PRECISE FORMULATION AND PROOF OF DIRAC'S LARGE NUMBERS HYPOTHESIS

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Abstract

Three mass dimension quantities have been derived by dimensional analysis by means of fundamental parameters – the speed of light in vacuum (c), the gravitational constant (G), the Planck constant ($\hbar$) and the Hubble constant (H). The extremely small mass $m_1 \sim \hbar H/c^2 \sim 10^{-33}\,\text{eV}$ has been identified with the Hubble mass m_H, which seems close to the graviton mass m_G. The enormous mass $m_2 \sim c^3/(GH) \sim 10^{53}\,\text{kg}$ is close to the mass of the Hubble sphere and practically coincides with the Hoyle-Carvalho formula for the mass of the observable universe. The third mass $m_3 \sim \sqrt[5]{H\hbar^3/G^2} \sim 10^7\,\text{GeV}$ could not be unambiguously identified at present time.

By two steps of approximation, it has been found that the ratio of the Hubble sphere mass M_H and the Planck mass m_{Pl} is equal to the substantial large number N definite from the equation $N = \sqrt{c^5/(2G\hbar H^2)} \approx 5.73 \times 10^{60}$. Besides, the large number N defines the ratio of Planck mass m_{Pl} and the Hubble mass m_H, the ratio of the Hubble distance cH^{-1} and the Planck length l_{Pl}, the ratio of Hubble time (age of the universe) H^{-1} and the Planck time t_{Pl} and the square root of the ratio of the Planck density ρ_{Pl} and the total density of the universe $\overline{\rho}$. Therefore, the substantial large number N relates cosmological parameters (mass, density, age and size of the observable universe) and fundamental microscopic properties of the matter (Planck units and Hubble mass). Thus, a precise formulation and proof of Dirac's Large Numbers Hypothesis (LNH) has been found and a new fundamental physical law has been established connecting micro and mega world.

PACS numbers: 06.20.fa, 06.30.Dr, 14.80. -j, 98.80.Es

Keywords: Dirac's large numbers hypothesis, Planck's units, dimensional analysis, mass of the observable universe, Hubble mass

1. INTRODUCTION

The Planck mass $m_{Pl} \sim \sqrt{\hbar c/G}$ has been introduced in [1] by means of three fundamental constants – the speed of light in vacuum (c), the gravitational constant (G) and the reduced Planck constant ($\hbar$). Since the constants c, G and $\hbar$ represent three very basic aspects of the universe (i.e. the relativistic, gravitational and quantum phenomena), the Planck mass appears to a certain degree a unification of these phenomena. The Planck mass have many important aspects in the modern physics. One of them is that the energy equivalent of Planck mass $E_{Pl} = m_{Pl}c^2 \sim \sqrt{\hbar c^5/G} \sim 10^{19}\,\mathrm{GeV}$ appears unification energy of four fundamental interactions [2]. Also, the Planck mass can be derived by setting it as the mass whose Compton wavelength and gravitational radius are equal [3]. Analogously, formulae for Planck length $l_{Pl} \sim 10^{-35}\,\mathrm{m}$, Planck time $t_{Pl} = l_{Pl}/c$ and Planck density $\rho_{Pl} \sim 10^{96}\ \mathrm{kg\,/\,m^3}$ have derived by dimensional analysis. In quantum gravity models, the Planck length is the length scale at which the structure of spacetime becomes dominated by quantum effects.

The Planck mass formula has been derived by dimensional analysis using fundamental constants c, G and $\hbar$. The dimensional analysis is a conceptual tool often applied in physics to understand physical situations involving certain physical quantities [4–8]. It is routinely used to check the plausibility of the derived equations and computations. When it is known, the certain quantity with which other determinative quantities would be connected, but the form of this connection is unknown, a dimensional equation is composed for its finding. In the left side of the equation, the unit of this quantity q_0 with its dimensional exponent has been placed. In the right side of the equation, the product of units of the determinative quantities q_i rise to the unknown exponents n_i has been placed $[q_0] \sim \prod_{i=1}^{n} [q_i]^{n_i}$, where n is positive integer and the exponents n_i are rational numbers. Most often the dimensional analysis has applied in the mechanics and other fields of the modern physics where there are many problems having a few determinative quantities.Many interesting and important problems related to the fundamental constants have been considered in [9–13].

The discovery of the linear relationship between recessional velocity of distant galaxies, and distance $v = Hr$ [14] introduces new fundamental constant in physics and cosmology – the famous Hubble constant (parameter) H. Even seven years before, Friedman [15] derived his equations from the Einstein field equations [16], showing that the universe might expand at a rate calculable by the equations. The Hubble parameter determines the age of the universe H^{-1}, the Hubble distance cH^{-1}, the critical density of the universe $\rho_c = 3H^2/(8\pi G) \sim 10^{-26}\,\mathrm{kg}\,/\,\mathrm{m}^3$ [17], and other large-scale properties of the universe.

Because of the importance of the Hubble constant, in the present paper we include H in dimensional analysis together with c, G and $\hbar$ aiming to find the new mass dimension quantities $m_i \sim \prod_{j=1}^{3} q_j^{n_j}$, where every triad $q_1,\ q_2,\ q_3$ consists of three constants c, G, $\hbar$ or H. Thus, the Hubble parameter will represent cosmological phenomena in new derived fundamental masses. The attempt to compose a mass dimension quantity by means of the four constants together produces an undetermined system of linear equations and it has been neglected. According to the recent cosmology, the Hubble "constant" slowly decreases with the age of the universe, but there are indications that other constants, especially gravitational and fine structure constants also vary with comparable rate [18–20]. That is why, the Hubble parameter could deserve being treated on an equal level with the other three constants used from Planck.

Dirac [18] suggested the Large Numbers Hypothesis (*LNH*) pointing out that the ratio of the age of the universe H^{-1} and the atomic unit of time $\tau = e^2/(m_e c^3) \sim 10^{-23}\,\mathrm{s}$ is a large number of the order of 10^{40}. Besides, the ratio of electrostatic e^2/r^2 and gravitational forces $Gm_e m_p/r^2$ between proton and electron in a hydrogen atom is of the order of 10^{39} and the ratio of mass of the observable universe M and nucleon mass roughly is of the order of 10^{80}:

$$\frac{H^{-1}}{\tau} \sim \frac{e^2}{Gm_e m_p} \sim \sqrt{\frac{M}{m_p}} \sim N_D \tag{1}$$

where e is the charge of the electron, m_e is the electron mass, m_p is the proton mass and $N_D \sim 10^{40}$ is the Dirac's large number.

Relying on the ratios (1), he proposed that as a consequence of causal connections between macro and micro physical world, gravitational constant G slowly decreases with time whereas mass of the universe increases in result of slow creation of matter. Although the *LNH* is inconsistent with General Relativity, the former has inspired and continues to inspire a significant body of scientific literature.

Many other interesting ratios have been found approximately relating some cosmological parameters and microscopic properties of the matter. For example, Narlikar [21] shows that the ratio of radius of the observable universe and classical radius of the electron $e^2/(m_e c^2)$ is of the order of 10^{40}. Besides, the ratio of the electron mass and Hubble (mass) parameter

$\hbar H/c^2$ approximates to 10^{39} [22]. Jordan [23] noted that the mass ratio for a typical star and an electron is of the order of 10^{60}. The ratio of mass of the observable universe and Planck mass is of the order of 10^{61} [24]. Peacock [25] points out that the ratio of Hubble distance and Planck length is of the order of 10^{60}. Finally, the ratio of Planck density ρ_{Pl} and recent critical density of the universe ρ_c is found to be of the order of 10^{121} [26]. Most of these large numbers are rough ratios of astrophysical parameters and microscopic properties of the matter determined with accuracy of the order of magnitude.

2. DERIVATION OF THREE FUNDAMENTAL MASSES BY DIMENSIONAL ANALYSIS

A quantity m_1 having mass dimension could be composed by means of the fundamental constants c, $\hbar$ and H:

$$m_1 = k c^{n_1} \hbar^{n_2} H^{n_3} \tag{2}$$

where n_1, n_2 and n_3 are unknown exponents to be determined by matching the dimensions of both sides of the equation, and k is dimensionless parameter of an order of magnitude of a unit.

As a result we find the system of linear equations:

$$\begin{aligned} n_1 + 2n_2 &= 0 \\ -n_1 - n_2 - n_3 &= 0 \\ n_2 &= 1 \end{aligned} \tag{3}$$

The unique solution of the system is $n_1 = -2,\ n_2 = 1, n_3 = 1$. Replacing obtained values of the exponents in equation (2) we find formula (4) for the mass :

$$m_H = m_1 \sim \frac{\hbar H}{c^2} \tag{4}$$

The recent experimental values of c, $\hbar$ and H are used: $c = 299792458$ m / s, $\hbar = 1.054571596 \times 10^{-34}\,\mathrm{J\,s}$ [27] and $H \approx 70$ km / s Mps [28]. Replacing these values in (4) we obtain $m_1 \sim 2.70 \times 10^{-69}\,\mathrm{kg} = 1.52 \times 10^{-33}\,\mathrm{eV}$. This exceptionally small mass coincides with the so called "Hubble mass" $m_H = \hbar H/c^2$ [29, 30], which seems close to the graviton mass m_G obtained by different methods [31–34]. Evidently, the mass $m_1 = m_H \sim m_G$ is in several orders of magnitude smaller than the upper limit of the graviton mass, obtained by astrophysical constraints [35].

From equation (4) we find that the reduced Compton wavelength λ_H of the Hubble mass is equal to the Hubble distance cH^{-1}:

$$\lambda_H = \frac{\hbar}{m_H c} = cH^{-1} \sim 1.3 \times 10^{26}\,\mathrm{m} \tag{5}$$

The presence of an exceptionally small, yet nonzero mass of the graviton (gauge boson of gravity), involves Yukawa potential of the gravitational field $\phi(r) = -\frac{GM}{r}\exp(-r/\lambda_H)$ and a finite range of the gravity close to the Hubble distance cH^{-1}. Therefore the last gives the size of gravitationally connected (observed) universe for an arbitrary observer.

Analogously, by means of the fundamental constants c, G and H, a quantity m_2 having dimension of a mass could be composed:

$$m_2 = kc^{n_1}G^{n_2}H^{n_3} \tag{6}$$

where n_1, n_2 and n_3 are unknown exponents to be determined by matching the dimensions of both sides of the equation, and k is dimensionless parameter of an order of magnitude of a unit.

We determine the exponents $n_1 = 3,\ n_2 = -1,\ n_3 = -1$ by the dimensional analysis again. Replacing the obtained values of the exponents in equation (6) we find formula (7) for the mass m_2:

$$M \sim m_2 \sim \frac{c^3}{GH} \tag{7}$$

First of all, the formula (7) has been derived by dimensional analysis in [36]. This formula practically coincides with Hoyle formula for the mass of the observable universe $M = c^3/(2GH)$ [37] and perfectly coincides with Carvalho formula [38] for the mass of the observable universe, obtained by totally different approach.

The Hubble sphere is the sphere where the recessional velocity of the galaxies is equal to the speed of the light in vacuum c, and according to the Hubble law $v = c$ when $r = cH^{-1}$. Besides, the Hubble sphere coincides with gravitationally connected universe for an arbitrary observer. Thus, the Hubble sphere appears a three-dimensional sphere, centered on the observer, having radius $r = cH^{-1}$ and density $\overline{\rho} \approx \rho_c$. Evidently, the formula (7) is close to the mass of the Hubble sphere M_H:

$$m_2 \approx M_H = \frac{4}{3}\pi \frac{c^3}{H^3}\frac{3H^2}{8\pi G} = \frac{c^3}{2GH} \tag{8}$$

Replacing the recent values of the constants c, G and H in (7) we obtain $m_2 \sim 1.76 \times 10^{53}$ kg. Therefore, the enormous mass m_2 would be identified with the mass of the observable universe M.

From formulae (4) and (7) we find the important relation (9):

$$\sqrt{m_1 m_2} = \sqrt{m_H M} = \sqrt{\frac{\hbar c}{G}} = m_{Pl} \tag{9}$$

Therefore, the Planck mass appears geometric mean of the Hubble mass and the mass of the observable universe. As the physical quantity mass is among the most important properties of the matter, the formula (9) hints at a deep relation of the micro particles and the entire universe.

The third quantity m_3 having dimension of a mass could be constructed by means of the fundamental constants G, $\hbar$ and H:

$$m_3 = kG^{n_1}\hbar^{n_2}H^{n_3} \tag{10}$$

We determine the exponents $n_1 = -\frac{2}{5}$, $n_2 = \frac{3}{5}$, $m_3 = \frac{1}{5}$ by dimensional analysis again. Replacing the obtained values of the exponents in formula (10) we find formula (11) for the mass m_3:

$$m_3 \sim \sqrt[5]{\frac{H\hbar^3}{G^2}} \tag{11}$$

Replacing the recent values of the constants G, $\hbar$ and H, the mass m_3 takes value $m_3 \sim 1.43 \times 10^{-20}$ kg $\approx 8.0 \times 10^6$ GeV. This mass is a dozen of orders of magnitude lighter than the Planck mass and several orders of magnitude heavier than the heaviest known particles like the top quark $m_t \approx 174.3$ GeV [39]. On the other hand, the energy $m_3 c^2 \sim 8 \times 10^6$ GeV appears medial for the important GUT scale $E_{GUT} \sim 10^{16}$ GeV and electroweak scale $E_{EW} \sim 10^2$ GeV. Therefore, the mass/energy m_3 could not be unambiguously identified at the present time, and it could be considered as heuristic prediction of the suggested approach concerning unknown very heavy particle or fundamental energy scale. In the first time these three masses have been derived in [40].

Finally, I shall demonstrate the heuristic power of the suggested approach approximately estimating the total density of the universe by dimensional analysis. Actually, a quantity ρ having dimension of a density could be constructed by means of the fundamental constants c, G and H:

$$\rho = kc^{n_1}G^{n_2}H^{n_3} \tag{12}$$

where k is a dimensionless parameter of the order of magnitude of a unit.

By the dimensional analysis, we have found the exponents $n_1 = 0, \; n_2 = -1, \; n_3 = 2$. Therefore:

$$\rho \sim \frac{H^2}{G} \approx 7.9 \times 10^{-26} \;\; \mathrm{kg} \,/\, \mathrm{m}^3 \tag{13}$$

The recent Cosmic Microwave Background (CMB) observations show that the total density of the universe $\overline{\rho}$ is [41–43]:

$$\overline{\rho} = \Omega \rho_c \approx \rho_c = \frac{3H^2}{8\pi G} \sim 10^{-26} \;\; \mathrm{kg} \,/\, \mathrm{m}^3 \tag{14}$$

Evidently, the density ρ derived by means of the fundamental constants c, G and H coincides with formula (14) for the total density of the universe with an accuracy of a dimensionless parameter of an order of magnitude of a unit. Besides, the formula (13) could be derived by means of other triad of fundamental constants, namely G, $\hbar$ and H.

3. PRECISE FORMULATION AND PROOF OF DIRAC'S LARGE NUMBERS HYPOTHESIS

3.1. Approximate estimation of the large number N relating cosmological parameters and Planck units by means of dimensional analysis

The Planck mass $m_{Pl} \sim \sqrt{\hbar c/G}$ and formulae (4) and (7) for the Hubble mass and mass of the observable universe have been derived by dimensional analysis by means of the fundamental constants c, G, $\hbar$ and H. The Planck density $\rho_{Pl} \sim c^5/(\hbar G^2) \approx 5.2 \times$

$10^{96}\,\mathrm{kg}\,/\,\mathrm{m}^3$, the Planck length $l_{Pl} \sim \sqrt{G\hbar/c^3} \approx 1.61 \times 10^{-35}\,\mathrm{m}$, the Planck time $t_{Pl} = l_{Pl}/c \sim \sqrt{G\hbar/c^5} \approx 5.4 \times 10^{-44}\,\mathrm{s}$ and the approximate formula (13) for the total density of the universe ρ also are obtained by dimensional analysis. Taking into account above mentioned formulae and Hubble distance cH^{-1} and age of the universe H^{-1} we find remarkable ratios (15):

$$\sqrt{\frac{M}{m_H}} = \frac{M}{m_{Pl}} = \frac{m_{Pl}}{m_H} = \frac{cH^{-1}}{l_{Pl}} = \frac{H^{-1}}{t_{Pl}} = \sqrt{\frac{\rho_{Pl}}{\overline{\rho}}} = \sqrt{\frac{c^5}{G\hbar H^2}} = \widetilde{N} \approx 8.1 \times 10^{60} \qquad (15)$$

Therefore, the ratio of the mass of the observable universe M and the Planck mass m_{Pl} is equal to the large number $\widetilde{N}$ definite from the equation $\widetilde{N} = \sqrt{c^5/(G\hbar H^2)} \approx 8.1 \times 10^{60}$. Besides, the large number $\widetilde{N}$ defines the ratio of Planck mass m_{Pl} and the Hubble mass m_H, the ratio of the Hubble distance cH^{-1} and the Planck length l_{Pl} , the ratio of Hubble time (age of the universe) H^{-1} and the Planck time t_{Pl} and the square root of the ratio of the Planck density ρ_{Pl} and the approximate density of the universe ρ determined from (13). Evidently, these ratios appear very important because they relate cosmological parameters (mass, density, age and size of the observable universe) and the fundamental microscopic properties of the matter (Planck mass, Planck density, Planck time, Planck length and Hubble mass). In recent quantum gravity models, the Planck units imply quantization of spacetime at extremely short range. Thus, the ratios (15) represent connection between cosmological parameters and quantum properties of spacetime. Obviously, the ratios (15) appear a formulation of *LNH*.

3.2. Precise determination of the large number N and proof of Dirac's *LNH* by fine tuning of Planck units

As it has been mention in Section 1, the dimensional analysis allows to find unknown quantity with accuracy to dimensionless parameter k of the order of magnitude of unit. Below, we shall recalculate the ratios (15) using exact values of the respective quantities. The *exact* value of Planck mass could be found from definition of the Planck mass as the mass m, whose Compton wavelength λ and gravitational (Schwarzschild) radius r_S are equal:

$$\lambda = \frac{\hbar}{mc} = r_S = \frac{2Gm}{c^2} \qquad (16)$$

Thus, from (16) we find the exact value of Planck mass:

$$m_{Pl} = \sqrt{\frac{\hbar c}{2G}} \approx 1.54 \times 10^{-8}\,\mathrm{kg} \tag{17}$$

The exact value of Planck length l_{Pl} could be found from (16) and (17):

$$l_{Pl} = r_S = \sqrt{\frac{2G\hbar}{c^3}} \approx 2.28 \times 10^{-35}\,\mathrm{m} \tag{18}$$

Finally, the exact value of the Planck density is the density of a sphere possessing mass m_{Pl} and radius l_{Pl}:

$$\rho_{Pl} = \frac{3}{16\pi}\frac{c^5}{\hbar G^2} \approx 3.1 \times 10^{95}\,\mathrm{kg}\,/\,\mathrm{m}^3 \tag{19}$$

Taking into account (4) and precise formulae (8), (17), (18) and (19) as well as the Planck time $t_{Pl} = l_{Pl}/c = \sqrt{2G\hbar/c^5} \approx 7.59 \times 10^{-44}\,\mathrm{s}$, Hubble distance cH^{-1}, Hubble time ("age of the universe") H^{-1} and exact total density of the universe $\overline{\rho} \approx \rho_c = 3H^2/(8\pi G)$ we find the ratios (20):

$$\sqrt{\frac{M_H}{m_H}} = \frac{M_H}{m_{Pl}} = \frac{m_{Pl}}{m_H} = \frac{cH^{-1}}{l_{Pl}} = \frac{H^{-1}}{t_{Pl}} = \sqrt{\frac{\rho_{Pl}}{\overline{\rho}}} = \sqrt{\frac{c^5}{2G\hbar H^2}} = N = \widetilde{N}/\sqrt{2} \approx 5.73 \times 10^{60} \tag{20}$$

It is worth noting that the precise recalculations of the Planck units fit them with the exact total density of the universe $\overline{\rho} \approx \rho_c = 3H^2/(8\pi G)$ and Hubble sphere mass $M_H = c^3/(2GH)$. Besides, the large number $N = \sqrt{c^5/(2G\hbar H^2)} \approx 5.73 \times 10^{60}$ is not simply ratio of two quantities but it is a *formula* expressed by means of the fundamental constants c, G, $\hbar$ and H. Thus, the ratios (20) represent exact formulation of the Dirac's LNH while the ratios (15) are approximate.

The relation (21) could be found from (4), (8), (14) and (19):

$$v_0 = \frac{M_H}{\rho_{Pl}} = \frac{m_H}{\overline{\rho}} = \frac{8\pi}{3}\frac{G\hbar}{Hc^2} \approx 2.83 \times 10^{-43}\,\mathrm{m}^3 \tag{21}$$

The radius of the sphere having volume v_0 is $r_0 \approx 4.1 \times 10^{-15}\,\mathrm{m}$, that is of the order of size of the atomic nucleus. Therefore, the formula (21) shows that when the size of the universe was of the order of atomic nucleus its density was close to the Planck density ρ_{Pl}. Besides, the volume v_0 of the recent universe having average density $\overline{\rho} \approx \rho_c \sim 10^{-26}\,\mathrm{kg}\,/\,\mathrm{m}^3$ holds matter and energy equivalent to the Hubble (graviton) mass m_H. It follows from equations (21) and (9):

$$v_0^2 = \frac{M_H}{\rho_{Pl}} \frac{m_H}{\overline{\rho}} = \frac{m_{Pl}^2}{\overline{\rho}\rho_{Pl}} \tag{22}$$

Therefore, $m_{Pl} = v_0\sqrt{\overline{\rho}\rho_{Pl}}$, i.e. the atomic nucleus volume v_0 having geometric mean density $\rho_{gm} = \sqrt{\overline{\rho}\rho_{Pl}} \approx 5.4 \times 10^{34}$ kg / m^3 contains mass equal to the Planck mass m_{Pl}. Finally, a sphere having Planck volume $v_{Pl} = (4/3)\pi l_{Pl}^3 \approx 5 \times 10^{-104}$ m^3 and density ρ_{gm} holds matter equal to the Hubble mass m_H:

$$m = \frac{4}{3}\pi\sqrt{\left(\frac{2G\hbar}{c^3}\right)^3}\sqrt{\frac{3H^2}{8\pi G}\frac{3c^5}{16\pi\hbar G^2}} = \frac{\hbar H}{c^2} \equiv m_H \tag{23}$$

As the large number N is inverse proportional to H, the former increases during the expansion. The ratios (20) show that the mass of the observable universe M increases linearly with the cosmological time H^{-1}, whereas Hubble (graviton) mass decreases. Besides, the total density of the universe $\overline{\rho} \approx \rho_c$ decreases quadratic with cosmological time. However, the time variations of these quantities are negligible:

$$\frac{\dot{M}}{M} = -\frac{\dot{m_H}}{m_H} = -\frac{1}{2}\frac{\dot{\overline{\rho}}}{\overline{\rho}} = \frac{\dot{N}}{N} \sim H \approx 7.3 \times 10^{-11} yr^{-1} \tag{24}$$

In addition, the large number N and Dirac's large number N_D are connected by the approximate formula (25):

$$N_D \sim N^{2/3} = \sqrt[3]{\frac{c^5}{2G\hbar H^2}} \approx 3.2 \times 10^{40} \tag{25}$$

4. CONCLUSIONS

Three mass dimension quantities m_i have been derived by dimensional analysis, in addition to the Planck mass $m_P \sim \sqrt{\hbar c/G} \approx 2.17 \times 10^{-8}$ kg. Four fundamental parameters – the speed of light in vacuum (c), the gravitational constant (G), the reduced Planck constant ($\hbar$) and the Hubble constant (H) have been involved in the dimensional analysis. The first derived mass dimension quantity $m_1 \sim \hbar H/c^2 \sim 10^{-33}$ eV has been identified with the Hubble mass, which seems close to the graviton mass. The enormous mass $m_2 \sim c^3/(GH) \sim 10^{53}$ kg is close to the mass of the Hubble sphere that appears gravitationally connected universe for an arbitrary observer. Besides, this formula practically coincides with the Hoyle-Carvalho formula for the mass of the universe obtained by totally different approach. The identification of the two derived masses reinforces the trust in the suggested approach. It is remarkable that the Planck mass appears geometric mean of the masses m_1 and m_2, i.e. $m_{Pl} = \sqrt{m_1 m_2}$. The third derived mass $m_3 \sim \sqrt[5]{H\hbar^3/G^2} \sim 10^7$ GeV could not be identified unambiguously at present time, and it could be considered as heuristic prediction of the suggested approach concerning unknown very heavy particle or fundamental energy scale. Besides, the order of magnitude of the total density of the universe has been estimated by means of the suggested approach.

By two steps of approximation, it has been found that the ratio of the Hubble sphere mass M_H and the Planck mass m_{Pl} is equal to the substantial large number N definite from the equation $N = \sqrt{c^5/(2G\hbar H^2)} \approx 5.73 \times 10^{60}$. Besides, the large number N defines the ratio of Planck mass m_{Pl} and the Hubble mass m_H, the ratio of the Hubble distance cH^{-1} and the Planck length l_{Pl}, the ratio of Hubble time (age of the universe) H^{-1} and the Planck time t_{Pl} and the square root of the ratio of the Planck density ρ_{Pl} and the total density of the universe $\overline{\rho}$.

Therefore, the large number N relates cosmological parameters (mass, density, age and size of the observable universe) and fundamental microscopic properties of the matter (Planck mass, Planck density, Planck time, Planck length and Hubble mass). Thus, a precise formulation and proof of *LNH* has been found and a new fundamental physical law has been established connecting micro and mega world.

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