On a Simpler and Truly Marvellous General Proof of Fermat's Last Theorem For $(\mathbf{n > 5})$

Golden Gadzirayi Nyambuya
Department of Applied Physics, National University of Science and Technology, Bulawayo, Republic *of* Zimbabwe
Email: physicist.ggn@gmail.com

Received ****; revised ****; accepted ****

English mathematics Professor, Sir Andrew John Wiles of the University *of* Cambridge finally and conclusively proved in 1995 Fermat's Last Theorem which had for 358 years notoriously resisted all gallant and spirited efforts to prove it even by three of the greatest mathematicians of all time – such as Euler, Laplace and Gauss. Sir Professor Andrew Wiles's proof employs very advanced mathematical tools and methods that were not at all available in the known World during Fermat's days. Given that Fermat claimed to have had the 'truly marvellous' proof, this fact that the proof only came after 358 years of repeated failures by many notable mathematicians and that the proof came from mathematical tools and methods which are far ahead of Fermat's time, this has led many to doubt that Fermat actually did possess the 'truly marvellous' proof which he claimed to have had. In this short reading, *via* elementary arithmetic methods which make use of Pythagoras theorem, we demonstrate conclusively that Fermat's Last Theorem actually yields to our efforts to prove it. This proof is so elementary that anyone with a modicum of mathematical prowess in Fermat's days and in the intervening 358 years could have discovered this very proof. This brings us to the tentative conclusion that Fermat might very well have had the 'truly marvellous' proof which he claimed to have had and his 'truly marvellous' proof may very well have made use of elementary arithmetic methods.

Keywords: Fermat's Last Theorem, Proof, Pythagoras theorem, Pythagorean triples.

"*Subtle is the Lord.*
Malicious He is not."

Albert Einstein $(1879 - 1955)$.

1. Introduction

The pre-eminent French lawyer and amateur mathematician, Pierre *de* Fermat $(1607 - 1665)$ in 1637, famously in the margin of a copy of the famous book *Arithmetica*, he wrote:

> "*It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvellous proof of this, which this margin is too narrow to contain.*"

In the parlance of mathematical symbolism, this can be written succinctly as:

$$\nexists\ (x, y, z, n) \in \mathbb{N}^{+}: \quad x^n + y^n = z^n \ \text{ for } \ (n > 2), \qquad (1)$$

where the triple $(x, y, z) \neq 0$, is piecewise coprime, and $\mathbb{N}^{+}$ is the set of all positive integer numbers. This theorem is classified among the most famous theorems in all History *of* Mathematics and prior to 1995, proving it was – and is; ranked in the *Guinness Book of World Records* as one of the "*most difficult mathematical problems*" known to humanity. Fermat's Last Theorem is now a true theorem since it has been proved, but prior to 1995 it was only a *conjecture*. Before it was proved in 1995, it is only for historic reasons that it was known by the title "*Fermat's Last Theorem*".

Rather notoriously, it stood as an unsolved riddle in mathematics for well over three and half centuries. Many amateur and great mathematicians tried but failed to prove the conjecture in the intervening years $1637 - 1995$; including three of the World's greatest mathematicians such as Italy's Leonhard Euler $(1707 - 1783)$, France's Pierre-Simon, marquis *de* Laplace $(1749 - 1827)$, and the celebrated genius and Crown Prince *of* Mathematics, Germany's Johann Carl Friedrich Gauss $(1777 - 1855)$,

[*] G. G. Nyambuya

amongst many other notable and historic figures of mathematics.

Without any doubt, the conjecture or Fermat's Last Theorem is in-itself – as it stands as a bare statement, deceptively simple mathematical statement which any agile 10 year old mathematical prodigy can fathom with relative ease. Fermat famously – *via* his bare marginal note; stated he had solved the riddle around 1637. His claim was discovered some 30 years later, after his death in 1665, as an overly simple statement in the margin of the famous copy *Arithmetica*. Fermat wrote many notes in the margins and most of these notes were 'theorems' he claimed to solved himself. Some of the proofs of his assertions were found. For those that were not found, all the proofs save for one resisted all intellectually spirited efforts to prove it and this was the marginal note pertaining the so-called Fermat's Last Theorem.

This marginal note dubbed Fermat's Last Theorem, was the last of the assertions made by Fermat whose proof was needed, and for this reason that it was the last of Fermat's statement that stood unproven, it naturally found itself under the title 'Fermat's Last Theorem'. Because all of the many of Fermat's assertions were eventually proved, most people believed that this last assertion must – too; be correct as Fermat had claimed. Few – if any; doubted the assertion may be false, hence the confidence to call it a theorem. Simple, the proof Fermat claimed to have had, had to be found!

Did Fermat actually posses the so-called 'truly marvellous' proof which he claimed to have had? This is the question many have justly and rightly asked over the years and this reading makes the temerarious endeavour to vindicate Fermat, that he very well might have had the 'truly marvellous' proof he claimed to have had and this we accomplish by providing a proof that employs elementary arithmetic methods that were available in Fermat's day.

Surely, there are just reasons to doubt Fermat actually had the proof and this is so given the great many notable mathematicians that tried and monumentally failed and aswell, given the number of years it took to find the first correct proof. The first correct proof was supplied only 358 years later by the English Professor of mathematics at the University *of* Cambridge – Sir Andrew John Wiles $(1953-)$, in 1995 [1].

To add salt to injury *i.e.* add onto the doubts on whether or not Fermat actually had his so-called 'truly marvellous' proof is that Sir Professor Andrew Wiles's proof* employs highly advanced mathematical tools and methods that were not at all available in the known World during Fermat's days. Actually, these tools and methods were invented (discovered) in the relentless effort to solve this very problem. Herein, we supply a very simple proof of Fermat's Last Theorem.

That said, we must hasten to say that, as a difficult mathematical problem that so far yielded only to the difficult, esoteric and advanced mathematical tools and methods of Sir Professor Andrew Wiles – Fermat's Last Theorem, as any other difficult mathematical problem in the History *of* Mathematics, it has had a record number of incorrect proofs of which the present may very well be an addition to this long list of incorrect proofs. In the words of historian of mathematics – Howard Eves [2]:

> *"Fermat's Last Theorem has the peculiar distinction of being the mathematical problem for which the greatest number of incorrect proofs have been published."*

With that in mind, allow us to say, we are confident the proof we supply herein is water-tight and most certainly correct and that, it will stand the test of time and experience.

As stated in the *ante penultimate* above is that, in this rather short reading, we make the temerarious endeavour to answer this question – of whether or not Fermat actually possessed the proof he claimed to have had. This we accomplish by supplying a simple and elementary proof that does not require any advanced mathematics but mathematics that was available in the days of Fermat. Sir Professor Andrew Wiles's acclaimed proof, is at best very difficult and to the chagrin of they that seek a simpler understanding – the proof is nothing but highly esoteric. The question thus 'forever' hangs in there to the searching and inquisitive mind: "*Did Fermat really possess the proof he claimed to have had?*" The proof that we supply herein leads us to strongly believe that Fermat might have had the proof and this proof most certainly employed elementary methods of arithmetics!

2. Proof

The proof that we are going to provide is a proof by contradiction. We assume that the statement:

$$\exists\ (x, y, z, n) \in \mathbb{N}^{+}:\ \ x^n + y^n = z^n,\ \text{ for }\ (n > 2), \tag{2}$$

to be true. The tripple (x, y, z) is piecewise *coprime*, the meaning of which is that the greatest common divisor $[\gcd()]$ of this triple or any arbitrary pair of the triple is unity. For our proof, we shall proceed in a general way to show that the statement (2) can never be true for $(n > 5)$. The cases for $n = (3, 4, 5)$ is assumed to have been proved by others as will be discussed soon. In our approach, we split the problem into two parts, *i.e.*:

1. **Case (I)** **:** This case proves for all powers of $(n > 5) \in \mathbb{E}^{+}$ where $\mathbb{E}^{+}$ is the set of all positive even integer numbers.

* The proof by Sir Professor Wiles is well over 100 pages long and consumed about seven years of his research time. For this notable achievement of solving Fermat's Last Theorem, he was Knighted *Commander of the Order of the British Empire* in 2000 by Her Majesty Queen Elizabeth (II), and received many other honours around the World.

2. **Case (II) :** This case proves for all powers of $(n > 5) \in \mathbb{O}^+$ where $\mathbb{O}^+$ is the set of all positive odd integer numbers.

Since the set $(n > 5) \in \mathbb{N}^+$ contains only odd and even values of n, to prove that there does not exist an even and odd $(n > 5) \in \mathbb{N}^+$ that satisfies (2) is a proof that there does not exist $(x, y, z, n) \in \mathbb{N}^+ : \quad x^n + y^n = z^n, \quad (n > 5)$ and combining this result with the well established proofs for $n = (3, 4, 5)$, it means that there does not exist $(x, y, z, n) \in \mathbb{N}^+ : \quad x^n + y^n = z^n, \quad (n > 2)$. This is a proof of the original statement (1).

Proof for the Case $\mathbf{n = (3, 4 \,\&\, 5)}$

As is well known, the case for $(n = 3)$, for all non-zero (x, y, z) and $(x, y, z) \in \mathbb{N}^+$, the equation $x^3 + y^3 = z^3$ has no solutions. This was first proved by the great Italian mathematician Leonhard Euler in 1770 [3], that is, 133 years after Fermat set into motion Fermat's Last Theorem. Euler used the technique of *infinite descent*, a technique that we also use for part of our proof. Euler's proof is not the only proof possible as other authors have published their independent proofs [see *e.g.* Refs. 4, 5, 6, 7, 8, amongst many others].

Fermat was the first to provide a proof for the case $(n = 4)$ which stated that for all non-zero piecewise coprime triple $(x, y, z) \in \mathbb{N}^+$, the equation $x^4 + y^4 = z^4$ admits no solutions. This proof by Fermat is the only surviving proof of Fermat's Last Theorem and as is the case with Euler's proof for the case $(n = 3)$, Fermat's proof makes use of the technique of infinite descent. If having gone through the proof that we provide in this reading, one will come to wonder if Fermat's proof for the case $(n = 4)$ was conducted by Fermat as part of a more general proof to Fermat's Last Theorem. Further, as is the case with Euler's proof for $(n = 3)$, Fermat's proof is not the only proof possible as other authors have published their independent proofs [see *e.g.* Refs. 5, 6, 9, 10, 11, amongst many others]. Even after Sir Professor Andrew Wiles's 1995 breakthrough [1], researchers are still publishing variants of the proof for the case $(n = 4)$ [see *e.g.* 12, 13, 14].

The case $(n = 5)$ was first proved independently by the French mathematician Adrien-Marie Legendre (1752 - 1833) and the German mathematician Johann Peter Gustav Lejeune Dirichlet (1805 - 1859) around 1825 and alternative and independent proofs were developed in the later years by others [see *e.g.* Refs. 5, 15, 16, 17, 18, 19, 20, 21, 22, amongst many others]. In this work, we take the cases $n = (3, 4, 5)$ as having been proven, thus prove only for the case $(n > 5)$.

-oOo-

Now, we provide a theorem on which our entire proof hinges on. The proof will be accompanied by a Lemma which is necessary in proving the theorem. As stated in the penultimate of the previous paragraph, our proof assumes that the cases $n = (3, 4, 5)$ have been proven by others as indicated above, thus we take on the proof only for the general case $(n > 5)$.

Theorem 1: *If $(x, y, z, n) \in \mathbb{N}^+$ such that the triple (x, y, z) is piecewise coprime, then, the equation*:

$$z^n = x^2 + y^2, \tag{3}$$

admits no solutions for $(n > 2)$.

-oOo-

Lemma 1: *If $(a, b) \in \mathbb{N}^+$ such that*:

$$a\sqrt{b} = c + d, \tag{4}$$

for some numbers (c, d), then, insofar as whether or not $\sqrt{b}$ is an integer or not, there are two conditions, and these are:

1. $\sqrt{b} \in \mathbb{N}^+$.
2. $\sqrt{b} \notin \mathbb{N}^+$. *That is, $\sqrt{b}$ is an irrational number.*

—

1. *If, $\sqrt{b} \in \mathbb{N}^+$, then, $(c, d) \in \mathbb{N}^+$.*
2. *If, $\sqrt{b} \notin \mathbb{N}^+$, then $\sqrt{b}$ is a surd – it is an irrational number and $(c, d) \notin \mathbb{N}^+$; and there must exist some $((c_1 < c) \,\&\, (d_1 < d)) \in \mathbb{N}^+$ such that $c = c_1\sqrt{b}$ and $d = d_1\sqrt{b}$ so that $a\sqrt{b} = c_1\sqrt{b} + d_1\sqrt{b}$, which implies that*:

$$a = c_1 + d_1. \tag{5}$$

The above stated Lemma is a self evident truth which is not only necessary but vital and pivotal for the proof that we now give below. Part of the proof is a proof by infinite descent and it is a proof for Theorem (1).

-oOo-

Proof of Theorem (1): First we must realise that for $(n > 2)$, n is either odd or even. We shall separate the proofs for even and odd n. We will begin with the proof for even n.

General Proof for the Case $(\mathbf{n > 2}) \in \mathbb{E}^+$: Now, we are going to prove that the equation:

$$z^n = x^2 + y^2, \tag{6}$$

admits no solutions for non-zero and piecewise coprime $(x, y, z) \in \mathbb{N}^+$.

General Case for Even $(\mathbf{n} > \mathbf{2})$: If $(n > 2)$ is even, then, we can write $n = 2k$ for some $k = 2, 3, 4, 5, \ldots$ *etc*. This means that for an even $(n > 2)$, equation (6) can be rewritten as:

$$z^{2k} = x^2 + y^2. \tag{7}$$

Now, let us remind ourself that, the three non-zero integers (x, y, z) are piecewise coprime and that they are all positive. One of the three must be even, whereas the other two are odd. Without loss of generality, z may be assumed to be even. Since x and y are both odd, they cannot be equal because if $x = y$, then $z^{2k} = 2x^2$, which implies that $\sqrt{2} = z^k/x$, the meaning of which that, the square-root of 2 must be rational. We know it, $\sqrt{2}$, is not, hence, by way of contradiction, $x \neq y$. Besides, our initial assumption is that (x, y, z) are piecewise coprime – this makes triple unique; the meaning of which is that $x \neq y$ in the first place.

Since x and y are both odd, their sum and difference are both even numbers, *i.e.*:

$$\left.\begin{array}{l} x+y=2u \\ x-y=2v \end{array}\right\} \Rightarrow \begin{array}{l} x=u+v \\ y=u-v \end{array}, \tag{8}$$

where the non-zero integers u and v are coprime and have different parity (*i.e.*, one is even and, the other is odd).

Now, substituting x and y as given in equation (8) into equation (6), we will have:

$$z^{2k} = 2(u^2 + v^2). \tag{9}$$

Since (u, v) have opposite parity, $u^2 + v^2$ is odd. In order for equation (9) to hold with the constraints placed on the piecewise coprime integers (u, v, z), there is need for $u^2 + v^2$ to be a factor of 2 if $z \in \mathbb{N}^+$, *i.e.* we need to have $u^2 + v^2 = 2^{2ak-1}b^4$ for some positive integers (a, b) in-order for (9) to hold, especially for $z \in \mathbb{N}^+$. If $u^2 + v^2 = 2^{2ak-1}b^4$, then $z = 2^a b$, which is a positive integer since $(a, b) \in \mathbb{N}^+$. Now, if $u^2 + v^2$ is to be a factor of 2, then, $u^2 + v^2$ must be even, the meaning of which is that it must both be even and odd since (u, v) have opposite parity. This is a clear contradiction. Hence, by way of contradiction, equation (6) admits no non-zero piecewise coprime positive integers (x, y, z) for $(n > 2) \in \mathbb{E}^+$.

Q.E.D.

General Case for Odd $(\mathbf{n} > \mathbf{2})$: If n is odd, then, we can write $n = 2k + 1$ for some $k = 1, 2, 3, 4, 5, \ldots$ *etc*. This means that for an odd $(n > 2)$, equation (6) can be rewritten as:

$$z^{2k+1} = x^2 + y^2. \tag{10}$$

As before, our proof shall proceed by way of contradiction and as-well by use of the concept of infinite descent, the meaning of which is that we shall assume that equation (10) admits solutions for non-zero, piecewise coprime triple $(x, y, z) \in \mathbb{N}^+$, and that, if it [equation (10)] did, then, there must exist some $z \in \mathbb{N}^+$ such that $z^{2^{-m}} \in \mathbb{N}^+$ for all $m = 1, 2, 3, \ldots$ *etc*; we know very well that there must be a finite limit for m for which $z^{2^{-m}} \in \mathbb{N}^+$; m can not increase infinitely. That said, let us rewrite (10) as:

$$\left(z^k\sqrt{z}\right)^2 = x^2 + y^2. \tag{11}$$

We know that there must exist some numbers (x_1, y_1), such that:

$$\left(\begin{array}{c} x \\ y \\ z^k\sqrt{z} \end{array}\right) = \left(\begin{array}{c} x_1^2 - y_1^2 \\ 2x_1y_1 \\ x_1^2 + y_1^2 \end{array}\right). \tag{12}$$

The pair of numbers $(x_1, y_1 : x_1 > y_1)$ are not necessary integers, it is either a pair of irrational numbers or integers. The proof for (12) is that:

$$\begin{array}{ccccc} \left(z^k\sqrt{z}\right)^2 & = & x^2 & + & y^2 \\ \Downarrow & & \Downarrow & & \Downarrow \\ \left(x_1^2+y_1^2\right)^2 & \equiv & \left(x_1^2-y_1^2\right)^2 & + & \left(2x_1y_1\right)^2 \end{array}. \tag{13}$$

Note that* $(x, y) > (x_1, y_1)$. There is no need for (x_1, y_1) to be integers as is the case with Pythagorean triples. Actually (x_1, y_1) are either both integers or surds (*i.e.*, irrational numbers).

Now, for $\sqrt{z}$, there are two and only two cases (conditions), and these are:

1. In the first case, we have $\sqrt{z} \notin \mathbb{N}^+$, it is an irrational number – a surd, like $\sqrt{27}$, $\sqrt{12}$ and $\sqrt{1977}$; for example.
2. In the second case, we have $\sqrt{z} \in \mathbb{N}^+$.

In case (1), we have according to Lemma (1), that for the z-component of (12) *i.e.* $z^k\sqrt{z} = x_1^2 + y_1^2$, we must have some $(p, q : p > q) \in \mathbb{N}^+$ such that $x_1^2 = p\sqrt{z}$ and $y_1^2 = q\sqrt{z}$. Substituting $x_1^2 = p\sqrt{z}$ and $y_1^2 = q\sqrt{z}$ into (12), we will have:

$$\left(\begin{array}{c} x \\ y \\ z^k\sqrt{z} \end{array}\right) = \left(\begin{array}{c} (p-q)\sqrt{z} \\ 2\sqrt{p}\sqrt{q}\sqrt{z} \\ (p+q)\sqrt{z} \end{array}\right). \tag{14}$$

What does equation (14) as a whole mean? Well, we know that $x \in \mathbb{N}^+$ but (14) is telling us that $x = (p-q)\sqrt{z}$ is an irrational number since $(p - q) \in \mathbb{N}^+$ and $\sqrt{z}$ is irrational number. This means that x must both be an integer and in irrational number. We know this to be impossible, hence, by way of contradiction, it follows that our initial assertion is wrong as it has lead us to an illogical conclusion.

Now, in the second case, where $\sqrt{z} \in \mathbb{N}^+$, it follows that there must exist some number $z_1 \in \mathbb{N}^+$ such that $\sqrt{z} = z_1$. This means we can rewrite (12) as:

* From (12), $x = x_1^2 - y_1^2$, it follows that $x/x_1 = x_1 - y_1^2/x_1 > 1$, which implies that $x > x_1$. Again, from (12), $y = 2x_1y_1$, it follows that $y/y_1 = 2x_1 > 1$, which implies that $y > y_1$.

$$\begin{pmatrix} x \\ y \\ z_1^{2k+1} \end{pmatrix} = \begin{pmatrix} x_1^2 - y_1^2 \\ 2x_1y_1 \\ x_1^2 + y_1^2 \end{pmatrix}. \tag{15}$$

From equation (15), as before, we can pluck-out the z-component, *i.e.*:

$$z_1^{2k+1} = x_1^2 + y_1^2. \tag{16}$$

Now, from equation (16), it is clear that we can begin the same process as we did at the point of equation (11), that is, rewrite (16) as:

$$\left(z_1^k\sqrt{z_1}\right)^2 = x_1^2 + y_1^2. \tag{17}$$

As before, we know that there exists some numbers $(x_2, y_2 : x_2 > y_2)$ which are either integers or irrational numbers, such that:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1^k\sqrt{z_1} \end{pmatrix} = \begin{pmatrix} x_2^2 - y_2^2 \\ 2x_2y_2 \\ x_2^2 + y_2^2 \end{pmatrix}. \tag{18}$$

Note that* $(x_1, y_1) > (x_2, y_2)$.

Now, as before, for $\sqrt{z_1}$, there are two and only two cases (conditions), and these are:

1. In the first case, we have $\sqrt{z_1} \notin \mathbb{N}^+$, it is an irrational number – a surd, like $\sqrt{2}$ for example.
2. In the second case, we have $\sqrt{z_1} \in \mathbb{N}^+$.

If the first case, then, as we have shown before, it follows that equation (18) has no solution. If the case is the second case, *i.e.*, we have $\sqrt{z_1} \in \mathbb{N}^+$, then, there must exist some number $z_2 \in \mathbb{N}^+$ such that $\sqrt{z_1} = z_2$. This means we can rewrite (18) as:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_2^{2k+1} \end{pmatrix} = \begin{pmatrix} x_2^2 - y_2^2 \\ 2x_2y_2 \\ x_2^2 + y_2^2 \end{pmatrix}. \tag{19}$$

From this equation, we can – as before; pluck-out the z-component, *i.e.*:

$$z_2^{2k+1} = x_2^2 + y_2^2. \tag{20}$$

What this means is that we can – again; begin the same process as we did at the point of equation (11), that is, rewrite (20) as:

$$\left(z_2^k\sqrt{z_2}\right)^2 = x_2^2 + y_2^2. \tag{21}$$

As before, we know that there exists some numbers $[(x_3, y_3) : (x_3 > y_3)$ & $(x_2, y_2) > (x_3, y_3)]$ such that:

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2^k\sqrt{z_2} \end{pmatrix} = \begin{pmatrix} x_3^2 - y_3^2 \\ 2x_3y_2 \\ x_3^2 + y_3^2 \end{pmatrix}. \tag{22}$$

Notice that $(x > x_1 > x_2)$, $(y > y_1 > y_2)$ and $(z > z_1 > z_2)$; and that in general $z_m = z^{2^{-m}}$ where $m = 1, 2, 3 \ldots etc$. If the process would go on and on, then $(x_m > x_{m+1})$, $(y_m > y_{m+1})$ and $(z_m > z_{m+1})$. We could continue this process, but not for ever! There will come a point in our descent where $\sqrt{z_m}$ is an irrational number – at which point, the corresponding z-component equation for $\sqrt{z_m}$ and its corresponding (x_m, y_m) will be $z_m^{2k+1} = x_m^2 + y_m^2$. As we have demonstrated, once $\sqrt{z_m}$ is irrational, there is no solution to the equation $z_m^{2k+1} = x_m^2 + y_m^2$. This means that our original assertion or assumption that equation (10) has solutions for any non-zero piecewise coprime $(x, y, z) \in \mathbb{N}^+$ for even $(n > 2)$, is wrong. Hence, from all this, it follows that equation (10) has no solution for any non-zero piecewise coprime $(x, y, z) \in \mathbb{N}^+$ for $(n > 2) \in \mathbb{O}^+$.

Q.E.D.

2.1. Summary of the Two Proofs

We have proved that the equation $z^n = x^2 + y^2$ admits no solutions for both odd and even $(n > 2)$ where $(x, y, z) \in \mathbb{N}^+$ are piecewise coprime. Combing the two proofs implies that, in general, for any piecewise coprime positive integers (x, y, z), the equation $z^n = x^2 + y^2$ admits no solutions for $(n > 2) \in \mathbb{N}^+$.

Q.E.D.

2.2. Case (I): Even Powers of $(n > 5)$

If $(n > 5) \in \mathbb{E}^+$, then we can write $n = 2k$ were $k = 3, 4, 5, \ldots etc \Rightarrow (k > 2)$. In this case, the equation $x^n + y^n = z^n$, will read:

$$x^{2k} + y^{2k} = z^{2k}, \tag{23}$$

and this can be rewritten as:

$$(x^k)^2 + (y^k)^2 = (z^k)^2. \tag{24}$$

The non-zero piecewise coprime numbers (x^k, y^k, z^k) are all integers, thus, the triple (x^k, y^k, z^k), is a Pythagorean triple in the true sense of a Pythagorean triple. As is well known from Euclid's formula for generating primitive Pythagorean triples, if (p, q) are any arbitrary integers *i.e.* $(p, q : p > q) \in \mathbb{N}^+$ such that p and q are coprime and $p - q$ is odd, the triple (x^k, y^k, z^k) is such that:

$$\begin{pmatrix} x^k \\ y^k \\ z^k \end{pmatrix} = \begin{pmatrix} p^2 - q^2 \\ 2pq \\ p^2 + q^2 \end{pmatrix}. \tag{25}$$

Taking the z-component of the above equation, we will have:

* From (18), $x_1 = x_2^2 - y_2^2$, it follows that $x_1/x_2 = x_2 - y_2^2/x_2 > 1$, which implies that $x_1 > x_2$. Again, from (18), $y_1 = 2x_2y_2$, it follows that $y_1/y_2 = 2x_2 > 1$, which implies that $y_1 > y_2$.

$$z^k = p^2 + q^2. \quad (26)$$

Since $(k > 2)$, it follows from Theorem (1), that there is no solution to the equation (26) for $(x, y, z) \in \mathbb{N}^+$ and $(k > 2)$. Since we have no solution to equation (26), it follows that (24) has no solutions, hence (2) has no integer solutions for all non-zero piecewise coprime (x, y, z) for all powers of $(n > 5) \in \mathbb{E}^+$. Combining this with the separate proofs for $(n = 4)$, it follows that (2) has no integer solutions for all non-zero piecewise coprime integers (x, y, z) for all powers of $(n > 2) \in \mathbb{E}^+$.

Q.E.D.

2.3. Case (II): Odd Powers of $(\mathbf{n} > \mathbf{5})$

Now, we have to prove for the case were $(n > 5) \in \mathbb{O}^+$. As before, we are going to employ Pythagoras theorem. We begin by rewriting $x^{2k+1} + y^{2k+1} = z^{2k+1}$ as:

$$\left(x^k\sqrt{x}\right)^2 + \left(y^k\sqrt{y}\right)^2 = \left(z^k\sqrt{z}\right)^2. \quad (27)$$

The fact that $(n > 5) \in \mathbb{O}^+$, this implies that we can set $n = 2k + 1$ where $k = 3, 4, 5, \ldots, etc \Rightarrow (k > 2)$. The triplet, trio or the three numbers $(x^k\sqrt{x}, y^k\sqrt{y}, z^k\sqrt{z})$ are not necessarily integers, thus this triple is not a Pythagorean triple in the traditional parlance of mathematics. However, this handicap does not stop us (or anyone for that matter) from finding real and irrational numbers $(p, q : p > q)$ which are not necessarily integers, where these numbers (p, q) are such that:

$$\begin{pmatrix} x^k\sqrt{x} \\ y^k\sqrt{y} \\ z^k\sqrt{z} \end{pmatrix} = \begin{pmatrix} p^2 - q^2 \\ 2pq \\ p^2 + q^2 \end{pmatrix}. \quad (28)$$

We are now going to look at the z-component of equation (28). For $\sqrt{z}$, we have two an only two cases (conditions) and these are:

1. $\sqrt{z} \in \mathbb{N}^+$.
2. $\sqrt{z} \notin \mathbb{N}^+$, is an irrational number.

In case (1) where $\sqrt{z} = w \in \mathbb{N}^+$, it follows that for the z-component of the equation (28), we will have:

$$w^{2k+1} = p^2 + q^2. \quad (29)$$

Now, since in (29) the exponent $(2k + 1 > 2)$ of w, it follows from Theorem (1), that this equation admits no solutions. Hence our initial supposition leading to this equation is wrong.

In case (2) where $\sqrt{z} \notin \mathbb{N}^+$ is an irrational number, it follows from Lemma (1) that for the z-component of the equation (28), there must exist some $(a, b : a > b) \in \mathbb{N}^+$, such that $p^2 = a\sqrt{z}$ and $q^2 = b\sqrt{z}$ *i.e.*, $z^k\sqrt{z} = a\sqrt{z} + b\sqrt{z}$. From $p^2 = a\sqrt{z}$ and $q^2 = b\sqrt{z}$ it follows that $p = \sqrt{a\sqrt{z}}$ and $q = \sqrt{b\sqrt{z}}$. Substituting all this into (28), we will have:

$$\begin{pmatrix} x\sqrt{x} \\ y\sqrt{y} \\ z^k\sqrt{z} \end{pmatrix} = \begin{pmatrix} (a-b)\sqrt{z} \\ 2\sqrt{a}\sqrt{b}\sqrt{z} \\ (a+b)\sqrt{z} \end{pmatrix}. \quad (30)$$

What does equation (30) as a whole mean? Well, we know that $x \in \mathbb{N}^+$ but (30) is telling us that $x = (a - b)\sqrt{z/x}$. Since $(a - b) \in \mathbb{N}^+$, for $x = (a - b)\sqrt{z/x} \in \mathbb{N}^+$, $\sqrt{z/x} = s \in \mathbb{N}^+$ *i.e.* $z = s^2x$. This means that x and z share a common factor s^2, the meaning of which is that the triple (x, y, z) is not piecewise coprime. Since our initial assertion runs contrary to our final conclusion, hence, by way of contradiction, it follows that our initial assertion is wrong as it has lead us to an illogical conclusion. Hence, (2) has no integer solutions for all non-zero piecewise coprime (x, y, z) for all powers of $(n > 5) \in \mathbb{O}^+$. Combining this with the separate proofs for $(n = 3, 5)$, it follows that (2) has no integer solutions for all non-zero piecewise coprime integers (x, y, z) for all powers of $(n > 2) \in \mathbb{O}^+$.

Q.E.D.

2.4. Summary of the Two Proofs

In §(2.2.) and (2.3.), we have proved that (2) has no integer solutions for any $(x, y, z) > 0$ and $(x, y, z) \in \mathbb{N}^+$ for all powers of $(n > 2) \in \mathbb{E}^+$ and for all powers $(n > 2) \in \mathbb{O}^+$. Combining these two proofs, it follows from the foregoing as stated and outlined at the beginning of this section, that (2) has no integer solutions for any $(x, y, z) > 0$ and $(x, y, z) \in \mathbb{N}^+$ for all powers of $(n > 2) \in \mathbb{N}^+$. Hence Fermat's Last Theorem is here proved in a simpler and truly marvellous general manner for the case $(n > 5)$. Combining this with the separate proofs for $n = (3, 4, 5)$, we concluded that Fermat's Last Theorem is here proved in for $(n > 2)$.

Q.E.D.

3. Discussion and Conclusion

If the proof we have provided herein stands the test of time and experience, then, it is without a doubt that Fermat's claim to have had a 'truly marvellous' proof may very well resonate with truth. If this proof employed the use of Pythagoras theorem as in the present case, then, for any book, the standard 'margin is too narrow' to contain the present proof, the meaning of which is that Fermat was most certainly right in his famous claim. This is especially true given that the proof contains separate proofs for $n = (3, 4)$ were by themselves, these proofs can not be contained in a standard margin of a book.

Clearly, the problem with the proof is not that it is difficult and only accessible to the highly esoteric, no! We ourselves (*i.e.*, amateur and seasoned mathematicians alike) have made this problem appear very difficult, highly esoteric and only accessible to foremost and advanced mathematical minds. Without the historic and personal encodes that will soon follow, this proof (*i.e.*, the morass substance of the present reading) can be typed using a standard font size of between $10 - 12$, *back-to-back* on a single standard a4-page. Few – if any; would believe that this is possible. The level difficulty and esoteric nature associated with this problem has been – until the present reading, very high.

What could have happened leading to the elevation of this problem to a point where it came to become one of the most difficult problems in all History *of* Mathematics is that – perhaps; the plethora of maiden failures to provide a proof must have led people to think that this problem must be very difficult. Failure after failure and especially so by great mathematicians must then have led to it [Fermat's Last Theorem] achieving 'international, worldwide and historic notoriety' as a very difficult problem that eluded even great minds like Euler, Laplace and Gauss. With this kind of background, certainly, when people approached this problem, they most probably did so with in mind that it was a very difficult problem probably to be solved by 'real super geniuses' and not mortals of modest means *e.g.* ourself.

If someone told you that a given problem is so difficult, so much that it has thus far eluded the finest, advanced and most esoteric minds that have attempted to find its solution, one naturally tries to use higher advanced methods to prove it. Further, if someone told you that a given problem is so difficult, so much that it have eluded the finest, advanced and most esoteric minds that have attempted to find its solution, one naturally is discouraged from using simple elementary methods to prove it because the feeling one has is that, if it can be solved *via* a simple method, surely, advanced minds before me must have discovered this, thus leading one to try and climb higher than those before them. If what we have presented stands the test of time and experience, then, the way we approach difficult problems may need recourse, especially the way the public media projects and posts the level difficulty and the supposed esoteric effort required in-order to solve these problems.

Our approach to solving so-called outstanding problems is that one must not be let down by the public media projections of the level difficult and the supposed esoteric effort required in-order to solve the problem. First, as we climb the ladder of level difficultly, we tackle it [problem] from a level simplicity accessible to the 'layman' and step-by-step as we move up the ladder. To us, we have come to realise that this has helped us in understanding the problem at a much deeper level. At each level, we make sure we exhaust 'all' the possible avenues. As to how one knows they have exhausted all the possible avenues, this is a difficult question to answer but the most potent and virile tool for us has been a deep and strong inner intuition, unshakable confidence in the solubility of the problem and singular conviction that victory is certain if one persists.

As we anxiously await the World to judge our proof, effort and work, we must — if this be permitted at this point of closing, say that, we are confident that – simple as it is or may appear, this proof is flawless, it will stand the test of time and experience. It strongly appears that the great physicist and philosopher – Albeit Einstein $(1879 - 1955)$, was probably right in saying that "*Subtle is the Lord. Malicious He is not.*" because in Lemma (1), there exists deeply embedded therein, a subtlety that resolves and does away with the malice and notoriety associated with Fermat's Last Theorem in a simpler and truly marvellous and general manner.

Conclusion

We hereby make the following conclusion:

1. By use of the method of 'Pythagorean triples', we have demonstrated that a solution to Fermat's Last Theorem exists in the realm of elementary arithmetic.
2. This proof employs elementary arithmetic tools and methods that were certainly accessible to Fermat, thus making it highly likely that Fermat's claim that he possessed a 'truly marvellous' proof may very be true.

REFERENCES

[1] Andrew Wiles. Modular elliptic curves and fermat's last theorem. *Annals of Mathematics*, 141(3):443–551, 1995. doi:10.2307/2118559.

[2] T. Koshy. *Elementary Number Theory With Applications*. New York: Academic Press (ISBN 978-0124211711), UK, 2001. p. 544.

[3] Euler Leonhard. Vollständige anleitung zur algebra. *Royal Academy of Sciences* (St. Petersburg), 1770.

[4] C. F. Kausler. Nova demonstratio theorematis nec summam, nec differentiam duorum cuborum cubum esse posse. *Novi Acta Acad. Petrop*, 13:245–253, 1802.

[5] D. Gambioli. Memoria bibliographica sull'ultimo teorema di fermat. *Period. Mat.*, 16:145–192, 1928.

[6] A. M. Legendre. Recherches sur quelques objets d'analyse indéterminée, et particulièrement sur le théorème de fermat. *Mém. Acad. Roy. Sci. Institut France*, 6:1–60, 1823.

[7] A. M. Legendre. *Théorie des Nombres*, volume II. Paris: Firmin Didot Frères, 3rd edition, 1930. Reprinted in 1955 by A. Blanchard (Paris).

[8] F. J. Duarte. Sobre la ecuaci on $x^3+y^3+z^3=0$. *Ciencias Fis. Mat. Naturales* (Caracas), 8:971–979, 1944.

[9] D. Hilbert. Die theorie der algebraischen zahlkörper. 4:175–546, 1897. Reprinted in 1965 in Gesammelte Abhandlungen, Vol. I by New York: Chelsea.

[10] V. A. Lebesgue. Rèsolution des Équations biquadratiques $z^2=x^4\pm2^my^4, z^2=2^mx^4y^4, 2^mz^2=x^4\pm y^4$. *J. Math. Pures Appl.*, 18:73–86, 1853. Lebesgue, V. A. (1859). Exercices d'Analyse NumÃl'rique. Paris: Leiber et Faraguet. pp. 83-84, 89. Lebesgue, V. A. (1862). Introduction à la Théorie des Nombres. Paris: Mallet-Bachelier. pp. 71-73.

[11] L. Kronecker. Vorlesungen über zahlentheorie. I:33–38, 1901. Reprinted by New York: Springer-Verlag in 1978.

[12] M. Grant and M. Perella. Descending to the irrational. *Mathematical Gazette*, 83:263–267, July 1999.

[13] S. Dolan. Fermat's method of descente infinie. *Mathematical Gazette*, 95:269–271, July 2011.

[14] R. Barbara. Fermat's last theorem in the case $n=4$. *Mathematical Gazette*, 91:260–262, July 2007.

[15] J. C. F. Gauss. *Neue Theorie der Zerlegung der Cuben*, volume II. (Zur Theorie der complexen Zahlen, Werke) Königl. Ges. Wiss. Göttingen, 2^{nd} edition, 1875. (Published posthumous).

[16] V. A. Lebesgue. Thèorémes nouveaux sur l'équation indéterminée $x^5+y^5=az^5$. *J. Math. Pures Appl.*, 8:49–70, 1843.

[17] G. Lamé. Mémoire sur la résolution en nombres complexes de léquation $a^5+b^5+c^5=0$. *J. Math. Pures Appl.*, 12(137-171), 1847.

[18] D. Gambioli. Intorno all'ultimo teorema di fermat. *Pitagora*, II(10):11–13, 41–42., 1903/4.

[19] A. S. Werebrusow. On the equation $x^5+y^5=az^5$ (in russian). *Moskov. Math. Samml.*, 25:466–473, 1905.

[20] K. Rychlik. On fermat's last theorem for $n=5$ (in bohemian). *Časopis Pěst. Mat.*, 39:185–195, 305–317, 1910.

[21] J. G. van der Corput. Quelques formes quadratiques et quelques Ãl'Équations indéterminées. *Nieuw Archief Wisk.*, pages 45–45, 1915.

[22] G. Terjanian. Sur une question de v. a. lebesgue. *Ann. Inst. Fourier*, 37(3):19–37, 1987. doi:10.5802/aif.1096.