PRECISE FORMULATION AND PROOF OF DIRAC LARGE NUMBERS HYPOTHESIS

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ABSTRACT

Three mass dimension quantities have been derived by dimensional analysis by means of fundamental parameters – the speed of light in vacuum (c), the gravitational constant (G), the Planck constant ($\hbar$), and the Hubble parameter (H). The extremely small mass $m_1 \sim \hbar H / c^2$ ~ 10^{-33} eV has been identified with the Hubble mass, which is analogous to the graviton mass. The enormous mass $m_2 \sim c^3/(GH)$ ~ 10^{53} kg is close to the mass of the Hubble sphere and practically coincides with the Hoyle-Carvalho formula for the mass of the observable universe. Astonishingly, the Planck mass is the geometric mean of the extreme masses m_1 and m_2. In two stage of calculation, it has been found the large number $N = \sqrt{\dfrac{c^5}{2G\hbar H^2}} \approx 5.73 \times 10^{60}$, connecting mass, density, age and size of the observable universe and Planck mass, density, time and length, respectively. Thus, a precise formulation and proof of Dirac Large Numbers Hypothesis (*LNH*) has been found and a new fundamental physical law has been established connecting the micro and the mega world.

Keywords: Dirac large numbers hypothesis; Planck units; dimensional analysis; mass of the observable universe; Hubble mass

1. INTRODUCTION

The Planck mass $m_{Pl} \sim \sqrt{\dfrac{\hbar c}{G}}$ has been introduced in [1] by means of three fundamental constants – the speed of light in vacuum (c), the gravitational constant (G), and the reduced Planck constant ($\hbar$). Since the constants c, G and $\hbar$ represent three very basic aspects of the universe (i.e. the relativistic, gravitational and quantum phenomena), the Planck mass appears to a certain degree a unification of these phenomena. The energy equivalent of Planck mass $E_{Pl} = m_{Pl}c^2$ ~ 10^{19} GeV represents unification energy of the fundamental interactions [2]. Also, the Planck mass can be derived by setting it as a mass, whose Compton wavelength and gravitational radius are equal [3]. Analogously, formulae for Planck length l_{Pl}, Planck time $t_{Pl} = l_{Pl}/c$ and Planck density ρ_{Pl} were derived by dimensional analysis. In quantum gravity models, the Planck length is the length scale at which the structure of spacetime becomes dominated by quantum effects.

The Planck mass formula has been derived by dimensional analysis using fundamental constants c, G and $\hbar$. The dimensional analysis is a conceptual tool often applied in physics to understand physical situations involving certain physical quantities [4-7]. When it is known that quantities should be connected, but the form of this connection is unknown, a dimensional equation is composed. In the left side of the equation, the unit of this quantity q_0 with its dimensional exponent is placed. Most often, dimensional analysis is applied in mechanics and other fields of modern physics, where problems have few determinative quantities. Many interesting and important problems related to the fundamental constants have been considered in [8-11].

The discovery of the linear relationship between recessional velocity of distant galaxies, and distance $v = Hr$ [12] introduces new fundamental constant in physics and cosmology – the famous Hubble "constant" (parameter) H. The Hubble parameter determines the age of the universe H^{-1}, the Hubble distance cH^{-1}, the critical density of the universe $\rho_c = \dfrac{3H^2}{8\pi G} \sim 10^{-26} kg/m^3$ [13]. Because of the importance of the Hubble constant, in the present paper we include H in the dimensional analysis together with c, G and $\hbar$ aiming to find the new mass dimension quantities $m_i \sim \prod_{j=1}^{3} q_j^{n_j}$, where every triad q_1, q_2, q_3 consists of three constants c, G, H or $\hbar$. Thus, the Hubble parameter will represent the cosmological phenomena in new derived fundamental masses. The attempt to compose a mass dimension quantity by means of the four constants together produces an undetermined system of linear equations and it has been ignored. According to the recent cosmology, the Hubble "constant" slowly decreases with the age of the universe, but there are indications that other constants, especially gravitational and fine structure constants also vary with comparable rate [14-16]. That is why, the Hubble parameter could deserve being treated on an equal level with the other three constants used from Planck.

Dirac [14] suggested the Large Numbers Hypothesis (*LNH*) pointing out that the ratio of the age of the universe H^{-1} and the atomic unit of time $\tau = e^2/(m_e c^3) \sim 10^{-23} s$ is a large number of the order of 10^{40}. Besides, the ratio of electrostatic e^2/r^2 and gravitational forces $Gm_e m_p/r^2$ between proton and electron in a hydrogen atom is of the order of 10^{39} and the ratio of mass of the observable universe M and nucleon mass roughly is of the order of 10^{80}:

$$\frac{H^{-1}}{\tau} \sim \frac{e^2}{Gm_e m_p} \sim \sqrt{\frac{M}{m_p}} \sim N_D \sim 10^{40} \tag{1}$$

where e is the charge of the electron, m_e is the electron mass, m_p is the proton mass and $N_D \sim 10^{40}$ is the Dirac large number. Relying on the ratios (1), he proposed that as a consequence of causal connections between macro and micro physical world, gravitational constant G slowly decreases with time.

Many other interesting ratios have been found approximately relating some cosmological parameters and microscopic properties of the matter. For example, Narlikar [17] shows that the ratio of radius of the observable universe and classical radius of the electron $e^2/(m_e c^2)$ is of the order of 10^{40}. Also, the ratio of the electron mass and Hubble (mass) parameter $\hbar H/c^2$ approximates to 10^{39} [18]. Jordan [19] noted that the mass ratio for a typical star and an electron is of the order of 10^{60}. The ratio of mass of the observable universe and Planck mass is of the order of 10^{61} [20]. Peacock [21] points out that the ratio of Hubble distance and Planck length is of the order of 10^{60}. Finally, the ratio of Planck density ρ_{Pl} and recent critical density of the universe ρ_c is of the order of 10^{121} [22]. Most of these large numbers are rough ratios of astrophysical parameters and microscopic properties of the matter determined with accuracy of the order of magnitude.

2. DERIVATION OF THREE FUNDAMENTAL MASSES BY DIMENSIONAL ANALYSIS

A quantity m_1 having mass dimension can be created with the fundamental constants c, $\hbar$ and H:

$$m_1 = kc^{n_1}\hbar^{n_2}H^{n_3} \tag{2}$$

where n_1, n_2 and n_3 are unknown exponents to be determined by matching the dimensions of both sides of the equation, and k is dimensionless parameter of an order of magnitude of a unit.

As a result, we find the system of linear equations:

$$\begin{aligned} &n_1 + 2n_2 = 0 \\ &-n_1 - n_2 - n_3 = 0 \\ &n_2 = 1 \end{aligned} \tag{3}$$

The unique solution of the system is $n_1 = -2, n_2 = 1, n_3 = 1$. Replacing obtained values of the exponents in equation (2) we find formula (4) for the mass m_1:

$$m_H = m_1 \sim \frac{\hbar H}{c^2} \tag{4}$$

The recent experimental values of c, $\hbar$ and H are used: c = 299792458 *m/s*, $\hbar$ = $1.054571596\times 10^{-34}$ *J s* [23] and $H \approx 70$ *km/s Mps* [24]. Replacing these values in (4) we obtain $m_1 \sim 2.70\times 10^{-69}$ *kg* $=1.52\times 10^{-33}$ *eV*. This exceptionally small mass coincides with the so called "Hubble mass" $m_H = \hbar H / c^2$ [25, 26], which seems close to the graviton mass m_G obtained by different methods [27-30]. Evidently, the mass $m_1 \equiv m_H \sim m_G$ is several orders of magnitude smaller than the upper limit of the graviton mass, obtained by astrophysical constraints [31].
From equation (4) we find that the reduced Compton wavelength $\lambda\!\!\!^{-}_H$ of Hubble mass is equal to the Hubble distance cH^{-1}:

$$\lambda\!\!\!^{-}_H = \frac{\hbar}{m_H c} = cH^{-1} \sim 1.3\times 10^{26} m \tag{5}$$

The presence of an exceptionally small, yet nonzero mass of the graviton (gauge boson of gravity), involves Yukawa potential of the gravitational field $V(r) = -\dfrac{Gm}{r}\exp(-r/\lambda\!\!\!^{-}_H)$ and a finite range of the gravity close to the Hubble distance cH^{-1}. Therefore the Hubble distance is the size of gravitationally connected (observed) universe for an arbitrary observer $R \sim \lambda\!\!\!^{-}_H \approx$ 1.38×10^{10} light years.

Analogously, by means of the fundamental constants *c*, *G* and *H*, a quantity m_2 having dimension of a mass can be composed:

$$m_2 = kc^{n_1}G^{n_2}H^{n_3} \qquad (6),$$

where n_1, n_2 and n_3 are unknown exponents to be determined by matching the dimensions of both sides of the equation, and *k* is dimensionless parameter of an order of magnitude of a unit.

We determine the exponents $n_1 = 3, n_2 = -1, n_3 = -1$ by the dimensional analysis again. Replacing the obtained values of the exponents in equation (6) we find formula (7) for the mass m_2:

$$M \sim m_2 \sim \frac{c^3}{GH} \qquad (7)$$

First time, the formula (7) has been derived by means of dimensional analysis in [32]. This formula practically coincides with the Hoyle formula [33] for the mass of the observable universe $M = \frac{c^3}{2GH}$ and perfectly coincides with Carvalho formula [34] for the mass of the observable universe *M*, obtained by totally different approach.
The Hubble sphere is the sphere where the recessional velocity of the galaxies is equal to the speed of the light in vacuum *c*, and according to the Hubble law *v* = *c* when $r = cH^{-1}$. Besides, the Hubble sphere coincides with gravitationally connected universe for an arbitrary observer. Thus, the Hubble sphere appears a three-dimensional sphere, centered on the observer, having radius $R = cH^{-1}$ and density $\overline{\rho} \approx \rho_c$. Clearly, the formula (7) is close to the mass of the Hubble sphere M_H:

$$m_2 \approx M_H = \frac{4}{3}\pi\frac{c^3}{H^3}\frac{3H^2}{8\pi G} = \frac{c^3}{2GH} \qquad (8)$$

Replacing the recent values of the constants *c*, *G* and *H* in (7) we obtain $m_2 \sim 1.76\times10^{53}$ *kg*. Therefore, the enormous mass m_2 is close to the mass of the observable universe *M*.

From formulae (4) and (7) we find the important relation (9):

$$\sqrt{m_1 m_2} = \sqrt{m_H M_H} = \sqrt{\frac{\hbar c}{G}} = m_{Pl} \qquad (9)$$

Therefore, the Planck mass is the geometric mean of the graviton mass and the mass of the observable (gravitationally connected) universe. As the physical quantity mass is among the most substantial properties of the matter, the formula (9) hints at a deep relation of the micro particles and the entire universe.

The third quantity m_3, having mass dimension can be composed by means of the fundamental constants G, $\hbar$ and H:

$$m_3 = kG^{n_1}\hbar^{n_2}H^{n_3} \tag{10}$$

We determine the exponents $n_1 = -\frac{2}{5}, n_2 = \frac{3}{5}, n_3 = \frac{1}{5}$ by dimensional analysis again. Replacing the obtained values of the exponents in equation (10) we find formula (11) for the mass m_3:

$$m_3 \sim \sqrt[5]{\frac{H\hbar^3}{G^2}} \tag{11}$$

Replacing the recent values of the constants G, $\hbar$ and H, the mass m_3 takes value $m_3 \sim 1.43\times10^{-20}$ *kg* $\approx 8.0\times10^6$ *GeV*. This mass is a dozen of orders of magnitude lighter than the Planck mass and several orders of magnitude heavier than the heaviest known particles like the top quark $m_t \approx 174.3$ *GeV* [35]. On the other hand, the energy $m_3c^2 \sim 8\times10^6$ GeV appears medial for the important Grand Unified Theory (GUT) scale $E_{GUT} \sim 10^{16}$ GeV and electroweak scale $E_{EW} \sim 10^2$ GeV. Therefore, the mass/energy m_3 yet could not be identified but could be considered as a heuristic prediction of a very heavy particle or fundamental energy scale. First time these three masses have been derived in [36].

Finally, the heuristic power of dimensional analysis can estimate the total density of the universe. Actually, a quantity ρ having dimension of density can be composed by means of the fundamental constants c, G and H:

$$\rho = kc^{n_1}G^{n_2}H^{n_3} \qquad (12),$$

where k is a dimensionless parameter of the order of magnitude of unit.

By the dimensional analysis, we have found the exponents $n_1 = 0, n_2 = -1, n_3 = 2$. Therefore:

$$\rho \sim \frac{H^2}{G} \approx 7.93\times10^{-26}\ kg/m^3 \tag{13}$$

The recent Cosmic Microwave Background (*CMB*) observations show that the total density of the universe $\overline{\rho}$ is [37-39]:

$$\overline{\rho} = \Omega\rho_c \approx \rho_c = \frac{3H^2}{8\pi G} \sim 10^{-26}\ kg/m^3 \tag{14}$$

Clearly, the density ρ derived by means of the fundamental constants c, G and H coincides with formula (14) for the total density of the universe with an accuracy of a dimensionless parameter of order of unit. Besides, the formula (13) could be derived by means of other triad of fundamental constants, namely G, $\hbar$ and H.

3. PRECISE FORMULATION AND PROOF OF DIRAC LARGE NUMBERS HYPOTHESIS

3.1. Approximate estimation of the large number *N* relating cosmological parameters and Planck units by means of dimensional analysis

The Planck mass $m_{Pl} \sim \sqrt{\frac{\hbar c}{G}}$ and formulae (4) and (7) for the Hubble mass and mass of the observable universe have been derived by dimensional analysis by means of the fundamental constants c, G, $\hbar$ and H. The Planck density $\rho_{Pl} \sim \frac{c^5}{\hbar G^2} \approx 5.2 \times 10^{96}\, kg/m^3$, the Planck length $l_{Pl} \sim \sqrt{\frac{G\hbar}{c^3}} \approx 1.61\times10^{-35}\ m$, the Planck time $t_{Pl} = \frac{l_{Pl}}{c} \sim \sqrt{\frac{G\hbar}{c^5}} \approx 5.37 \times 10^{-44} s$ and the approximate formula (13) for the total density of the universe ρ also are obtained by dimensional analysis. Taking into account above mentioned formulae and Hubble distance cH^{-1} and age of the universe H^{-1} we find remarkable ratios (15):

$$\sqrt{\frac{M}{m_H}} = \frac{M}{m_{Pl}} = \frac{m_{Pl}}{m_H} = \frac{cH^{-1}}{l_{Pl}} = \frac{H^{-1}}{t_{Pl}} = \sqrt{\frac{\rho_{Pl}}{\overline{\rho}}} = \sqrt{\frac{c^5}{G\hbar H^2}} = \widetilde{N} \approx 8.1 \times 10^{60} \tag{15}$$

Therefore, the ratio of the mass of the observable universe M and the Planck mass m_{Pl} is equal to the large number $\widetilde{N}$ defined from the equation $\widetilde{N} = \sqrt{\frac{c^5}{G\hbar H^2}} \approx 8.1 \times 10^{60}$. Besides, the large number $\widetilde{N}$ defines the ratio of Planck mass m_{Pl} and the Hubble mass m_H, the ratio of the Hubble distance cH^{-1} and the Planck length l_{Pl}, the ratio of Hubble time (age of the universe) H^{-1} and the Planck time t_{Pl} and the square root of the ratio of the Planck density ρ_{Pl} and the approximate density of the universe ρ determined from (13). These ratios appear very important because they connect cosmological parameters (mass, density, age and size of the observable universe) and the fundamental microscopic properties of the matter (Planck mass, Planck density, Planck time, Planck length and Hubble mass). In recent quantum gravity models, the Planck units imply quantization of spacetime at extremely short range. Thus, the ratios (15) represent connection between cosmological parameters and quantum properties of spacetime. Obviously, the ratios (15) appear a formulation of *LNH*.

3.2. Precise determination of the large number *N* and proof of Dirac *LNH* by fine tuning of Planck units

As mentioned in Section 1, dimensional analysis allows findings unknown quantities with accuracy of the dimensionless parameter *k*, unit order of magnitude. Below, we recalculate the ratios (15) using exact values of the respective quantities. The exact value of Planck mass is the mass, whose Compton wavelength $\lambdabar$ and gravitational (Schwarzschild) radius r_S are equal:

$$\lambdabar = \frac{\hbar}{mc} = r_S = \frac{2Gm}{c^2} \tag{16}$$

Thus, from (16) we find the exact value of Planck mass:

$$m_{Pl} = \sqrt{\frac{\hbar c}{2G}} \approx 1.54 \times 10^{-8}\, kg \tag{17}$$

The exact value of Planck length l_{Pl} follows from (16) and (17):

$$l_{Pl} = r_S = \frac{2G}{c^2}\sqrt{\frac{\hbar c}{2G}} = \sqrt{\frac{2G\hbar}{c^3}} \approx 2.28\times 10^{-35}\ m. \tag{18}$$

Finally, the exact value of the Planck density is calculated as the density of a sphere possessing mass m_{Pl} and radius l_{Pl}:

$$\rho_{Pl} = \frac{3m_{Pl}}{4\pi l_{Pl}^3} = \frac{3}{16\pi}\frac{c^5}{\hbar G^2} \approx 3.1\times 10^{95}\, kg/m^3 \tag{19}$$

Taking into account (4) and precise formulae (8), (17), (18) and (19) as well as the Planck time $t_{Pl} = \frac{l_{Pl}}{c} = \sqrt{\frac{2G\hbar}{c^5}} \approx 7.59\times 10^{-44}\ s$, Hubble distance cH^{-1}, Hubble time ("age of the universe") H^{-1} and exact total density of the universe $\bar{\rho} \approx \rho_c = \frac{3H^2}{8\pi G}$ (14) we find the exact ratios (20):

$$\sqrt{\frac{M_H}{m_H}} = \frac{M_H}{m_{Pl}} = \frac{m_{Pl}}{m_H} = \frac{cH^{-1}}{l_{Pl}} = \frac{H^{-1}}{t_{Pl}} = \sqrt{\frac{\rho_{Pl}}{\bar{\rho}}} = \sqrt{\frac{c^5}{2G\hbar H^2}} = N = \tilde{N}/\sqrt{2} \approx 5.73\times 10^{60} \tag{20}$$

It is worth noting that the precise recalculations of the Planck units correspond with the exact total density of the universe $\bar{\rho} \approx \rho_c = \frac{3H^2}{8\pi G}$ and Hubble sphere mass $M_H = \frac{c^3}{2GH}$. Therefore, the equations (17-19) for the Planck mass, length and density, as well as the Planck time equation $t_{Pl} = \sqrt{\frac{2G\hbar}{c^5}}$, are exact whereas Planck units obtained by dimensional analysis are approximate. Besides, the large number $N = \sqrt{\frac{c^5}{2G\hbar H^2}} \approx 5.73\times 10^{60}$ is not simply ratio of two quantities but it is an exact formula expressed by means of the fundamental constants c, G, $\hbar$ and H. Thus, the ratios (20) represent precise formulation of the Dirac *LNH*. Evidently, the exact equations (4), (8) and (17) confirm that the Planck mass is the geometric mean of graviton mass and mass of the Hubble sphere.

The following relation (21) results from (4), (8), (14) and (19):

$$v_0 = \frac{M_H}{\rho_{Pl}} = \frac{m_H}{\overline{\rho}} = \frac{8\pi}{3}\frac{G\hbar}{Hc^2} \approx 2.83\times 10^{-43} m^3 \tag{21}$$

The radius of the sphere having volume v_0 is $r_0 \approx 4.1\times 10^{-15}$ m, i.e. of the order of size of the atomic nucleus. Therefore, the formula (21) shows that when the size of the universe was of the order of atomic nucleus its density was close to the Planck density ρ_{Pl}. Besides, the volume v_0 of the recent universe having average density $\overline{\rho} \approx \rho_c \sim 10^{-26} kg/m^3$ holds matter and energy equivalent to the Hubble mass $m_H \sim 10^{-33}$ eV. It follows from equations (21) and (9):

$$v_0 v_0 = v_0^2 = \frac{M_H}{\rho_{Pl}}\frac{m_H}{\overline{\rho}} = \frac{m_{Pl}^2}{\rho_{Pl}\overline{\rho}} \tag{22}$$

Therefore, $m_{Pl} = v_0\sqrt{\overline{\rho}\rho_{Pl}}$, i.e. the atomic nucleus volume v_0 having geometric mean density $\rho_{gm} = \sqrt{\overline{\rho}\rho_{Pl}} \approx 5.4\times 10^{34} kg/m^3$ contains mass equal to the Planck mass m_{Pl}. Besides, taking in account equations (14), (18) and (19) we find that a sphere having Planck volume $v_{Pl} = (4/3)\pi l_{Pl}^3 \approx 5\times 10^{-104} m^3$ and density ρ_{gm} holds matter equal to the Hubble mass m_H :

$$m = (4/3)\pi l_{Pl}^3\sqrt{\overline{\rho}\rho_{Pl}} = \frac{\hbar H}{c^2} \equiv m_H \tag{23}$$

As the large number N is inverse proportional to H, the former increases during the expansion. The equations (8), (4) and (14) show that the mass of the observable universe $M \approx M_H$ increased, whereas the Hubble (graviton) mass m_H and the total density of the universe $\overline{\rho} \approx \rho_c$ decreased with age of the universe H^{-1}. Nevertheless, the ratios (20) continue to be in force during cosmological extension. Furthermore, the time variations of these quantities are negligible:

$$\frac{\dot{M}}{M} = -\frac{\dot{m}_H}{m_H} = -\frac{1}{2}\frac{\dot{\overline{\rho}}}{\overline{\rho}} = \frac{\dot{N}}{N} \sim H \approx 7.26\times 10^{-11} yr^{-1} \tag{24}$$

Apparently, the large number N and Dirac large number N_D are connected by the approximate formula (25):

$$N_D \sim N^{2/3} = \sqrt[3]{\frac{c^5}{2G\hbar H^2}} \approx 3.2\times 10^{40} \tag{25}$$

4. CONCLUSIONS

Three mass dimension quantities m_i have been derived by dimensional analysis, in addition to the Planck mass $m_{Pl} \sim \sqrt{\frac{\hbar c}{G}} \approx 2.17\times 10^{-8}$ *kg*. Four fundamental parameters – the speed of light in vacuum (*c*), the gravitational constant (*G*), the reduced Planck constant ($\hbar$) and the Hubble constant (*H*) have been involved in the dimensional analysis. The first deduced mass dimension quantity $m_1 \sim \frac{\hbar H}{c^2} \sim 10^{-33}$ *eV* has been identified with the Hubble mass, which is close to the graviton mass. The enormous mass $m_2 \sim \frac{c^3}{GH} \sim 10^{53}$ *kg* is close to the mass of the Hubble sphere. This formula practically coincides with the Hoyle-Carvalho formula for the mass of the universe obtained by a totally different approach. The identification of the two derived masses reinforces the suggested approach. It is remarkable that the Planck mass is geometric mean of the extreme masses m_1 and m_2, i.e. $m_{Pl} = \sqrt{m_1 m_2}$. The third derived mass $m_3 \sim \sqrt[5]{\frac{H\hbar^3}{G^2}} \sim 10^7$ *GeV* yet could not be identified, and could be a heuristic prediction of a very heavy particle or fundamental energy scale. Besides, the order of magnitude of the total density of the universe has been estimated by means of the suggested approach.

In two stage of calculation, it has been found that the ratio of the Hubble sphere mass M_H and the Planck mass m_{Pl} is equal to the substantial large number *N* definite from the equation $N = \sqrt{\frac{c^5}{2G\hbar H^2}} \approx 5.73\times 10^{60}$. Besides, the large number *N* defines the ratio of Planck mass m_{Pl} and the Hubble mass m_H, the ratio of the Hubble distance cH^{-1} and the Planck length l_{Pl}, the ratio of Hubble time (age of the universe) H^{-1} and the Planck time t_{Pl} and the square root of the ratio of the Planck density ρ_{Pl} and the total density of the universe $\bar{\rho}$. Therefore, the large number *N* connects cosmological parameters (mass, density, age and size of the observable universe) and fundamental microscopic properties of the matter (Planck mass, Planck density, Planck time, Planck length and Hubble mass). Thus, a precise formulation and proof of Dirac *LNH* has been found and a new fundamental physical law has been established connecting the micro and the mega world, although the deep nature of Planck units and Hubble mass nowadays is not sufficiently clear. It is worth noting that the derived ratios (20) are not simply numbers of the same order of magnitude but a single large number *N*, represented by an exact equation by means of the fundamental constants – *c*, *G*, $\hbar$ and *H*.

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REFERENCES

[1] M. Planck, The Theory of Heat Radiation, Dover Publications, New York, 1959 (translation from German Ed. 1906).
[2] H. Georgi, H.R. Quinn, S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.
[3] P.G. Bergmann, The Riddle of Gravitation, Dover Publications, New York, 1992.

[4] P.W. Bridgman, Dimensional Analysis, Yale University Press, New Haven, 1922.
[5] R. Kurth, Dimensional Analysis and Group Theory in Astrophysics, Pergamon Press, Oxford, 1972.
[6] R. Bhaskar, A. Nigam, Artificial Intelligence 45 (1990) 73.
[7] G.W. Petty, Software – Practice and Experience 31 (2001) 1067.
[8] J.M. Levy-Leblond, Riv. Nuovo Cim. 7 (1977) 187.
[9] M.J. Duff, L.B. Okun, G. Veneziano, J. High En. Phys. Issue 03 (2002) 023.
[10] J.D. Barrow, The Constants of Nature: From Alpha to Omega, Jonathan Cape, London, 2002.
[11] H. Fritzsch, The Fundamental Constants, a Mystery of Physics, World Scientific, Singapore, 2009.
[12] E. Hubble, Proc. Nat. Acad. Sci. 15 (1929) 168.
[13] P.J. Peebles, Physical Cosmology, Princeton University Press, 1971.
[14] P.A.M. Dirac, Nature 139 (1937) 323.
[15] Y. Wu, Z. Wang, Phys. Rev. Lett. 57 (1986) 1978.
[16] J.K. Webb et al., Phys. Rev. Lett. 87 (2001) 091301.
[17] J.V. Narlikar, The Structure of the Universe, Oxford University Press, 1977.
[18] A. Cetto, L. de la Pena, E. Santos, Astron. Astrophys. 164 (1986) 1.
[19] P. Jordan, Die Herkunft der Sterne, Wiss.Verlagsges., Stuttgart, 1947.
[20] V.E. Shemi-zadeh, Preprint: arXiv:gr-qc/0206084.
[21] J.A. Peacock, Cosmological Physics, Cambridge University Press, 1999.
[22] A.Y. Andreev, B.V. Komberg, Astron. Rep. 44 (2000) 139.
[23] P. Mohr, B. Taylor, J. Phys. Chem. Ref. Data 28 (1999) 1713.
[24] J.R. Mould et al., Astrophys. J. 529 (2000) 786.
[25] I. Maor, R. Brustein, Phys. Rev. D 67 (2003) 103508.
[26] J.P. Gazeau, F. Toppan, Class. Quantum Grav. 27 (2010) 025004.
[27] J.F. Woodward, R. J. Crowley, W. Yourgrau, Phys. Rev. D 11 (1975) 1371.
[28] S.S. Gershtein, A.A. Logunov, M.A. Mestvirishvili, Dokl. Phys. 43 (1998) 293.
[29] D. Valev, Aerospace Res. Bulg. 22 (2008) 68; Preprint :arxiv:hep-ph/0507255.
[30] M.E. Alves, O.D. Miranda, J. C. de Araujo, Preprint :arXiv:0907.5190.
[31] A.S. Goldhaber, M.M. Nietto, Phys. Rev. D 9 (1974) 1119.
[32] D. Valev, Compt. Rend. Acad. Bulg. Sci. Spec. Issue (2009) 233; Preprint :arXiv:0909.2726.
[33] H. Kragh, Cosmology and Controversy: The Historical Development of Two Theories of the Universe, Princeton University Press, 1999.
[34] J.C. Carvalho, Int. J. Theor. Phys. 34 (1995) 2507.
[35] M. Mangano, T. Trippe, Europ. Phys. J. C 15 (2000) 385.
[36] Valev D., Preprint :vixra:1103.0096.
[37] A. Balbi et al., Astrophys. J. 545 (2000) L1.
[38] P. de Bernardis et al., Nature 404 (2000) 955.
[39] D.N. Spergel et al., Astrophys. J. Suppl. Series 148 (2003) 175.