The relation of colour charge to electric charge

Dirac has shown how the Klein-Gordon equation can be factored into two linear parts using 4x4 Dirac gamma matrices.

[Dirac, P.A.M., The Principles of Quantum Mechanics, 4th edition (Oxford University Press) ISBN 0-19-852011-5]

$$(\partial_t^2 - \partial_x^2 - \partial_y^2 - \partial_z^2 + m^2)\, I = (-i[s\gamma^0\partial_t + r\gamma^1\partial_x + g\gamma^2\partial_y + b\gamma^3\partial_z] - mI)\,(i[s\gamma^0\partial_t + r\gamma^1\partial_x + g\gamma^2\partial_y + b\gamma^3\partial_z] - mI)$$

where r,g,b and s equal +1 or -1.

For leptons r,g,b all equal -1 and for quarks two of r,g,b are equal to +1 and the third equals -1. The signs are all negated for anti-particles as in the equation above.

When s = +1, count the number of plus signs (say) for r,g,b which is 0 for leptons and 2 for quarks.

When s = -1, count the number of minus signs (say) for r,g,b which is 3 for leptons and 1 for quarks.

For material particles r,g,b all equal -1 which is always true for leptons and true for three distinct quarks with r,g,b equal to -1 separately or a quark and an appropriate anti-quark.

Let $\bar{\gamma} = \gamma_0\gamma_1\gamma_2\gamma_3\gamma_4$ where γ_0 , γ_1 , γ_2 , γ_3 , γ_4 are vectors which anti-commute and where:

$$\gamma_1^2 = \gamma_2^2 = \gamma_3^2 = \gamma_4^2 = -I \qquad\qquad \gamma_0^2 = I \quad .$$

$$\gamma_0\bar{\gamma} = \bar{\gamma}\gamma_0 \qquad \gamma_1\bar{\gamma} = \bar{\gamma}\gamma_1 \qquad \gamma_2\bar{\gamma} = \bar{\gamma}\gamma_2 \qquad \gamma_3\bar{\gamma} = \bar{\gamma}\gamma_3 \qquad \gamma_4\bar{\gamma} = \bar{\gamma}\gamma_4 \qquad\qquad \bar{\gamma}^2 = I$$

Let:

$$\hat{s} = \frac{1}{2}(I + s\bar{\gamma}) \qquad \hat{r} = \frac{1}{2}(I + r\bar{\gamma}) \qquad \hat{g} = \frac{1}{2}(I + g\bar{\gamma}) \qquad \hat{b} = \frac{1}{2}(I + b\bar{\gamma})$$

$$\hat{s}^2 = \hat{s} \qquad \hat{r}^2 = \hat{r} \qquad \hat{g}^2 = \hat{g} \qquad \hat{b}^2 = \hat{b}$$

$$\hat{s}\hat{r} = \hat{r}\hat{s} \qquad \hat{s}\hat{g} = \hat{g}\hat{s} \qquad \hat{s}\hat{b} = \hat{b}\hat{s} \qquad\qquad \hat{r}\hat{g} = \hat{g}\hat{r} \qquad \hat{g}\hat{b} = \hat{b}\hat{g} \qquad \hat{b}\hat{r} = \hat{r}\hat{b}$$

A charged particle moving in an electromagnetic field will have ∂_t , ∂_x , ∂_y , ∂_z modified by the scalar and vector potentials of the field, where ∂_t , ∂_x , ∂_y , ∂_z do not commute with each other. Thus:

$$(\hat{s}\gamma_0\partial_t + \hat{r}r\gamma_1\partial_x + \hat{g}g\gamma_2\partial_y + \hat{b}b\gamma_3\partial_z + i\hat{s}\gamma_4 m)\,(\hat{s}\gamma_0\partial_t + \hat{r}r\gamma_1\partial_x + \hat{g}g\gamma_2\partial_y + \hat{b}b\gamma_3\partial_z + i\hat{s}\gamma_4 m)$$

$$= \hat{s}\partial_t^2 - \hat{r}\partial_x^2 - \hat{g}\partial_y^2 - \hat{b}\partial_z^2 + \hat{s}m^2$$

$$+ \hat{s}\gamma_0[\hat{r}r\gamma_1(\partial_t\partial_x - \partial_x\partial_t) + \hat{g}g\gamma_2(\partial_t\partial_y - \partial_y\partial_t) + \hat{b}b\gamma_3(\partial_t\partial_z - \partial_z\partial_t)] \quad \text{(= 0 for a neutrino)}$$

$$+ \hat{r}r\hat{g}g\gamma_1\gamma_2(\partial_x\partial_y - \partial_y\partial_x) + \hat{g}g\hat{b}b\gamma_2\gamma_3(\partial_y\partial_z - \partial_z\partial_y) + \hat{b}b\hat{r}r\gamma_3\gamma_1(\partial_z\partial_x - \partial_x\partial_z)$$

$$= \hat{s}\partial_t^2 - \hat{r}\partial_x^2 - \hat{g}\partial_y^2 - \hat{b}\partial_z^2 + \hat{s}m^2$$

$$+ \hat{s}\gamma_0[\hat{r}r\gamma_1(\partial_t\partial_x - \partial_x\partial_t) + \hat{g}g\gamma_2(\partial_t\partial_y - \partial_y\partial_t) + \hat{b}b\gamma_3(\partial_t\partial_z - \partial_z\partial_t)] \quad \text{(= 0 for a neutrino)}$$

$$- \hat{r}\hat{g}b\gamma_1\gamma_2(\partial_x\partial_y - \partial_y\partial_x) - r\hat{g}\hat{b}\gamma_2\gamma_3(\partial_y\partial_z - \partial_z\partial_y) - \hat{r}g\hat{b}\gamma_3\gamma_1(\partial_z\partial_x - \partial_x\partial_z)$$