

# Physico-Mathematical Fundamentals of the Theory of Reference Frames

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*Not nature but observers need reference frames.*

## **Abstract**

This paper represents the first essay belonging to the "Trilogy on the Knowable Universe". Main physico-mathematical fundamentals of the Theory of Reference Frames concerning kinematic, dynamic, electrodynamic and electromagnetic aspects of the physical reality, already introduced in previous papers, are presented here methodically. We demonstrate the importance for observer of reference frame in order to understand and to describe a physical event; we use new definitions of space, time, simultaneity and introduce the new concept of electrodynamic mass. We demonstrate besides an important relation between time and mass and a new expression for the relativistic mass.

## **1. Introduction**

The mathematical formalization of space-time for inertial reference frames was achieved at first in classical physics with Galileo-Newton's transformations and later with Lorentz's transformations in Einstein's Special Relativity. These last transformations have been generalized later in General Relativity before a gravitational field for accelerated reference frames.

With this paper we want to reintroduce more methodically basic aspects of the Theory of Reference Frames (TR)<sup>[1,2,3,4,5]</sup>, which is valid for all reference frames, inertial and non-inertial, provided with any velocity. TR is a critical scientific theory towards main existing theories concerning the question of relativity: the Theory of Ether, Special and General Relativity, the Theory of Light Emission. In TR the concept of reference frame has decisive importance, it is different from the simple system of coordinates used for instance by Einstein. In TR the reference frame has a physico-mathematical structure, whose choice can have effects on the analysis of the considered event always however with due respect for general principles and laws of physics.

After the new definitions of space, time and simultaneity we define the important concept of reference frame distinguishing Einsteinian reference frames from Galilean reference frames with the primary object to reach new transformations of space-time, that are valid for all reference frames, inertial and non-inertial, with any relative velocity.

The introduction in TR of the innovative "Principle of Reference" allows to define a preferred reference frame which is different from the absolute reference frame of the classical physics. As per the new principle it is possible to distinguish the "**physical speed**" of light and e.m. waves from the "**relativistic speed**". The physical speed of light is constant with respect to all reference frames and it is the generally measured speed. The relativistic speed instead isn't constant, has vector nature and depends on the relative speed between the two considered reference frames: the reference frame of the source (primary or secondary) and the reference frame of the observer.

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## 2. Space, time and simultaneity of events

In classical physics space and time were considered two absolute quantities whose definition had a metaphysical meaning. Consequently space was Euclidean and filled with a strange material substance that coincided with the Aristotelian ether and had the main purpose to explain the propagation of light and electromagnetic waves. Time was the absolute time measured by the only cosmic clock of the whole universe. In Special Relativity (SR), under the influence of the negative result of the Michelson-Morley experiment, Einstein tried to give a physical meaning to time by the definition of time measured through identical and synchronized clocks placed in every point of space. Space in SR coincided with the Euclidean space but later in General Relativity (GR) Einstein accepted Riemann's elliptic non-Euclidean space, due to the gravitational field, and he gave a tensor definition of the non-linear element.

In the Theory of Reference Frames space is three-dimensional (3D) with three space coordinates (x,y,z). The three coordinates are mathematically independent each other and their interdependence can have only a physical meaning defined for instance by the kinematics of point.

The space considered in TR is at the moment the **empty geometric space**, without any physical property, defined only by a tern of Cartesian coordinates (x,y,z) and each time by an origin O.

In our view the empty geometric space, with respect to the three coordinates, is **infinite, infinitesimal and discrete**. It is **infinite** because, for every distance "d" it is always possible for instance to double that distance without an objective limit to this possibility exists.

It is **infinitesimal** because the same distance "d" can be for example time after time halved without an objective limit to this possibility exists. It is **discrete** because the distance "d" can be made always smaller without reach the zero distance which matches the geometric point.

The **empty physical space** instead is the empty geometric space defined by a few physical properties as the dielectric constant in a vacuum ( $\epsilon_0=8,86 \times 10^{-12}$ F/m) and the magnetic permeability in a vacuum ( $\mu_0=12,56 \times 10^{-7}$  tesla $\times$ m/ampere) with regard to electromagnetic properties and the resistance in a vacuum ( $k_0=0$ ) with regard to mechanical properties. It is suitable to underline that this definition of empty physical space is altogether different from the Aristotelian ether, which was considered an immobile material medium that filled the whole universe and whose no sure physical property was renowned.

After the cited M&M experiment new definitions of ether were excogitated (dragged ether, partially dragged ether, et cetera) but all these definitions were without avail.

The **physical space** is the physical space, limited or unlimited, which can be empty but can be also filled with known substances as air, water, etcetera, and which is venue for the considered physical event.

**Time** in TR is the **measured time** by a clock which is synchronized with the standard clock and is situated in the place where the physical event happens.

This time definition equals Einstein's definition, but the concept of simultaneity of events makes the difference between the Special Relativity and the Theory of Reference Frames. In fact in SR the simultaneity of two events that happen in two different points of the physical space is defined through paths that different rays of light follow with respect to two identical and synchronized clocks placed in the two considered points.

We can observe that this definition disregards possible violations of the symmetry condition that is necessary for its validity (fig.1).

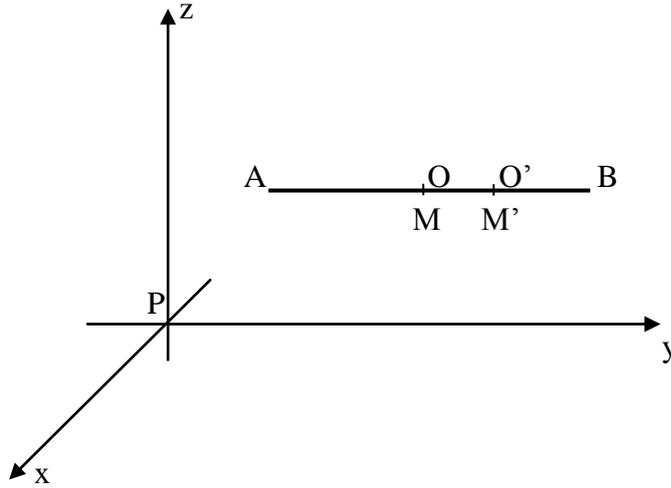


Fig.1 The observer O placed in the middle point of the path AB is symmetric with respect to points A and B from which two rays of light leave. The observer O' placed in the non-middle point M' on the contrary isn't symmetric with respect to two points.

In fact suppose that at the time  $t=0$  two synchronous rays of light leave the points A and B and reach the middle point O at the time  $t_0$ . Light travels at the same speed  $c$  towards the two directions and the three points A, B, M are motionless in the physical space  $(x,y,z,t)$ ; accordingly because of the symmetry state of the considered physical process the two rays of light reach the middle point M at the same time  $t_0$ . The observer O gauges that the two rays of light are simultaneous and therefore confirms the initial hypothesis, for which

$$\frac{AM}{c} = \frac{BM}{c} = t_0 \quad (1)$$

The observer O', placed in the non-middle point M', because of the symmetry breaking in the physical space  $(x,y,z,t)$ , observes the two rays of light aren't simultaneous because the ray coming from the point B reach him first. In fact

$$\frac{BM'}{c} < \frac{AM'}{c} \quad (2)$$

Consequently the observer O' doesn't confirm the initial hypothesis gauging an event that from his asymmetrical viewpoint isn't simultaneous contradicting the real initial hypothesis. As the whole process happens inside the same physical space, it is manifest that only the symmetric observer O measures a true fact while the observer O' measures a distorted fact due to the symmetry breaking of the considered physical process because of his asymmetrical position.

If we repeat the same reasoning supposing that at the same time  $t=0$  in which the two rays of light leave the two points, the observer O is moving with speed  $v$  towards the point B, this same observer who at rest gauged simultaneous rays, now gauges non-simultaneous rays because he sees first the ray coming from the point B, because of the finite speed of light (fig.2). We understand then the moving observer O reaches the same result gauged by the non-symmetrical observer O' and therefore

the motion state, involving a symmetry state breaking, must be considered like the non-symmetrical resting state of the observer  $O'$ .

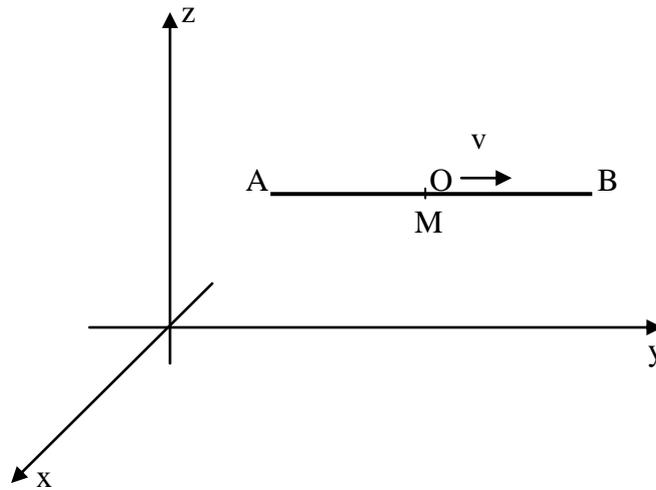


Fig.2 If the observer  $O$  placed in the middle point  $M$  of the path  $AB$ , at starting initial time of the two synchronous rays of light, is provided with speed  $v$  towards the point  $B$ , then he is in the same conditions of non-symmetry of the observer  $O'$  in fig.1.

This same result happens also if instead the rays of light we consider two any moving objects from the points  $A$  and  $B$  towards the points  $M$  and  $M'$ .

It induces us to understand the simultaneity state must be defined in physical situations in which symmetry conditions of process are fulfilled and any breaking of symmetry can imply an error of judgement in the measure and so in the simultaneity. We will see later the surmounting of this contradiction due to the symmetry breaking is possible introducing the Principle of Reference.

These considerations prove the concept of simultaneity is tied to the time behaviour of events but also the space situation of symmetry of process can have a decisive importance on the valuation of simultaneity by different observers.

Besides the claim that all observers are equivalent independently of their observations and that results of their observations on the simultaneity of two events are legitimate similarly means firstly to deny the initial hypothesis and secondly to certify the existence of a contradictory physical reality.

### 3. Reference Frame

In TR the Reference Frame isn't a simple system of coordinates but it is a physico-mathematical complex system composed of kinematic coordinates of the three-dimensional space and of time  $(x,y,z,t)$ , of one origin  $O$  and of the physical space, in the sense before defined, in which event happens. It is convenient to underline that the physical space is also composed of all physical elements that are necessary for a complete description of the considered physical event.

In order to clarify the concept of reference frame let us distinguish the following reference frames:

- a. Galilean reference frames
- b. Einsteinian reference frames

This distinction is substantial because we will see the choice of either between the two reference frames implies difference in the study of physical event. Anyway we will represent the reference frame with the paradigm 4D  $S[O,x,y,z,t]$  where  $x,y,z$  are coordinates of the three-dimensional space coinciding with the physical space,  $O$  is the common origin of three Cartesian axes and  $t$  is the characteristic time of the reference frame. A different reference frame  $S'$  will be represented with the paradigm 4D  $S'[O',x',y',z',t']$ .

**a.** The Galilean reference frame is closed, isolated and non-interacting with the rest of the universe. Galileo himself gets the reference frame across to us that he considers in one of his main works<sup>[6]</sup>:

***“ Shut yourself with a few friends in the gratest room that is below decks of some large ship and here, when the skip is still, observe with diligence..... Later get the skip moving with any constant and straight speed, you will not recognize changes..... ” .***

This definition of reference frame explains perfectly what we must mean by physical space connected with the Galilean reference frame. In particular Galilean physical space connected with the reference frame is limited, closed and isolated with respect to any interaction coming from the external physical space to the reference frame. Interferences of any physical nature, thermal, optical, electromagnetic, etcetera aren't allowed in the Galilean relativistic physics.

For instance the Doppler effect and electromagnetic interferences are typical examples of physical events that in the Galilean relativity cannot be considered: in fact when Galileo lived neither the Doppler effect nor electromagnetic interferences were known.

The concept of relativity was born by Galileo in order to underline that any physical event happens always similarly whether when the reference frame is still or when it is moving with inertial speed (straight and constant) and therefore the observer is unable to understand if he is still or in motion.

**b.** The Einsteinian reference frame on the contrary is open and interacting with the rest of the universe. Star reference frames are the most clear example about the Einsteinian reference frames in which light is born in the reference frame of a star and it is observed in the different reference frame of the earth's observer. This concept of reference frame is also present in the Principle of Constancy of the Speed of Light that is the second postulate of SR<sup>[7]</sup>:

***“ Every ray of light travels with respect to the resting coordinate system with the constant speed  $c$  independently of the speed of source ”***

In the physical event considered by Einstein light is born in the reference frame of the moving source and travels in the resting reference frame, changing reference frame and it is possible only with open reference frames. Besides we observe Einstein considers the coordinate system that in our definition has a more restricted meaning than the reference frame.

#### **4. General Principles of the physical space-time**

The Physical Principle describes a general behaviour of physical systems independently of cause and effect. In the Theory of Reference Frames the physical space-time is characterized by a reference frame  $S[O,x,y,z,t]$  in the meaning before specified; each time then we will clarify if the considered reference frame is Galilean or Einsteinian. In both types of reference frame a few general physical principles are valid:

#### ***4a. Principle of Reference***

This Principle is a typical principle concerning the Theory of Reference Frames. It derives from the necessity to define a new methodological and working procedure in order to analyse correctly the physical behaviour of considered events. The principle claims:

***“For any physical event a preferred reference frame exists and it coincides with the reference frame physically tied to the physical space in which the event happens. Moreover the privileged observer has to be placed symmetrically inside this preferred reference frame”.***

The Principle of Reference affirms the existence of both a preferred reference frame and a privileged observer. In the Theory of Reference Frames this preferred reference frame is completely different from the absolute reference frame used for instance in the Ether Theory.

In fact the preferred reference frame in TR isn't absolute but changes according to the physical space where the considered event happens. Moreover also inside the same preferred reference frame, not all observers are equivalent and privileged observers are placed symmetrically with respect to the considered physical event. With regard to the event considered in fig.1 the two observers are both in the same preferred resting reference frame, but only the observer O is privileged because of his symmetry state. In order to analyse then the same physical event with respect to another reference frame it is convenient to make use of the later Principle of Relativity.

#### ***4.b Principle of Inertia***

The Principle of Inertia is a principle of classical physics. We nevertheless have generalized<sup>[3]</sup> the validity of this principle also to different motions from the rectilinear constant motion for which in TR the Generalized Principle of Inertia affirms:

***“Every physical system tends to keep its resting state or its inertial motion with respect to a reference frame supposed at rest, until an external cause or force changes its state”***

The classical Principle of Inertia considered only constant and rectilinear motions (linear motions) with respect to a reference frame supposed at rest, we have expanded this definition to all motions which happen under the action of forces whose total resultant is null. The Generalized Principle of Inertia defines a physical equivalence between the resting state and the state of inertial motion which can be broken only by the appearance of causes or forces that are in all different from zero. A cause that can invalidate the inertial motion is for instance the onset of external resistant forces. It is manifest that all inertial reference frames exclude the presence of external forces that are in all different from zero.

#### ***4.c Principle of Relativity***

We accept the formulation of Galileo&Einstein's Principle of Relativity: Galileo formulated that principle only for mechanics and Einstein expanded it to electromagnetism and to all physics. Besides for the enunciation of the principle we will consider whether the Principle of Reference or the Principle of Inertia. The Principle of Relativity in TR claims:

***“Physical phenomena and mathematical laws that represent those phenomena are invariant with regard to all inertial reference frames considered with respect to the preferred reference frame in which the phenomenon happens”.***

As per the Principle of Inertia, inertial reference frames with respect to the preferred reference frame are all those reference frames whose motion doesn't introduce any additional force on the considered physical event.

While the Principle of Inertia defines a simple equivalence among all inertial reference frames, the Principle of Relativity affirms a further stronger property: the invariance of physico-mathematical description concerning the considered physical event with respect all inertial reference frames. Naturally this principle is valid whether for Galilean or Einsteinian reference frames, but in some physical situations it is convenient to specify well the reference frame in order to have its correct application because of possible interferences due to the open structure of Einsteinian reference frames.

It is also convenient to underline that the Principle of Relativity defines a property of invariance for inertial reference frames and therefore the General Relativity, that is based on a principle of covariance, cannot be considered an effective theory of relativity. GR is really a theory of equivalence between accelerated reference frames and gravitational reference frames.

### 5. Kinematics of linear Reference Frames

Linear reference frames move with rectilinear velocity. Let us consider then two reference frames  $S[O,x,y,z,t]$  and  $S'[O',x',y',z',t']$ , suppose that  $S$  is the resting reference frame and  $S'$  is the moving reference frame with linear speed  $\mathbf{u}$  which is variable anyway; besides  $(u_x, u_y, u_z)$  are the components of the speed vector  $\mathbf{u}$  with respect to  $S$ . Suppose still that the point  $P$  is inside the moving reference frame  $S'$ , its space coordinates are  $[x', y', z']$  with respect to  $S'$  and  $[x, y, z]$  with respect to  $S$ .

It's manifest all points belonging to the reference frame  $S'$  are endowed with the speed  $\mathbf{u}$  and what we will say is valid for every point  $P$  belonging to the reference frame  $S'$ .

Suppose that at the initial time  $t=t'=0$  the two reference frames coincide and have therefore the same space axes ( $x'=x, y'=y, z'=z$ ) and the same origin  $O'=O$ . With respect to  $S$  we have (fig.3)

$$\begin{aligned} x &= x' + \int_0^t u_x dt \\ y &= y' + \int_0^t u_y dt \\ z &= z' + \int_0^t u_z dt \end{aligned} \quad (3)$$

The (3) represent the equations of relativistic transformations of space coordinates for any linear speed  $\mathbf{u}$ . The equations (3) can be written also in a symbolic manner

$$\mathbf{P}[x,y,z] = \mathbf{P}[x',y',z'] + \mathbf{f}[u_x, u_y, u_z] \quad (4)$$

in which  $\mathbf{f}$  represents the integral function of variation of space coordinates caused by the speed  $\mathbf{u}$ . In this first part of our study the speed  $\mathbf{v}'$  isn't considered.

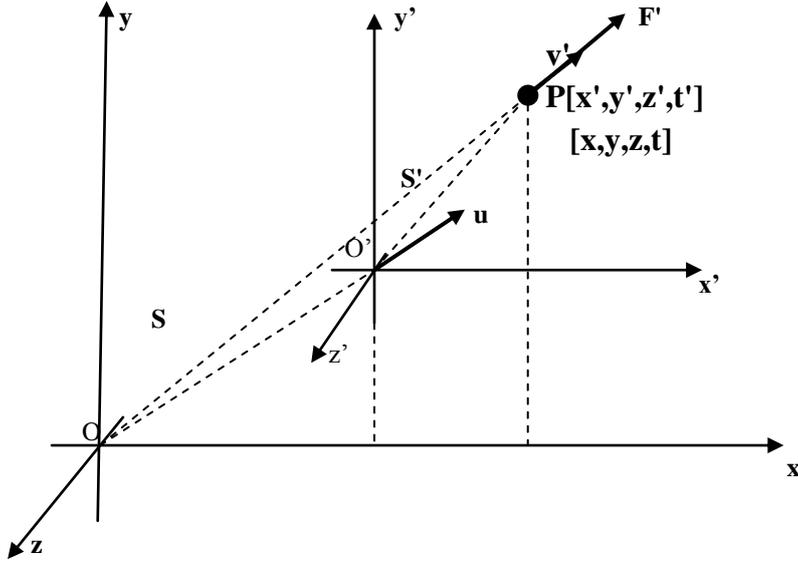


Fig.3 The reference frame  $S'$  is moving with linear speed  $u$  with respect to the resting reference frame  $S$ . The point  $P$  is inside the moving reference frame  $S'$ .

Symmetrically we have

$$\begin{aligned} x' &= x - \int_0^t u_x dt \\ y' &= y - \int_0^t u_y dt \\ z' &= z - \int_0^t u_z dt \end{aligned} \quad (5)$$

$$\mathbf{P}'[x', y', z'] = \mathbf{P}[x, y, z] - \mathbf{f}[u_x, u_y, u_z] \quad (6)$$

Deriving a first time the (3) with respect to time  $t$ , because the coordinates  $x', y', z'$  of the point  $P$  are constant, we obtain

$$\frac{dx}{dt} = u_x \quad \frac{dy}{dt} = u_y \quad \frac{dz}{dt} = u_z \quad (7)$$

from which  $\mathbf{v}(\mathbf{P}) = \mathbf{u} \quad (8)$

where  $\mathbf{v}(\mathbf{P})$  represents the speed of the point  $P$  with respect to  $S$  and its vector components are given by the (7). Deriving a second time we have

$$\frac{d^2x}{dt^2} = \frac{du_x}{dt} \quad \frac{d^2y}{dt^2} = \frac{du_y}{dt} \quad \frac{d^2z}{dt^2} = \frac{du_z}{dt} \quad (9)$$

from which  $\mathbf{a}(\mathbf{P}) = \mathbf{a}_u \quad (10)$

where  $\mathbf{a}(\mathbf{P})$  represents the acceleration of the point P with respect to S with the vector components given by the (9). The accelerated linear motion of the reference frame S' introduces for all points of S' a linear acceleration  $\mathbf{a}_u = \mathbf{a}(\mathbf{P})$  with respect to S. If the linear speed  $\mathbf{u}$  is constant the linear acceleration is zero

$$\mathbf{a}_u = \mathbf{0} \quad (11)$$

Suppose now that the material point P, which has mass  $m'$  with respect to S', is subjected to the force  $\mathbf{F}'$  that has the same direction  $\mathbf{u}$ ; the point takes on a speed  $\mathbf{v}'$  with respect to S' and a speed  $\mathbf{v}$  with respect to S and the two speeds  $\mathbf{v}'$  and  $\mathbf{v}$  have the same direction  $\mathbf{u}$  (fig.3).

In that case we can write, in the absence of external resistant forces to motion,  $\mathbf{F}' = m' d\mathbf{v}'/dt'$ .

The reference frame S' is non-inertial with respect to S because the speed  $\mathbf{u}$  is variable; then for the Principle of Inertia a linear force  $\mathbf{F}_L$  necessarily must cause this non-inertial motion  $\mathbf{u}$  and this force works on all points that are inside the reference frame S' and therefore also on the material point P, which has mass  $m$  with respect to S. We can write  $\mathbf{F}_L = m d\mathbf{u}/dt$  where  $\mathbf{a}_u = d\mathbf{u}/dt$  represents the linear acceleration of S' with respect to S.

The two forces  $\mathbf{F}'$  and  $\mathbf{F}_L$  have the same direction and the resultant force  $\mathbf{F} = m d\mathbf{v}/dt$  that acts on the material point with respect to the resting reference frame S is  $\mathbf{F} = \mathbf{F}' + \mathbf{F}_L$ . Replacing we have

$$m \frac{d\mathbf{v}}{dt} = m' \frac{d\mathbf{v}'}{dt'} + m \frac{d\mathbf{u}}{dt} \quad (12)$$

and because  $\mathbf{v} = \mathbf{v}' + \mathbf{u}$  we have then

$$m \frac{d\mathbf{v}'}{dt} = m' \frac{d\mathbf{v}'}{dt'} \quad (13)$$

from which

$$dt' = \frac{m'}{m} dt \quad (14)$$

The equations of space-time relativistic transformations for reference frames, also non-inertial, with any linear speed with respect to the resting reference frame, in the passage from S to S' are so

$$\begin{aligned} x' &= x - \int_0^t u_x dt \\ y' &= y - \int_0^t u_y dt \\ z' &= z - \int_0^t u_z dt \\ dt' &= \frac{m'}{m} dt \end{aligned} \quad (15)$$

If  $u_y = u_z = 0$ , the speed  $u = u_x$  has the direction of the axis x, and the (15) become

$$\begin{aligned}
x' &= x - \int_0^t u dt \\
y' &= y \\
z' &= z \\
dt' &= \frac{m'}{m} dt
\end{aligned}
\tag{16}$$

If the speed  $u$  is constant we have  $\mathbf{F}_L=0$  and

$$\begin{aligned}
x' &= x - u t \\
y' &= y \\
z' &= z \\
dt' &= \frac{m'}{m} dt
\end{aligned}
\tag{17}$$

If then the mass of the material point doesn't change with speed, as it happens for masses of classical bodies of mechanical type, we have  $m'=m$  and therefore

$$\begin{aligned}
x' &= x - u t \\
y' &= y \\
z' &= z \\
t' &= t \quad (\text{inertial time})
\end{aligned}
\tag{18}$$

The equations of space-time transformations represented by (18), valid in very restricted physical conditions, match Galileo's equations of space-time transformations with one only difference: time considered in the Galilean transformations coincided with the Galilean-Newtonian absolute time while in the (18) the considered time is the common inertial time to all inertial reference frames with respect to  $S$ . The inertial time is valid for inertial masses which don't change with the speed. For massive elementary particles we have introduced<sup>[3,5,8]</sup> the electrodynamic mass and we have proved (see also afterwards paragraph 7.b) there is a real variation of electrodynamic mass caused by the motion given by

$$m = m' \left( 1 - \frac{u^2}{2c^2} \right)
\tag{19}$$

in which  $m'$  is the resting electrodynamic mass and  $m$  is the moving electrodynamic mass. Put  $\beta=u/c$ , the (17) become for  $u$  constant

$$\begin{aligned}
x' &= x - u t \\
y' &= y \\
z' &= z \\
t' &= \frac{t}{1 - \frac{\beta^2}{2}}
\end{aligned}
\tag{20}$$

In particular the equation of time transformation in the (20) demonstrates that a time relativistic effect on electrodynamic particles exists in the sense that if  $t'$  is particle's its own time, the calculated or measured time  $t$  of the same particle with respect to the resting reference frame  $S$ , which is the observer's reference frame, is given by  $t=(1-\beta^2/2)t'$ . It means that average lives of particles measured by the laboratory's observer are different from the real average lives which nevertheless are physically little revealing and can be calculated using the last of (20). From this relationship we deduce also that real average lives, for smaller speeds than the critical speed  $u < \sqrt{2}c$ , are greater than measured those, and therefore measured average lives of particles undergo a contraction with respect to real average lives. It explain well the behaviour of cosmic muons<sup>[4]</sup>. At the critical speed  $u = \sqrt{2}c$  the particle's real average life is infinite and it involves a highest degree of intrinsic stability of particle. At this speed in fact the electrodynamic particle changes generating two energy quanta which have infinite intrinsic average life with a highest degree of stability. The residue of the initial particle has zero electrodynamic mass and also it is stable until is at the critical speed. The residual particle becomes unstable when it becomes a free particle and therefore subject to the Decay Principle<sup>[8,9,10]</sup>.

For  $u > \sqrt{2}c$  the (own) intrinsic time becomes negative, as electrodynamic mass, the free particle is strongly unstable and it is subject to physical consequences defined in previous papers<sup>[8,9,10,11]</sup>.

For inertial reference frames, and only for mechanical systems for which  $m'=m$ , we deduce from (18) two important kinematic properties. In the first place deriving with respect to coordinate  $t'=t$  the first of (18) we have

$$\mathbf{v}' = \mathbf{v} - \mathbf{u} \quad (21)$$

The relation (21) defines the "theorem of not invariance of speeds for inertial reference frames". In actuality this theorem is valid in different shape also for electrodynamic systems (from (17)):

$$\mathbf{v}' = \frac{m}{m'} (\mathbf{v} - \mathbf{u}) \quad (22)$$

Deriving again the (21) with respect to the coordinate  $t$  we obtain

$$\mathbf{a}' = \mathbf{a} \quad (23)$$

The relation (23) defines the "theorem of invariance of accelerations for inertial reference frames", valid only for mechanical systems.

In fact as per the (22) for electrodynamic systems we have

$$\mathbf{a}' = \frac{m}{m'} \mathbf{a} \quad (24)$$

and therefore accelerations for electrodynamic systems are non-invariant.

Electromagnetic systems (waves and energy particles) don't have mass and therefore for they there isn't mass variation and time variation: they have the same inertial time with respect to any reference frame. For these systems the (21) becomes  $\mathbf{c}' = \mathbf{c} - \mathbf{u}$  in which  $\mathbf{c}'$  is the physical speed in  $S'$  and  $\mathbf{c}$  is the relativistic speed<sup>[1,2]</sup> in  $S$ .

## 6. Kinematics of non-linear Reference Frames

Non-linear reference frames are reference frames which move with non-rectilinear speed. Generally curvilinear (non-rectilinear) motion happens around an axis whose point of central symmetry is called "pole". We distinguish two cases:

- 6a. Rotary Reference Frames
- 6b. Orbital Reference Frames

### 6a. Rotary Reference Frames

Rotary Reference Frames are those reference frames  $S'[O',x',y',z',t']$  which are endowed with rotary motion with angular velocity  $\omega_r$  with respect to the resting reference frame  $S[O,x,y,z,t]$ , in which  $O=O'$  represents the pole of motion.

Suppose that the rotary motion happens around the axis  $z$  and therefore only the coordinates  $x$  and  $y$  are revealing and besides that the angular velocity  $\omega_r$  is constant (fig.4). Suppose also that at the initial time  $t=t'=0$  the two reference frames are perfectly coincident with regard to both the origin and axes and therefore, if  $r$  is the distance of the point  $P$  from the origin, we have for  $t=t'=0$

$$\begin{aligned} x &= x' = r \cos \alpha_0 \\ y &= y' = r \sin \alpha_0 \\ z &= z' \end{aligned} \quad (25)$$

For time instants  $t > 0$  the physical situation is represented in fig.4. We deduce

$$\begin{aligned} x &= r \cos(\alpha_0 + \alpha) \\ y &= r \sin(\alpha_0 + \alpha) \\ z &= z' \end{aligned} \quad (26)$$

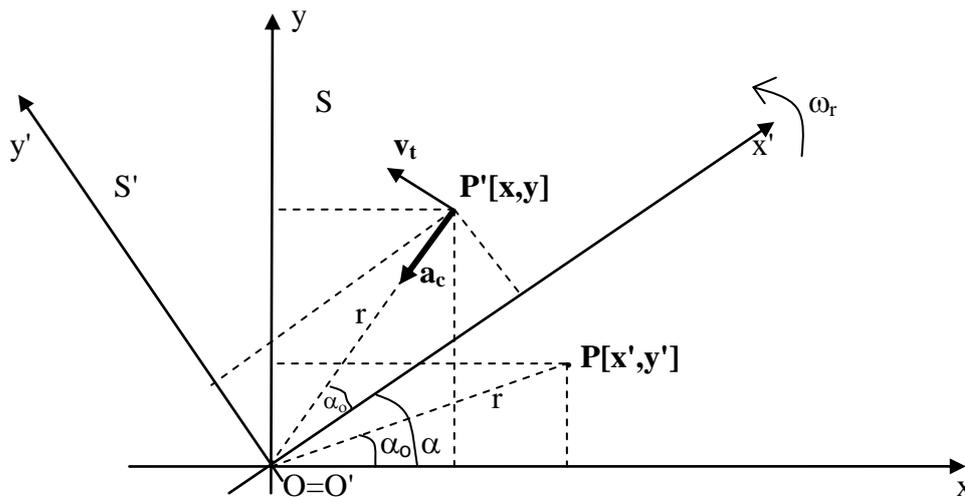


Fig.4 The reference frame  $S'$  is moving with angular velocity  $\omega_r$  in the plane  $(x,y)$  with respect to the axis  $z$ , which isn't represented in figure. The point  $P$  is inside the reference frame  $S'$ . At the time  $t=t'=0$  the two reference frames and the points  $P$  and  $P'$  coincide. At the instant  $t > 0$   $S'$  undergoes a rotation and  $P$  moves to  $P'$ .

Consequently, being  $\alpha=\omega_r t$ , equations of transformations of space coordinates for rotary reference frames with constant angular velocity  $\omega_r$  are

$$\begin{aligned}x &= x' \cos \omega_r t - y' \sin \omega_r t \\y &= y' \cos \omega_r t + x' \sin \omega_r t \\z &= z'\end{aligned}\tag{27}$$

Deriving with respect to time  $t$  the space coordinates  $x$  and  $y$ , because  $x',y',z'$  are constant, we have

$$v_x = -\omega_r y ; \quad v_y = \omega_r x ; \quad v_z = 0\tag{28}$$

The (28) represent the components of the linear kinematic speed in the rotary motion with respect to the resting reference frame. Deriving a second time we obtain the components of the linear kinematic acceleration generated by the rotary motion with respect to the resting reference frame S

$$a_x = -\omega_r^2 x \quad a_y = -\omega_r^2 y \quad a_z = 0\tag{29}$$

Analysing the (28) and (29) we understand the (28) represent the components of the tangential velocity  $\mathbf{v}_t$  and the (29) represent the components of Coriolis acceleration  $\mathbf{a}_c = -\omega_r^2 \mathbf{r}$ , which matches the centripetal acceleration of the motion. We notice still that while in linear reference frames acceleration is zero when the relative speed between the two reference frames is constant, the Coriolis acceleration is always different from zero also when the relative angular velocity between the two rotary reference frames is constant.

In order to achieve the equation of time transformation in the rotary reference frames it is necessary to consider what happens if the material point P' undergoes in S' the action of a torque (of forces) with moment of couple  $C'=J'd\omega'/dt'$  where  $\omega'$  is the angular velocity around the axis  $z'=z$  and J' is the moment of inertia in S'. With respect to the resting reference frame it is  $C=Jd\omega/dt$  where J and  $\omega$  are the moment of inertia of the material point and the angular velocity in S with respect to the axis of rotation  $z=z'$ .

Because the centripetal force, which is generated by the Coriolis acceleration, is balanced in the rotary motion by the centrifugal force, we can write  $C=C'$ ; considering that  $\omega=\omega'+\omega_r$ , we have

$$dt = \frac{J}{J'} dt'\tag{30}$$

The (30) represents the equation of time relativistic transformation with respect to rotary reference frames. The space-time relativistic transformations for rotary reference frames are therefore

$$\begin{aligned}x &= x' \cos \omega_r t - y' \sin \omega_r t \\y &= y' \cos \omega_r t + x' \sin \omega_r t \\z &= z' \\dt &= \frac{J}{J'} dt'\end{aligned}\tag{31}$$

and because the moment of inertia, for mechanical classical bodies, undergoes no variation when the angular velocity changes, we have the following equations of space-time transformations for rotary reference frames

$$\begin{aligned}
 x &= x' \cos \omega_r t - y' \sin \omega_r t \\
 y &= y' \cos \omega_r t + x' \sin \omega_r t \\
 z &= z' \\
 t &= t'
 \end{aligned}
 \tag{32}$$

in which  $t=t'$  represents the inertial time. We achieve the important result that rotary reference frames around the same axis are inertial with respect to the resting reference frame. This conclusion proves the validity of the Generalized Principle of Inertia and at the same time explains the result of the Sagnac experiment.

If the rotating system is an elementary particle rotating around its own axis, the moment of couple  $p_s = \hbar/2\pi r$  and the angular momentum (spin)  $q_s = \hbar/2\pi$  (the spin quantum number  $s$  equals  $1/2$  when particle is free<sup>[8,12]</sup>) are independent of the angular velocity. Consequently the particle's own time is inertial with respect all rotary reference frames around its own axis.

**6b. Orbital Reference Frames**

Orbital reference frames are those reference frames  $S'[O',x',y',z',t']$  that are endowed with orbital motion and angular velocity  $\omega_o$  with respect to the resting reference frame  $S[O,x,y,z,t]$ , where  $O$  represents the pole of motion. Suppose that orbital motion happens around the axis  $z$  and therefore only coordinates  $x$  and  $y$  are revealing. More suppose that motion is circular with radius  $d$  and that orbital angular velocity  $\omega_o$  is constant. Suppose at last that at the time  $t=t'=0$ , the origin  $O'$  of the moving reference frame  $S'$  is in the point  $Q$  (fig.5).

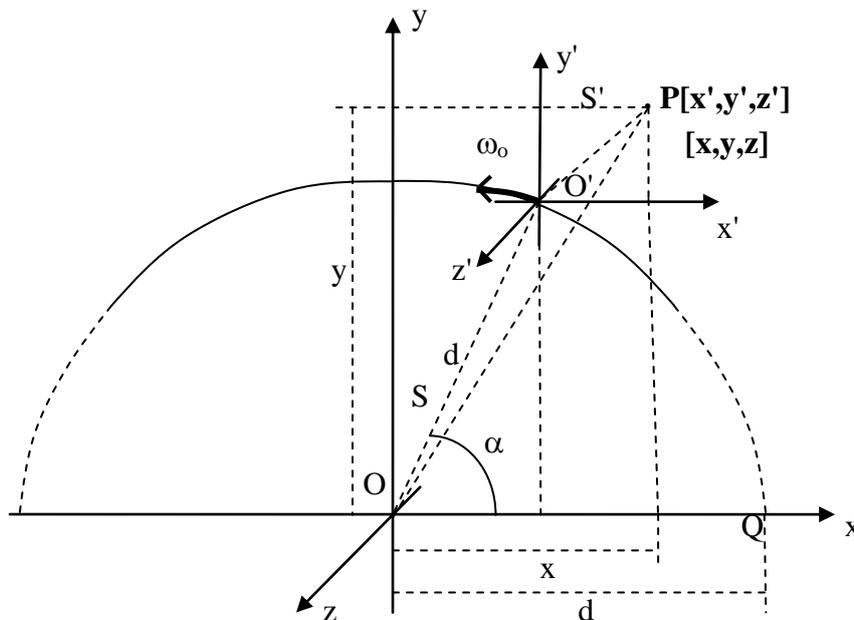


Fig.5 Representation of orbital motion of the reference frame  $S'$  with respect to the resting reference frame  $S$ . Suppose that orbital motion is circular and that the orbital angular velocity is constant.

If the orbital radius is  $d$  and  $\alpha = \omega_0 t$ , from the figure we deduce

$$\begin{aligned}x &= x' + d \cos \omega_0 t \\y &= y' + d \sin \omega_0 t \\z &= z'\end{aligned}\tag{33}$$

The (33) represent the transformations equations of space coordinates for orbital reference frames with constant circular orbital velocity  $\omega_0$ . Deriving with respect to time  $t$  we have the components of the tangential velocity

$$\begin{aligned}v_x &= -d\omega_0 \sin \omega_0 t = -\omega_0(y - y') \\v_y &= d\omega_0 \cos \omega_0 t = \omega_0(x - x') \\v_z &= 0\end{aligned}\tag{34}$$

and deriving a second time we achieve the components of the kinematic acceleration produced by the orbital motion with respect to the resting reference frame  $S$

$$\begin{aligned}a_x &= -\omega_0^2 (x - x') \\a_y &= -\omega_0^2 (y - y') \\a_z &= 0\end{aligned}\tag{35}$$

The (35) represent the components of the Coriolis orbital acceleration, which is different from zero also when the orbital velocity is constant.

Repeating for orbital reference frames the same reasoning which has been made for rotary reference frames, we have space-time relativistic transformations for orbital reference frames

$$\begin{aligned}x &= x' + d \cos \omega_0 t \\y &= y' + d \sin \omega_0 t \\z &= z' \\dt &= \frac{J}{J'} dt'\end{aligned}\tag{36}$$

and because moment of inertia, for mechanical classical bodies, undergoes no change during the process of revolution, also when the orbital angular velocity changes, we achieve the following equations of space-time transformations for orbital reference frames

$$\begin{aligned}x &= x' + d \cos \omega_0 t \\y &= y' + d \sin \omega_0 t \\z &= z' \\t &= t'\end{aligned}\tag{37}$$

in which  $t = t'$  represents the inertial time. Also orbital motions are inertial with respect to  $S$ .

In real physical situations S is endowed with a rotary motion  $\omega_r$  but in the geostationary satellite orbital reference frame the effect concerning this rotary motion cancels out assuming  $\omega_o = \omega_r$ . Although both the reference frame S and the orbital reference frame S' are inertial and synchronized at first with the same inertial time, a periodical synchronization of two system clocks<sup>[1]</sup> needs because of loss of synchronization for various reasons independent of motion. In fig.6 O represents the pole of orbital motion and P=O' represents the position of the geostationary satellite at the time  $t_o$ , c is the speed of electromagnetic signal with respect to the resting reference frame. The signal leaves the pole O at time  $t=0$  and reaches the point P in the time  $t_1=d/c$  in S. With respect to the moving reference frame, which moves with tangential velocity v, the signal in order to cover the same distance d spends a time

$$t_1' = \frac{d}{\sqrt{c^2 + v^2}} < t_1 \quad (38)$$

For the periodical synchronization of the orbital clock a synchronization delay is therefore necessary given by  $\Delta t = t_1 - t_1'$ . For  $v \ll c$  the synchronization delay is with good approximation

$$\Delta t = \frac{t_1 v^2}{2c^2} \quad (39)$$

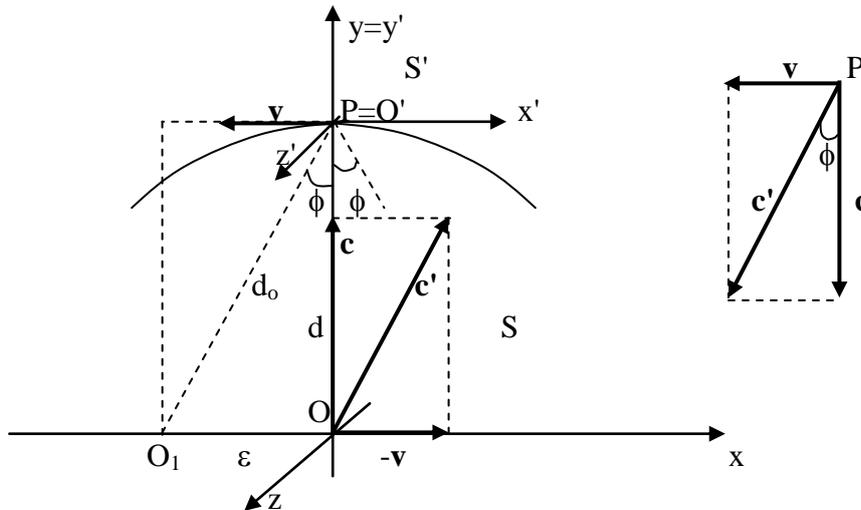


Fig.6 The geostationary satellite station is in the point P at the time  $t_o$ .

Suppose now that the orbital moving reference frame S' (geostationary station) sends from the position P an electromagnetic signal towards the pole O. In the resting reference frame the signal covers the distance  $d_o$  and therefore it reaches the position  $O_1$  instead of the position O with a space relativistic error given by  $\epsilon = vd/c$ . For a tangential orbital velocity  $v = 14000 \text{ km/h}$  and for an orbital distance  $d = 20000 \text{ km}$  the space relativistic error is  $\epsilon \approx 259 \text{ m}$ . In order to eliminate this error the signal must be sent with a "retro-displacement angle"

$$\phi = \text{arctg} \frac{v}{c} \quad (40)$$

## 7. Dynamics and Electrodynamics of physical processes with linear symmetry

Physical Process is described by a Physical Law. The Physical Law describes the evolution of a physical system through a mathematical relation between cause and effect that represents the mathematical model of the process. Let us consider now a few fundamental physical processes with linear symmetry whose motion is caused by a linear accelerating force.

For analysis of these processes we suppose, as per the Principle of Reference, process happens inside a reference frame that represents the laboratory reference frame and consequently the preferred reference frame. The description of the same process with respect to another reference frame is made as per the Theory of Reference Frames and in the event of inertial systems in obedience to the Principle of Relativity.

### 7a. Law of linear motion for mechanical systems

We have demonstrated<sup>[3]</sup> the general law of rectilinear motion of a mechanical system with mass  $m$ , under the action of a constant force  $F$ , considering whether the internal resistant force (force of inertia) or external resistant forces (resistant forces of medium where motion happens), is given by the following equation

$$F = m \frac{dv(t)}{dt} + kv(t) \quad (41)$$

in which  $k$  is the "resistant coefficient of medium" and  $v(t)$  is the system speed.

The general equation of motion is given therefore by a linear differential equation of first order and it represents the mathematical model of the considered system with respect to the supposed resting reference frame  $S[O,x,y,z,t]$ , which coincides with the laboratory reference frame.

Supposing that the mechanical system is initially resting and so  $v(0)=0$  for  $t=0$ , the solution of the differential equation is given by

$$v(t) = \frac{F}{k} \left( 1 - e^{-kt/m} \right) \quad (42)$$

in which  $V_o=F/k$  is the final speed of the system: it follows that the final speed of a mechanical system can be limited only by the applied force or by the resistant coefficient of medium. Besides the quantity

$$T = \frac{m}{k} \quad (43)$$

is the "time mechanical constant" of the system.

Charting the motion law we have the diagram of fig.7.

By the diagram we observe the physical effect of a constant force, applied to a mechanical system with mass  $m$ , is in real physical conditions a variable acceleration and therefore the considered motion is a non-uniform various motion.

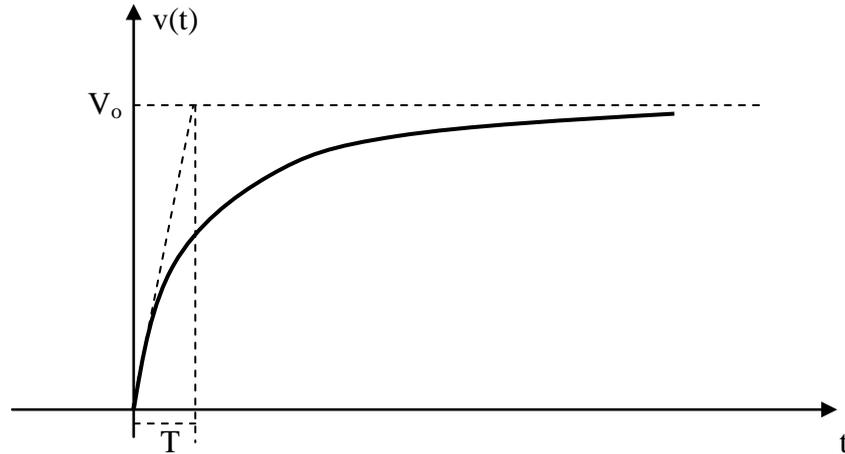


Fig.7 Time graphic trend of the speed of a mechanical system under the action of a constant force F.

We can observe also the speed of the system depends strongly on the mass only in the initial phase of acceleration and this dependence decreases as the system speed increases; it tends towards a practically constant final velocity which depends only on the resistant coefficient  $k$ , besides on the applied force  $F$ .

As per the Principle of Relativity the general law of linear motion for mechanical systems must be invariant for all inertial reference frames. With respect to any reference frame  $S'[O',x',y',z',t']$ , endowed with inertial motion with respect to  $S$ , the general law of motion is therefore

$$F' = m' \frac{dv'(t')}{dt'} + k'v'(t') \quad (44)$$

Supposing that reference frames  $S$  and  $S'$  tally at the initial time  $t=t'=0$ , supposing still that the moving system  $S'$  moves with constant speed  $u$  towards increasing  $x$  and that also the speed  $v$  is directed towards increasing  $x$  (fig.8), the equations of motion transformation from  $S$  to  $S'$  are, according to the (18),

$$\begin{aligned} t' &= t && \text{(inertial time)} \\ x' &= x - u t \\ y' &= y \\ z' &= z \\ m' &= m \\ F' &= F \\ v' &= v - u \end{aligned} \quad (45)$$

As per (45) the (44) becomes

$$F = m \frac{dv(t)}{dt} + k'(v(t) - u) \quad (46)$$

and so making a comparison between (46) and (41) we have

$$k'(v - u) = k v \quad (47)$$

Since  $u/v = \beta$ , it is

$$k' = \frac{k}{1 - \beta} \quad (48)$$

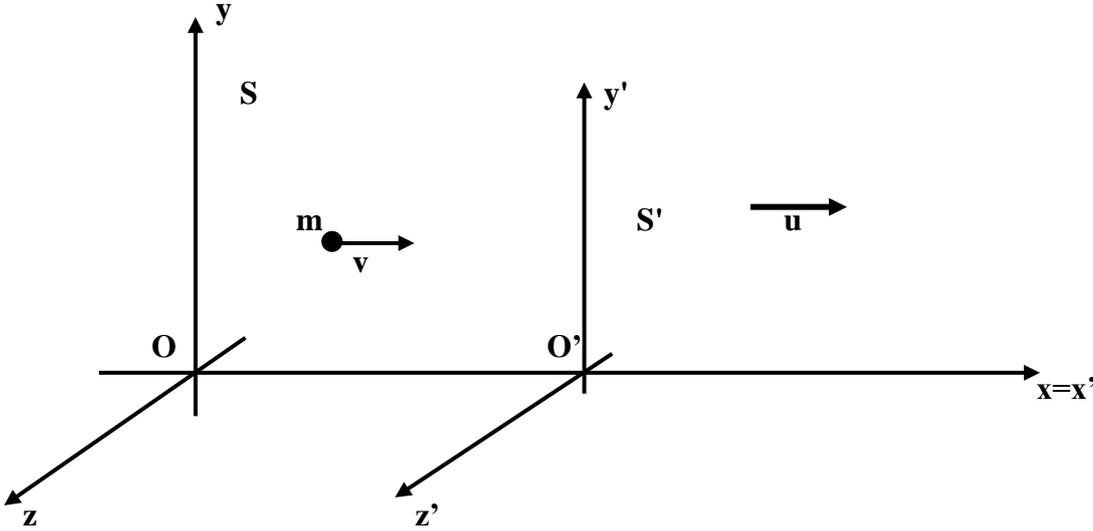


Fig.8 Graphic representation of motion of a mechanical system with mass  $m$  with respect to two reference frames  $S$  and  $S'$ , in which the resting reference frame  $S$  coincides with the laboratory frame.

The (48) is the transformation equation of the resistant coefficient for inertial systems.

When external resistant forces are null ( $k=0$ ) or smallest and insignificant ( $k \approx 0$ ) the general law of motion (41) matches Newton law  $F = m dv(t)/dt$ . The motion general law (41) represents a synthesis between the Newtonian physics and the Aristotelian physics.

### 7b. Law of linear motion for electrodynamic systems

Let us consider now the motion of massive elementary particles that are different microphysical systems from mechanical systems as they are endowed also with an elementary electric charge<sup>[3,5,8]</sup>. We know electrodynamic mass of charged elementary particles is sensitive to electric or magnetic forces and therefore it is different from mass of mechanical systems which is instead sensitive to mechanical forces.

If a charged elementary particle, with resting electrodynamic mass  $m'$ , is accelerated through a constant electric force, it emits radiant energy in the shape of energy quanta under physical conditions that we will clarify afterwards. We have demonstrated it in preceding papers<sup>[3,5]</sup>: here we reproduce now the brief shape.

Let us consider a stable charged massive elementary particle, for instance an electron, which has a resting electrodynamic mass  $m'$  and consequently an intrinsic energy  $E_i' = m'c^2$ . If electron is accelerated by an electric force it gains a speed  $v$  and an equivalent kinetic energy  $E_c = m'v^2/2$ ; we know also it emits electromagnetic energy and it can happen only at the expense of electrodynamic mass of particle. With respect to the resting reference frame  $S[O, x, y, z, t]$  the moving particle has therefore an electrodynamic mass  $m$  and an intrinsic energy  $E_i = mc^2$  for which effecting the energy balance we have

$$mc^2 = m'c^2 - \frac{m'v^2}{2} \quad (49)$$

from which

$$m = m' \left( 1 - \frac{v^2}{2c^2} \right) \quad (50)$$

The (50) coincides with the (19) and it represents the variation law of electrodynamic mass of an elementary particle with respect to the speed.

For  $v=0$   $m=m'$ , at any speed  $v$  different from zero particle's moving mass is different from resting mass and the mass difference is in the shape of stored electromagnetic energy in the particle.

For  $v<c$  moving mass  $m$  is positive and smaller than resting mass. When accelerating force finishes its action particle returns to its resting state with the emission of an e.m. energy quantum whose frequency is given by

$$f = \frac{\Delta m c^2}{h} = \frac{m' v^2}{2h} \quad (51)$$

Emission frequency depends on the final speed of particle and generally it belongs to the frequency band of X rays (for  $v<c$ ).

For  $v=c$  the particle's transformation begins and under the action of accelerating force it emits a first energy quantum at the speed of light and a second energy quantum at the critical speed  $v_c = \sqrt{2} c$  and these emissions happen at the expense of electrodynamic mass of particle which at the speed of light becomes half and at the critical speed becomes zero: consequently we understand the particle doesn't have disappeared but it has only experienced a transformation<sup>[8]</sup> and this process has quantum nature. The two energy quanta emitted at the speed of light and at the critical speed belong to the frequency band of gamma rays.

For greater speeds than the critical speed ( $v>v_c$ ) the particle's electrodynamic mass becomes negative with the initiation of instability problems, consequent processes of decay and emission of energy quanta belonging to the frequency band of delta rays<sup>[8,9,10]</sup>.

If accelerated particle instead of being a lepton, like electron, is a baryon, like proton, all process happens similarly with the only difference that the two emitted energy quanta at the speed of light and to the critical speed belong to delta radiation<sup>[11]</sup>.

## 8. Dynamics and Electrodynamics of physical processes with central symmetry

The motion with central symmetry<sup>[3]</sup> is generated by a force field in which a central pole causes the system symmetry. Motion in that case can be linear, like for the gravitational motion, or non-linear, as for orbital and rotary motions.

The central pole can be characterized physically by rotating mass of a body or by an electric charge. In our considerations the resting reference frame  $S[O,x,y,z,t]$  can have the origin  $O$  coinciding with the fixed pole of the field with central symmetry or coinciding with a moving point.

Points of the axis of rotation are immobile and don't take part in the motion.

**8a. Dynamics of gravitational motion**

If the central system is a system endowed with rotating static mass  $M_0$ , the force field generated by this mass is the gravitational field which causes an attraction force at distance on any secondary system endowed with static mass  $m_0$  where  $M_0 \gg m_0$  (fig.9).

Let us consider a supposed resting reference frame  $S[O,x,y,z,t]$  placed on the surface of the rotating mass so as simulating the laboratory reference frame.

In that case we must ignore the Coriolis force because the supposed resting reference frame joins in the rotary motion and the Coriolis acceleration is zero.

If the secondary mass  $m_0$  is free and subject to no bond, the gravitational force  $F_g = GM_0m_0/r^2$  generates the fall of the mass  $m_0$  with speed  $v(t)$ , with respect to the laboratory reference frame  $S$ , on the surface of the central mass in accordance with the direction which joins barycentres of two masses.

The free fall of the mass  $m_0$  is described by the general law of gravitational motion<sup>[3]</sup>

$$m_0 \frac{dv(t)}{dt} + kv(t) = \frac{G M_0 m_0}{r^2} \tag{52}$$

The (52) represents the mathematical model in scalar shape of the gravitational motion,  $G$  is the "gravitational constant",  $k$  is the resistant coefficient of medium and  $r$  is the distance between the barycentres of two masses.

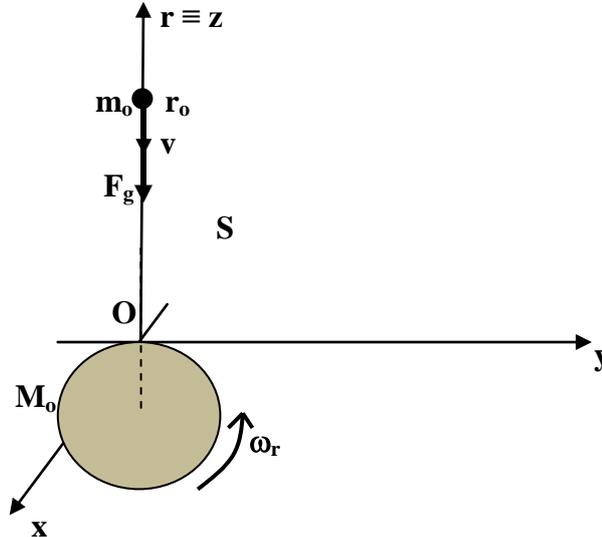


Fig.9 Representation of gravitational motion in which the supposed resting reference frame is on the surface of the mass  $M_0$  and therefore rotating with it.

From the equation (52) we deduce that the fall speed  $v(t)$  depends on the mass  $m_0$  of the secondary system because of the presence of external resistant forces. If these are null ( $k=0$ ), the fall speed is independent of the mass  $m_0$ , as it is proved by Newton's experience. In these conditions ( $k=0$ ) we have

$$\frac{dv(t)}{dt} = \frac{G M_o}{r^2} \quad (53)$$

Supposing that at the initial time  $t=0$  the mass  $m_o$  is at the distance  $r_o$  with null initial speed  $v(0)=v(r_o)=0$ , and considering that  $dv/dt=-v dv/dr$ , the integration of the (53) in the domain  $r$  has the following result

$$v(r) = \sqrt{\frac{2 G M_o}{r_o} \frac{r_o - r}{r}} \quad (54)$$

Charting the (54) we have the graph of fig.10. We can observe in figure that in the initial part of fall (part A-B) the mass motion is with good approximation accelerated in a constant manner<sup>[3]</sup>. Besides  $dv/dt$  is the time acceleration and  $dv/dr$  is the space acceleration.

If instead the resting reference frame  $S$  has the origin in the fixed point coinciding with the barycentre of the pole it doesn't participate in the rotary motion and then in that case it is necessary to consider the Coriolis force that depends on the latitude of the initial point of fall. The ideal observer placed in this reference frame would observe a curved non-linear motion.

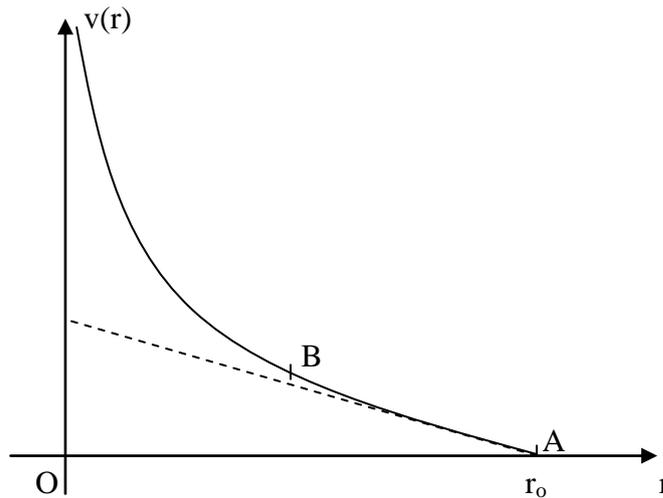


Fig.10 Trend of the speed of a body in free fall in the absence of external resistant forces.

### 8b. *Elettrodynamics of an elementary particle in a central field*

The central system is composed of a constant positive electric charge  $+Q$  (pole) which generates an electrostatic field with central symmetry<sup>[3]</sup>. An elementary particle with charge  $-q$  experiences an attractive force given by Coulomb's law

$$F = \frac{Q q}{4 \pi \epsilon_o r^2} \quad (55)$$

The motion law of elementary particle in the field is

$$m_0 \frac{dv(t)}{dt} + kv(t) = \frac{Q q}{4 \pi \epsilon_0 r^2} \quad (56)$$

where  $m_0$  is the particle's resting electrodynamic mass. Supposing of neglecting external resistant forces ( $k=0$ ) we have

$$m_0 \frac{dv(t)}{dt} = \frac{Q q}{4 \pi \epsilon_0 r^2} \quad (57)$$

Supposing then that at the initial time  $t=0$  particle is at practically infinite distance from the central charge ( $r_0=\infty$ ) with null initial speed  $v(t=0)=v(r_0=\infty)=0$ , the (57) gives for every distance  $r$  the following solution relative to the space speed

$$v(r) = \sqrt{\frac{Q q}{2 \pi \epsilon_0 m_0 r}} \quad (58)$$

Elementary particle motion isn't influenced by the Coriolis acceleration if we consider the motion with respect to the laboratory resting reference frame.

The (58) claims particle's speed increases as it nears the central charge. If an elementary particle is accelerated its electrodynamic mass changes with the speed in accordance with the relation

$$m = m_0 \left[ 1 - \frac{v^2}{2c^2} \right] \quad (59)$$

coinciding with the con la (50) when  $m'=m_0$ , from which

$$m = m_0 \left[ 1 - \frac{Q q}{4\pi \epsilon_0 m_0 r c^2} \right] \quad (60)$$

In mechanical systems the work performed by the force field transforms to kinetic energy, for elementary particles the work performed by the electric force field transforms to radiant energy at the expense of particle's electrodynamic mass. We observe in addition that at the critical distance

$$r_c = \frac{Q q}{4\pi \epsilon_0 m_0 c^2} \quad (61)$$

electrodynamic mass is null ( $m=0$ ) and it transforms completely to energy. If both the central charge  $+Q$  and the secondary charge  $-q$  have elementary electric charge equal to electron charge we have  $r_c=2,8 \times 10^{-15} \text{m}$ , which coincides with the radius of atomic nucleus.

The distance  $r_c$  represents the critical distance because at this distance from the central charge the particle's electrodynamic mass disappears completely, for greater distances electrodynamic mass is positive and for smaller distances is negative. At the critical distance the particle's speed equals the critical speed  $v_c = \sqrt{2} c$ .

## 9. Electromagnetic Process

As electromagnetic field is described generally in vector shape it is suitable to do a few considerations on symbols of vector calculus that are used in this paper:

- considering any vector  $\mathbf{A}$ , "the divergence of A" is represented by  $\text{div}\mathbf{A}$
- "the curl of  $\mathbf{A}$ " is represented by  $\text{rot}\mathbf{A}$
- considering two vectors  $\mathbf{A}$  and  $\mathbf{B}$ , vector product is represented by  $\mathbf{A}\wedge\mathbf{B}$ .

According to the Lorentz force and neglecting the equation  $\text{div}\mathbf{B}=0$ , which is little revealing as it expresses an always satisfied identity, the Maxwell equations can be written with respect to the supposed resting Einsteinian reference frame  $S[O,x,y,z,t]$  in the rationalized shape<sup>[1,13]</sup>

$$\text{div } \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (\text{Gauss-Poisson's law}) \quad (62)$$

$$\text{rot } \mathbf{E} = - \frac{\delta \mathbf{B}}{\delta t} \quad (\text{Faraday-Neumann-Lenz's law}) \quad (63)$$

$$\text{rot } \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\delta \mathbf{E}}{\delta t} \quad (\text{Ampere-Maxwell's law}) \quad (64)$$

$$\mathbf{E}_L = \mathbf{u}\wedge\mathbf{B} \quad (\text{Lorentz's law}) \quad (65)$$

in which  $\mathbf{u}$  is the speed of charged electrodynamic particle (for example electron) and  $c$  is the "physical speed" of light and e.m. waves with respect to the reference frame  $S$ , where the electromagnetic process is born and propagates. This group of equations is said rationalized because it is complete and it is able to describe any electromagnetic phenomenon which isn't possible by the historical group of Maxwell's equations where the equation (65) is absent.  $\mathbf{E}_L$  represents the Lorentz field.

As per the Principle of Relativity the Maxwell equations with respect to the moving inertial reference system  $S'[O',x',y',z',t']$  are (fig.11)

$$\text{div } \mathbf{E}' = \frac{\rho'}{\epsilon_0} \quad (66)$$

$$\text{rot } \mathbf{E}' = - \frac{\delta \mathbf{B}'}{\delta t'} \quad (67)$$

$$\text{rot } \mathbf{B}' = \mu_0 \mathbf{J}' + \frac{1}{c'^2} \frac{\delta \mathbf{E}'}{\delta t'} \quad (68)$$

$$\mathbf{E}_L' = (\mathbf{u}+\mathbf{v})\wedge\mathbf{B} \quad (69)$$

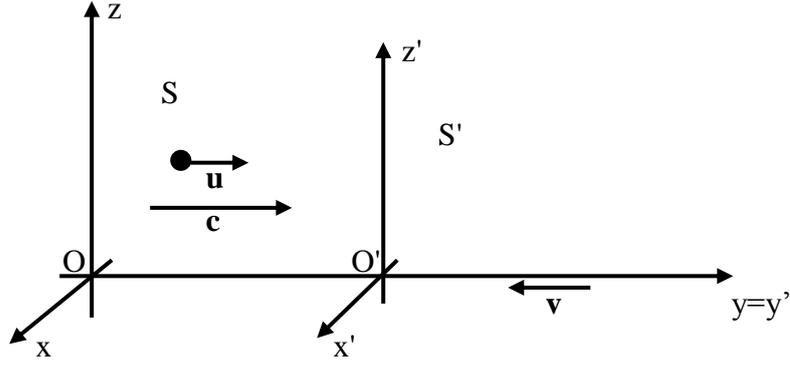


Fig.11 The Einsteinian reference frame S is at rest and the Einsteinian reference frame S' is moving with speed  $v$ .

The (69) gives the transformation equation of the Lorentz field for inertial reference frames. Considering then the Lorentz field  $\mathbf{E}_L$  like additional induced electric field due to cut flux, in the absence of particle ( $\mathbf{u}=0$ ), the (69) becomes

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \wedge \mathbf{B} \quad (69')$$

We have demonstrated<sup>[1,13]</sup> that Maxwell's equations are invariant with respect to the two reference frames through the following transformations

$$\begin{aligned} t' &= t && \text{(inertial time)} \\ c' &= c + v && \text{(relativistic speed)} \\ \rho' &= \rho + \rho_L \end{aligned}$$

$$\mathbf{B}' = \mathbf{B} - \int_0^t \text{rot} \mathbf{v} \wedge \mathbf{B} \, \delta t \quad (70)$$

$$\mathbf{J}' = \mathbf{J} + \epsilon_0 \frac{\delta \mathbf{E}}{\delta t} \left( 1 - \frac{1}{\left(1 + \frac{v}{c}\right)^2} \right) - \frac{\epsilon_0}{\left(1 + \frac{v}{c}\right)^2} \frac{\delta}{\delta t} \mathbf{v} \wedge \mathbf{B} - \frac{1}{\mu_0} \text{rot} \int_0^t \mathbf{v} \wedge \mathbf{B} \, \delta t \quad (71)$$

where  $\rho_L = \epsilon_0 \text{div} \mathbf{v} \wedge \mathbf{B}$ .

In a vacuum, where field sources are absent, it is necessary to put in equations  $\rho=0$  and  $\mathbf{J}=0$ . In the Theory of Reference Frames all elementary particles which have electromagnetic nature, from infrared rays until  $\delta$ -Y radiation, are electromagnetic nanowaves whose origin (fig.12) is described by the following equations deduced from Maxwell's equations<sup>[13]</sup>

$$\text{rot} \mathbf{b} = \frac{u_z}{c} \frac{\delta \mathbf{e}}{\delta z} + \mu_0 \mathbf{j} \quad (72)$$

$$\mathbf{e} = \mathbf{e}_0 + \mathbf{u} \wedge \mathbf{b} \quad (73)$$

where  $\mathbf{j}$  is the nanocurrent density and  $\mathbf{e}_0$  ( $\mathbf{e}_0 = \rho_t \mathbf{j}$ ,  $\rho_t$  is the electric resistivity) is the electric nanofield connected with the moving electron. In our case  $|\mathbf{u} \wedge \mathbf{b}| = ub$ , because the vectors  $\mathbf{u}$  and  $\mathbf{b}$  are perpendicular (it corrects  $\mathbf{u} \wedge \mathbf{b} = 0$  in ref.[13], page 7).

The following equations (74) and (75) describe instead the propagation of the electromagnetic nanowave connected with the single energy quantum with respect to the S resting reference system

$$\text{rot } \mathbf{e} = - \frac{\delta \mathbf{b}}{\delta t} \quad (74)$$

$$\text{rot } \mathbf{b} = \frac{1}{c} \frac{\delta \mathbf{e}}{\delta t} \quad (75)$$

The equations (72) and (73) represent "photon equations" in generation phase while the equations (74) and (75) represent "photon equations" in propagation phase.

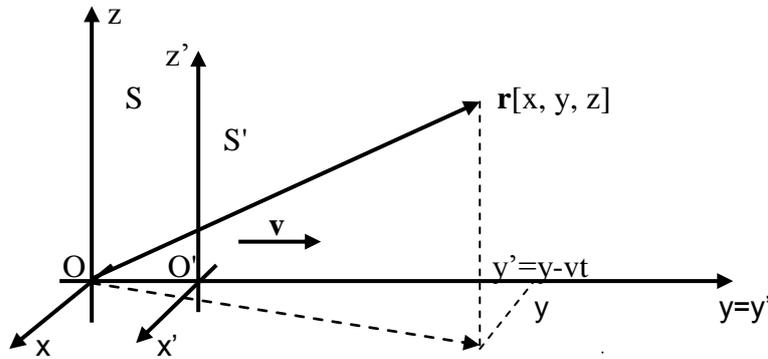


Fig.12 The accelerated electron, which generates the single energy quantum, moves with speed  $u_z = a_z t$  along the axis z. At initial time  $t=t'=0$  the two reference systems S and S' coincide. The single energy quantum moves along the axis y like the S' reference frame.

## 10. Frequency and Wavelength Shifts

Frequency and wavelength shifts for all electromagnetic phenomena have relativistic nature and they are due generally to the Doppler effect.

In fig.13 S[O,x,y,z,t] is the resting Einsteinian reference frame and S'[O,x',y',z',t'] is the moving reference frame, the relative speed  $\mathbf{v}$  is for the sake of argument parallel to the direction of the axis x,  $\Phi_1$  is the angle between the direction of the ray of light (or e.m. waves) and the direction  $\mathbf{v}$  when source is in O' and the observer is in O,  $\Phi_2$  is on the contrary the angle between the direction of the ray of light (or e.m. waves) and the direction  $\mathbf{v}$  when source is in O and the observer is in O'.

Analysis of frequency and wavelength shifts due to the Doppler effect<sup>[1]</sup> demonstrates equal results are attained in the two considered cases: a) resting observer and moving source, b) resting source and moving observer. It proves there is complete symmetry between the two physical situations; besides it proves also frequency and wavelength shifts depend only on the relative speed  $\mathbf{v}$  and don't depend on whom is moving.

If  $f_m$  and  $\lambda_m$  are frequency and wavelength measured by observer and  $f_s$  and  $\lambda_s$  are frequency and wavelength emitted by source, frequency and wavelengths shifts for every direction depend on the angle  $\Phi_2 = \pi - \Phi_1$  and are given by

$$f_m = f_s \sqrt{1 + \frac{v^2}{c^2} - \frac{2v}{c} \cos\Phi_2} \quad (76)$$

$$\lambda_m = \frac{\lambda_s}{\sqrt{1 + \frac{v^2}{c^2} - \frac{2v}{c} \cos\Phi_2}} \quad (77)$$

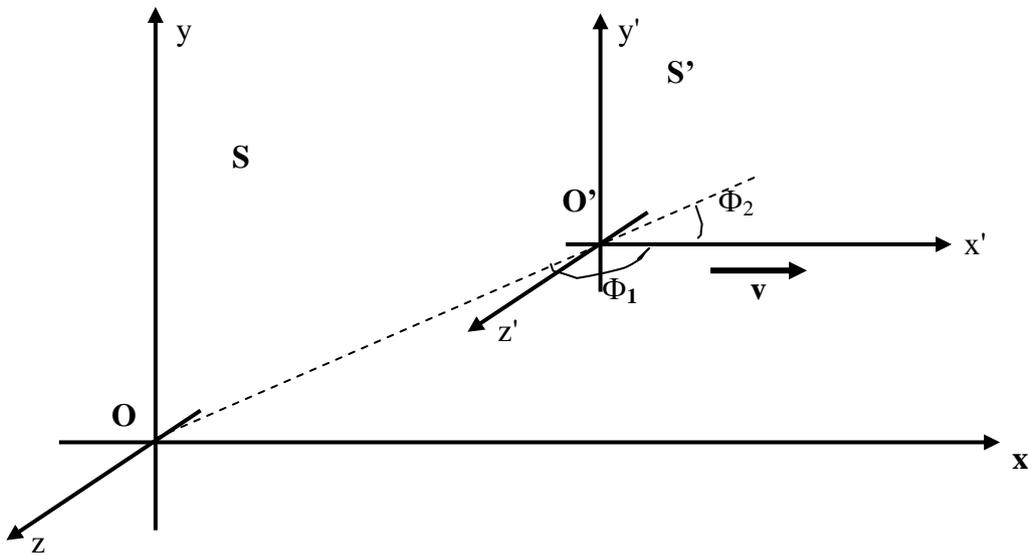


Fig.13 S[O,x,y,z] is the resting Einsteinian reference frame and S'[x',y',z',t'] is the moving reference frame. In the graph is  $\Phi_2 = \pi - \Phi_1$ .

Relations (76) and (77) can be demonstrated in the two cases making use of the fig.13.

**a. resting observer and moving source**

Moving source emits an electromagnetic radiation (waves or rays) with frequency  $f_s$  and wavelength  $\lambda_s$  so as  $\lambda_s f_s = c$  in which  $c$  is the physical speed of the radiation with respect to the moving reference frame  $S'$ . With respect to the resting reference frame  $S$  the relativistic speed  $c_r$  of the same radiation is (fig.14a)

$$c_r = \sqrt{c^2 + v^2 - 2vc \cos\Phi_2} \quad (78)$$

and therefore the frequency measured by the resting observer is

$$f_m = \frac{c_r}{\lambda_s} = f_s \sqrt{1 + \frac{v^2}{c^2} - \frac{2v}{c} \cos\Phi_2} \quad (79)$$

Because the physical speed of the radiation in S is  $c=\lambda_m f_m$  we have

$$\lambda_m = \frac{\lambda_s}{\sqrt{1 + \frac{v^2}{c^2} - \frac{2v}{c} \cos\Phi_2}} \quad (80)$$

The (79) and (80) coincide with the (76) and (77) and the Doppler effect in the case a. is proved.

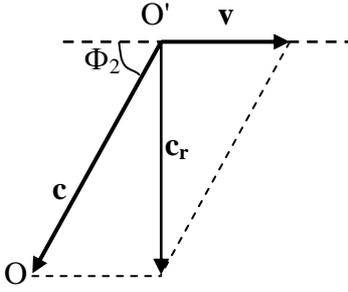


Fig.14a Observer at rest and moving source

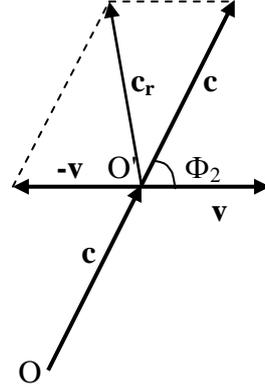


Fig.14b Source at rest and moving observer

**b. resting source and moving observer**

Resting source emits an electromagnetic radiation (waves or rays) with frequency  $f_s$  and wavelength  $\lambda_s$  so as  $\lambda_s f_s = c$  in which  $c$  is the physical speed of the radiation with respect to the resting reference frame S. With respect to the moving reference frame S' the relativistic speed  $c_r$  is (fig.14b)

$$c_r = \sqrt{c^2 + v^2 - 2vc \cos\Phi_2} \quad (81)$$

and repeating the same reasoning as in the case a., we achieve that the frequency and the wavelength measured by the moving observer are given again by the (76) and (77), where in that case the physical speed of the radiation in S' is  $c=\lambda_m f_m$ . The Doppler effect is therefore demonstrated also in the case b..

From relations (76) and (77) we can deduce in both cases, setting  $\Phi_2=\pi/2$ , the transversal Doppler effect for frequency and wavelength

$$f_m = f_s \sqrt{1 + \frac{v^2}{c^2}} \quad \text{e} \quad \lambda_m = \frac{\lambda_s}{\sqrt{1 + \frac{v^2}{c^2}}} \quad (82)$$

Similarly the longitudinal Doppler effect in frequency and wavelength, in both cases, is given by

$$f_m = f_s (1 \pm \beta) \quad \text{e} \quad \lambda_m = \frac{\lambda_s}{1 \pm \beta} \quad (83)$$

with  $\beta=v/c$ . The departure longitudinal Doppler effect happens for  $\Phi_2=0$ , in relations (83) it involves the negative sign and the physical effect is the redshift. The approach longitudinal Doppler effect happens for  $\Phi_2=\pi$ , in relations (83) it involves the positive sign and the physical effect is the blueshift.

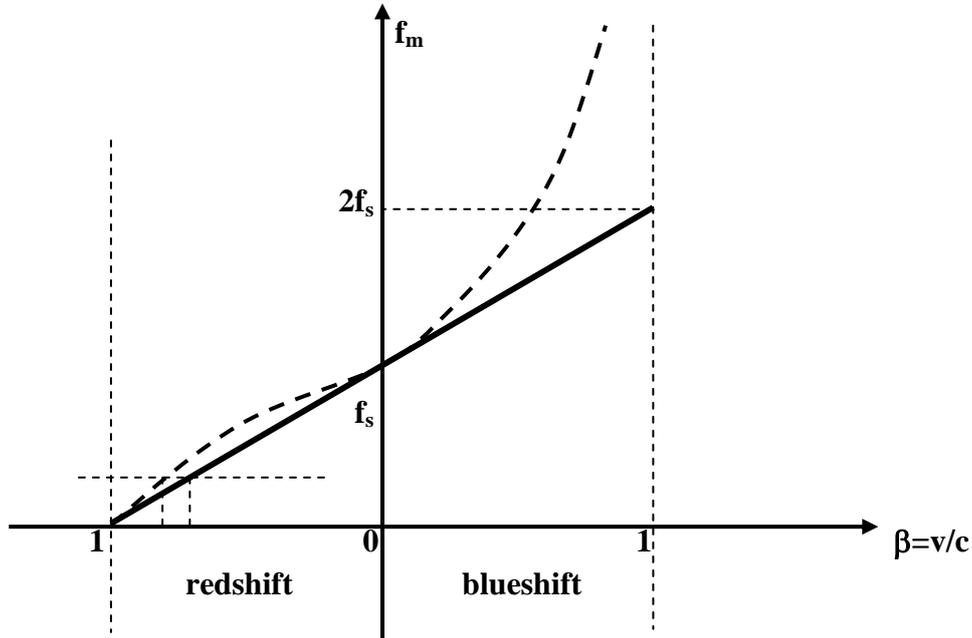


Fig.15 The dotted graph refers to Special Relativity (SR), the continuous graph refers to the Theory of Reference Frames (TR).  $c$  is the speed of light,  $v$  is the relative speed between observer and source.

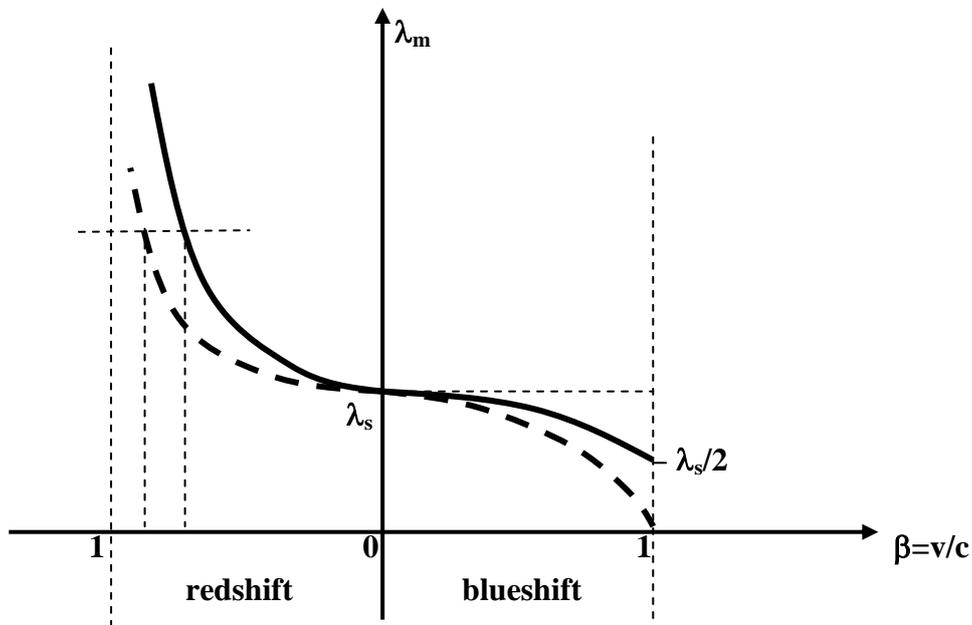


Fig.16 The dotted graph refers to Special Relativity (SR), the continuous graph refers to the Theory of Reference Frames (TR).  $c$  is the speed of light,  $v$  is the relative speed between observer and source.

Drawing in a diagram (f-β) and in a diagram (λ-β) results obtained in frequency and wavelength, whether for Special Relativity or for the Theory of Reference Frames, we have graphs as in fig.15 and in fig.16. We observe in those graphs frequency shifts in TR are smaller than in SR and wavelength shifts in TR are greater than in SR.

Graphs in fig.15 and 16 are able to explain because explosion of remote supernovae with high redshift emits a smaller light intensity than the intensity calculated in SR and consequently in concordance with TR. In fact in both graphs we observe that at the same redshift the departure speed of supernova in TR is smaller than in SR; this implies the decelerating dying supernova explodes at a greater distance than the distance calculated in SR because of its smaller departure speed: it explains the measured smaller intensity of light.

In General Relativity Einstein proved also the existence of a cosmological redshift that he explained through the gravitational field. A few experiments executed in the period 1921-1928 have proved the cosmological redshift (called also "Einstein effect") is really present in the light that comes to the earth after the emission from the sun and effected measures gave for the sun Einstein effect the value  $\Delta f/f_T = -0,2 \times 10^{-5}$ , where  $\Delta f = f_S - f_T$ ,  $f_T$  is the frequency of the spectral line emitted by the earth and  $f_S$  is the frequency of the same spectral line emitted by the sun.

In TR the cosmological redshift has an explanation based on the atomic theory<sup>[14]</sup>.

The Doppler effect is a variation of frequency and wavelength of electromagnetic radiation caused by the relative speed between emitting source and receiving observer. The Doppler effect is generated therefore after that electromagnetic radiation has been emitted. In order to explain the cosmological redshift, and in particular the sun redshift, the cause must be pursued in the same act of emission.

We know light is composed of photons that are generated by quantum jumps of electrons among energy levels in the atom. Quantum jumps on the surface of the earth depend on the electrodynamic mass  $m_T$  of electron that is on the surface of the earth. Because a variation of frequency in the radiation emitted on the sun and measured on the earth has been proved experimentally, this implies electron mass on the sun or on any other star has a different value  $m_S$  from  $m_T$ .

It is possible to demonstrate<sup>[14]</sup> that this frequency variation is given by

$$\frac{\Delta f}{f_T} = \frac{f_S - f_T}{f_T} = \frac{m_S - m_T}{m_T} \quad (84)$$

Assuming for electrodynamic mass a sensitive behaviour to the gravitational field and consequently an equivalent behaviour of gravitational mass, because the variation of electrodynamic mass is connected with both the variation of intrinsic energy and the variation of potential energy relative to the surface of the two celestial masses  $\Delta E_p = E_p(S) - E_p(T) = m_S U(S) - m_T U(T)$ , we have with good approximation, being  $m_S \approx m_T$ ,

$$\frac{\Delta f}{f_T} = \frac{f_S - f_T}{f_T} = \frac{m_S - m_T}{m_T} = \frac{G}{c^2} \left( \frac{M_T}{r_T} - \frac{M_S}{r_S} \right) \quad (85)$$

As in the event of sun and earth is  $M_S/r_S \gg M_T/r_T$ , we have

$$\frac{\Delta f}{f_T} = \frac{f_S - f_T}{f_T} = - \frac{GM_S}{c^2 r_S} \quad (86)$$

Replacing the well-known values of  $G$ ,  $M_S$ ,  $c$ ,  $r_S$ , we obtain just  $\Delta f/f_T = -0,2 \times 10^{-5}$  which is the measured experimental value with regard to the cosmological redshift produced by the sun light (this value corrects the value  $\Delta f/f_T = -0.2 \times 10^{-6}$  in ref.[14]).

Also in the Theory of Reference Frames "the Einstein effect" is accurately demonstrated. In the Theory of Reference Frames besides new theories and explanations concerning the astronomical aberration<sup>[15]</sup> and black holes<sup>[16]</sup> have been developed and they can be looked at papers specified into references.

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