What is maths differance

——With three kinds of new maths

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Abstract: This paper presents a new framework for the maths development which is called maths differance. There are three typical maths differance: proof, axiom and shift, which is corresponding to three kinds of new maths roughly: the fluid maths, the model maths and the novel maths.

1. Differance and maths differance

What is the essential part of maths? Maybe non-maths students will say calculation,and some maths students will say proof, but the mathematician G.H.Hardy says that strictly speaking, there is no proof, in the final analysis, we can only point.

I think Hardy is right, but we can go further. In fact, Derrida's differance is more suitable for this job. Jacques Derrida is a philosopher, but not a mathematician. He wants to express a special kind of difference, not only a difference with A and B, but a difference with A and A', which contains a significance of the chain, an extension of meaning, so he modifies the word difference and creates a new word differance. Maybe this is not the original meaning of Derrida, but it is a clear explanation of the differance and also enough for this paper, the differance of differance.

Many things contains meaning, so differance can be widely used. But I think maths is the best stages for differance because it contains the pure and holonomy meaning. When the maths develops, you can imagine the crystal growth, which is a vivid picture of differance. Theoretically, you can grow from every point of the maths theory,, but some points have more priority than others and some great mathematician can change the value of the point in the whole maths theory. We can say that there is no fixed point in the maths.

2. Three typical maths differance

At least, there are three typical maths differance with the different lever in the mathematics, proof, axiom and shift. Now I will show an outline of them.

2.1 Proof

Proof is the basic part of maths, the typical example is the Euclidean geometry， you can prove a lot of propositions from some definitions with a little axioms. I think the main reasons for someone who like the proof is that proof is exact and powerful, which makes the maths total different from the science. Some persons treat the maths as the model of truth and treat the proof as a model of the model.

But some mathematicians dislike the proof, they think proof is too stuffy and do not as exact as we have seen. Some of them emphasize the maths experience and some of them emphasize the maths intuition. I think proof is only the basic rule, if you play a chess, the wonderful thing is not the rule, but how do we use the rule. In my opinion, experience is the base of proof and proof is the base of intuition. Here the base is only material, but not the soul. I think the soul of maths is something as the genius of art and the most wonderful maths is created by intuition, maybe that is because it is hard for us to say how do we create it.

Some intuition refers the soft proof, which is not the standard syllogism, but some analogy with some strictly reasons. For example, you can get some claims from the dual claims in projective geometry and you can write a new proof follow the proof procedure. However, the proof makes the forward term and the back term at the same level.

2.2 Axiom

Axiom is also an important part in the maths, specially in the modern maths. The basic philosophy of axiom is upgrading the typical properties into a law which we call it axiom, then we can get some high level concepts which can do more representation. For example, the distance has three typical properties: nonnegative, symmetric and triangle inequality, we make them as axioms, we can get the metric space. According to this method, we can make the properties of absolute value into normed space and so on.

Sometimes, the result of the axioms maybe not a whole space, but only a concept. Give any group G, we can ask that whether we have a CW-complex whose fundamental group is G. The answer is yes and we call this CW-complex Eilenberg-MacLane space, many modern maths are relate with it such as Eilenberg-Ganea problem (see [1]) and higher algebraic K-theory (see [2]).

Now look at an similar story, Give any group G, can you find an extension field F over K, such that the Galois group Gal(F/K)=G. In fact, Galois groups are all profinite groups, even for the finite groups, there is a famous inverse Galois problem asking that whether every finite group appears as the Galois group of some Galois extension of the rational numbers Q. It is unsolved (see [3]), so we can not get an axiom definition.

In fact, the axiom definition and the inverse problem are two sides of a coin. The request of a properties is an inverse problem, if an inverse problem has a complete solutions, we can get an axiom definition from it, if not, we only have a unsolved inverse question. If there are some partial solutions, we can do this program locally.

2.3 Shift

The last differance is called Shift by myself. Let's imagine them vivid first. proof is run, you can catch a runner by your eyes, axiom is jump, you must raise your head to see the jumper. But shift is such a special ninja skill, disappearing at here then appearing there, you can not see the middle traces.

Traceless is an important character of shift. Let's take an example to explain it. The group theory is starting with solution question of the high algebraic equation. As we know, the general equation of degree 5 and higher degree are not always solvable with radicals, we can proof it with the group theory and Galois theory. But the group and Galois theory are totally different with the algebraic equation, we can imagine such a person who is an expert of group theory, but he can not solve the three or four degree algebraic equation.

For the same example, we also can compare the axiom and shift. If you do the axiom for the multiplication, then you can get the groups. If you follow the trace of the axiom, only require the abstract form, then you can catch the the theme, but if you do same thing for the shift, you will lost the target and trap into the details.

Traceless brings the non-historical of the shift. We can ask such an fundamental mathematical inevitable question: if there is the parallel earth whose civilization level is similar with us, comparing our maths history, which parts or which order of maths will be the same? We can imagine that some mathematicians of the parallel earth find group from the multiplication directly, we even can imagine that they get groups from the vector space just as we get the metric space from the Euclidean space, but it is hard for us to imagine that they get the root formula of three degree algebraic equations before than they can solve quadratic equations.

3. Other maths difference

Of course, there are some other kinds of differance in the maths. We have the relative differance, one example is to define the relative open or closed set in the topology, the other is to modify the homological group to the relative homological groups, then we have the long exact sequence. We also have the local differance, the stand example is local rings in the commutative algebras and algebraic geometry, we also can modify the theory locally, from the Banach space to the local Banach space with the holomorphic functional calculus (see [4]).

Some differance has no system name, but is also valuable. For example, I find that many modern maths branches related to change the coefficient into p-adic numbers, then we have many p-adic maths object such as p-adic Lie algebra, p-adic Banach space, p-adic cohomology, p-adic Hodge theory and even p-adic quantum mechanics. Why does people like p-adic numbers? It is complete as the real numbers. It related the prime numbers, so we do the factor job from it. At last, it is the balance point from abstract and concrete, if you change the coefficient into a field or a commutative ring directly, the theory will be poor, you can choose the p-adic numbers as your first step.

Reference:

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[4] Blackadar B. K-theory for operator algebras[M]. Cambridge University Press, 1998.

Appendix: Three kinds of new maths

We have three kinds of new maths: the fluid maths, the model maths and the novel maths, which is roughly corresponding to the three kinds of differance respectively.

1) The fluid maths

What is the fluid maths? It does not mean some maths method for the fluid mechanic, but its ideal really comes from the fluid physics. Since we have particle mechanics，rigid body mechanics and fluid mechanic in the physics， which focus particles, rigid bodies and fluid respectively， we also can have the point maths, body maths and fluid maths in the mathematics, which focus the points， metric and density respectively.

An typical example of the fluid is the Lebesgue theory in the real analysis, from that we can claim that almost every point have the metric density 1 in the measurable set， here the metric density(also call it density for short） means the limit of the average in the neighborhood. Strictly speaking， let E is a Lebesgue measurable set and $x \in E$, then the density of E at the point x is lim $m(E \cap B(x,r))/m(B(x,r))$, here m means Lebesgue measure and we can extend to other interesting measure, B is stand symbol for the metric ball(see [1]).

In fact, the Lebesgue theorem is a weighting for the claim a.e point have density 1 in the measurable set. From the direction， we can extend the measurable density into integral density， then we can get the maximal function which is bridge from the real analysis to the harmonic analysis. From the other direction, we can study the density more geometric, maybe you can find more examples in the geometric measure theory， from which the density maybe large than 1 because the dimension changes（see [2]).

2) The model maths

What is the model maths? Maybe you are more familiar with the mathematical model. Most of mathematical models are model for the science, statistics and so on, but the model maths is for the models of pure maths, models of models, then we have three levers of mathematics: model mathematics, pure mathematics and applied mathematics, the model maths to the pure math is just like the pure maths to the applied maths.

Which kind of maths is model maths? Metamathematics, axiomatic set theory, category theory are some typical examples. In the pure maths, we also have some local model maths, for example, the Steenrod-Eilenberg axioms for homology in the algebraic topology（see [3]) and the Tits systems in the Lie groups(see [4]) and algebraic groups(see [5]), they are the results of the axiomatization. In fact, the main way to reach the model maths is to do axiomatization, uploading to basic properties into axiomatic laws, if there laws are too dry as a model, then we can call it model maths.

How to decide whether it belongs to model maths. The main standard is from the logic， the model maths should be abstract and frame, above than others as a overpass, but there is also some cultural standards, when mathematicians find more and more relations and details about the model, it maybe be soft to the common maths. For example, the separation axioms T0-T6 in the general topology are models for the beginners, but when you study more and more, you will think they are only some background, but not typical models, maybe some algebraists also think that categories are not models. In fact, every parts of maths are models, but some maths are more model than other model, so we call them model maths by custom.

3) The novel maths

What is the novel maths? It must be new and unusual, not the nature development from the traditional maths， but a branch from the side and containing a theoretical shift. The typical examples of novel maths are fuzzy maths, fractional calculus，fractal geometry and so on. Maybe the S-divisor(see [6]) created by myself is also an original point of a (program style) novel maths.

There are two kind of novel maths, program style and object style. The program style novel is an extension of the classical maths, for example, the fuzzy maths extend the character function of the set from the value {0,1} to [0,1], then we can construct many new maths breaches in parallel with the traditional maths such as fuzzy topology, fuzzy measure theory and so on. The fractional calculus is similar, but a little weak. The object style changes the main studying object, for example, the fractal geometry refers to the breaking structure which is ignored by the traditional geometry. In general, the future development of program style novel maths will go back to the pure maths such as many p-adic theories and the future development of the object style novel maths will become a new separate area such as the probability theory and stochastic mathematics.

Maybe some people will think that fuzzy maths, fractional calculus and fractal geometry are all applied mathematics, but that is an old classification. In fact, the novel maths are different with the standard applied mathematics such as computer mathematics which study something outside of the maths. Of course, the edge is not exact, there are something similar between the novel maths and the applied maths, maybe they both have some motivation for application and are not as rich as the pure maths, the reason is that the novel maths is new, so it should depends on something outside more, but the direction is inside, that is the key difference.

Reference for the appendix:

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[6] Strongart：Every set has its S-divisor(2012) http://arxiv.org/abs/1205.5874

Special Remark: The paper is a summary of my original Chinese blog articles, you can see them at: http://blog.sina.com.cn/strongart.