0.8, 0.2, 0.7)$
- Got Extenics, since Contradiction: T+F = 0.8+0.7>1
- Can remove Contradiction by normalization:

NL(P) = (0.47, 0.12, 0.41); now T+F ≤ 1.

Example of Extenics in 4-Valued Neutrosophic Logic

- About a proposition Q, the first source of information provides the truth-value T=0.4, second source provides the false-value F=0.3, third source provides the undefined-value U=0.1, fourth source provides the contradiction-value C=0.2.
- Hence $NL_4(Q) = (0.4, 0.1, 0.2, 0.3)$.
- Got Extenics, since Contradiction $C = 0.2 > 0$.
- Since $C = T \wedge F$, we can remove it by transferring its value 0.2 to T and F (since T and F were involved in the conflict) proportionally w.r.t. their values 0. 4,0.3:

$x_T/0.4 = y_F/0.3 = 0.2/(0.4+0.3)$, whence $x_T=0.11$, $y_F=0.09$.

Thus T=0.4+0.11=0.51, F=0.3+0.09=0.39, U=0.1, C=0.

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