## Universal Gravitational Constant Via Proton


#### Abstract

Using a formula including the proton mass and Compton's wavelength for the proton, I obtained the value of the universal gravitational constant by two orders of magnitude more accurate than the recommended CODATA value [1].


## Introduction

The dimension of the universal gravitational constant $\mathbf{G}$ is: $\mathrm{M}^{-1} \mathrm{~L}^{3} \mathrm{~T}^{-2}$. If it is expressed in natural units [2], it has value by definition (in Planck units equals 1). The exact value of the constant is also possible in any other system in which G, or the values from which it could be directly derived, would by definition have exact values. That is not possible in the International System of Units [3] because in that system only the speed of light with dimensions $\mathrm{L}^{2} \mathrm{~T}^{-2}$ has exact value and can be used for determining G. For example, if in that system Planck mass and length would have the value by definition, then by using formula: $\mathbf{G}=\mathbf{c}^{2} \mathbf{1}_{\mathbf{p}} / \mathbf{m}_{\mathbf{p l}}$ ( $\mathrm{c}-$ speed of light, $1_{\mathrm{pl}}$ - Planck length, $\mathrm{m}_{\mathrm{pl}}$ - Planck mass), $\mathbf{G}$ would also have exact value. The same result could be obtained by applying some other combinations of the exactly defined values.

There is a large number of formulas which feature G, and still its value is known for its low accuracy in the SI. The reason for that is that the values which are included in the calculation of G are difficult to determine experimentally or cannot be determined at all. It is more common for those values to even be determined via the known G. Hence, in the following formulas at least one of the Planck values is always present:

$$
\begin{aligned}
\mathrm{G} & =\mathrm{c}^{2} \mathbf{1}_{\mathrm{p}} / \mathrm{m}_{\mathrm{pl}} \\
\mathrm{G} & =\mathrm{l}_{\mathrm{pl}}{ }^{3} / \mathrm{m}_{\mathrm{pl}} \mathrm{t}_{\mathrm{pl}}^{2} \\
\mathrm{G} & =\mathrm{hc} / \boldsymbol{\pi}^{\prime} \mathrm{m}_{\mathrm{pl}}
\end{aligned}
$$

Taken from [1]:

| Planck length | $1.616199 \mathrm{e}-35$ | $0.000097 \mathrm{e}-35 \mathrm{~m}$ |
| :--- | :--- | :--- |
| Planck mass | $2.17651 \mathrm{e}-8$ | $0.00013 \mathrm{e}-8 \mathrm{~kg}$ |
| Planck time | $5.39106 \mathrm{e}-44$ | $0.00032 \mathrm{e}-44 \mathrm{~s}$ |
| Planck constant | $6.62606957 \mathrm{e}-34$ | $0.00000029 \mathrm{e}-34 \mathrm{Js}$ |

Therefore we have:
Newtonian constant of gravitation $6.67384 \mathrm{e}-11 \quad 0.00080 \mathrm{e}-11 \mathrm{~m}^{\wedge} \mathrm{kg}^{\wedge-1} \mathrm{~s}^{\wedge-2}$
in the similar range of accuracy. On the right is the value of uncertainty expressed by $1 \sigma$, standard deviations. In the text below, the uncertainty will be shown in brackets, after the value of the physical quantity. Therefore, for the accurate determination of $G$ it is necessary to express this constant via the physical constants whose values can be determined experimentally with great accuracy.

## Formula for G

Starting from the statement 'Parts are dependent on the whole (Universe) and are also an integral part of the whole; therefore, the whole is also dependent on the parts!" I developed a
methodology which produced results in the articles published on [4]. Especially the article [5] shows the accuracy of determining the mass of tau particles by using the original formula.

Let's define the mathematical constants:

$$
t=\log (2 \pi, 2)=2.651496 \ldots, \text { Cycle, } c y=e^{2 \pi}=535.49165 \ldots, \text { Half cycle, } \mathrm{z}=\mathrm{e}^{2 \pi} / 2=267.74582776 \ldots
$$

The masses of the universe and proton are as follows:

$$
M_{u}=1.73944912 \mathrm{E}+53 \mathrm{~kg}[6], \mathrm{m}_{\mathrm{p}}=1.672621777 \mathrm{E}-27 \mathrm{~kg}[1]
$$

From [7], p - the constant related to the proton is:

$$
\begin{equation*}
p=\log \left(m_{u} / m_{p}, 2\right) \tag{1}
\end{equation*}
$$

And also:

$$
\begin{equation*}
z=\mathrm{e}^{2 \pi} / 2=\log \left(m_{u} / m_{z}, 2\right) \tag{2}
\end{equation*}
$$

Then we can define, let call it the proton shift, $\mathbf{z p}$ :

$$
\begin{equation*}
z p=z-p=\log \left(m_{p} / m_{z}, 2\right)=1.9350609435 \tag{3}
\end{equation*}
$$

We will also use physical constants $\boldsymbol{\mu}$ - proton-to-electron mass ratio and $\dot{\boldsymbol{\alpha}}$ - inverse fine-structure constant from [1]. They can also be used to determine the proton shift:

$$
\begin{equation*}
z p=\left(\mu / \alpha^{\prime}+1\right) /\left(\mu / \alpha^{\prime}+2\right)+1=1.9350609435 \tag{4}
\end{equation*}
$$

Or:

$$
\begin{equation*}
z p=\left[1+1 /\left(\mu / \alpha^{\prime}+1\right)\right]+1=1.9350609435 \tag{5}
\end{equation*}
$$

Or:

$$
\begin{equation*}
z p=\frac{1}{1+\frac{1}{\mu / \alpha^{\prime}+1}}+1=1.9350609435 \tag{5b}
\end{equation*}
$$

Also, from (3) and (5b):

$$
\begin{equation*}
p=e^{2 \pi}-\frac{1}{1+\frac{1}{\mu^{\prime} \alpha^{\prime}+1}}-1=265.8107668 \tag{6}
\end{equation*}
$$

If $\mathbf{m}_{\mathbf{p}}$ is the proton mass and $\boldsymbol{\lambda}_{\mathbf{p}}$ stands for the proton Compton wavelength, we obtain the following formula:

$$
\begin{equation*}
G=c^{2} m_{p}^{-1} * \lambda_{p} * 2^{(-c y / 4+3 z p / 2+t / 2)} \tag{7}
\end{equation*}
$$

Or:

$$
\begin{equation*}
G=c^{2} m_{p}^{-1} * \lambda_{p} * 2^{(z-3 p / 2+t / 2)} \tag{8}
\end{equation*}
$$

Or:

$$
\begin{equation*}
G=c^{2} m_{p}^{-1} * \lambda_{p} * \sqrt{2 \pi * 2^{(c y-3 p)}} \tag{9}
\end{equation*}
$$

All the physical quantities in (8) are related to the proton and are accurately determined experimentally.

## Testing the formula for $\mathbf{G}$

Here we will test the formula (8) by using the historical CODATA values. The CODATA values for $\dot{\boldsymbol{a}}, \mu, \lambda_{\mathrm{p}}, \mathbf{m}_{\mathrm{p}}$ are shown in Table $\mathbf{1}$, columns 1, 2, 4 and 5 . There, for example, we can see that each of the four physical constants in 2010 [1] have at least two significant digits more than $\mathbf{G}$, while the value of the speed of light $\mathbf{c}$ is exact by definition.

The seventh column of Table 1 shows the value of $G$ determined by the formula (8), so that once the upper value $\mathbf{G}^{\prime}$ is determined based on the CODATA values ( $\dot{\boldsymbol{\alpha}}, \boldsymbol{\mu}, \lambda_{\mathbf{p}}, \mathbf{m}_{\mathbf{p}}$ - for the corresponding year), and once the lower value $\underline{\mathbf{G}}$. The upper and lower values determine the uncertainty $+/-1 \sigma$, shown in brackets. Value $\left(\mathbf{G}^{\prime}-\underline{\mathbf{G}}\right) / \mathbf{2}$ is adopted to represent $\mathbf{1} \boldsymbol{\sigma}$.

Table 1
Determening the universal gravitational constant - G

|  | $\begin{aligned} & p=c y / 2-1 /\left[1+1 /\left(\mu^{\prime} / \dot{\alpha}^{\prime}+1\right)\right]-1 \\ & p^{\prime}=c y / 2-1 /\left[1+1 /\left(\mu^{\prime} \dot{\alpha}^{\prime}+1\right)\right]-1 \end{aligned}$ |  | $\begin{aligned} & \mathbf{G}^{\prime}=\mathbf{c}^{2} * \underline{m}_{p}^{-1} * \lambda_{p}^{\prime} * 2^{(\mathrm{cy} / 2-3 \mathrm{p} / 2+\mathrm{t} / 2)} \\ & \underline{\underline{G}}=\mathbf{c}^{2} * \mathbf{m}_{\mathrm{p}}^{\prime-1} * \underline{\lambda}_{\mathrm{p}} * 2^{(\mathrm{cy} / 2-3 \mathrm{p} / 2+\mathrm{t} / 2)} \end{aligned}$ |  |  |  | formula <br> value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year | $\begin{aligned} & \text { CODATA } \\ & \dot{\alpha}=1 / \text { alpha } \\ & \hline \end{aligned}$ | $\begin{gathered} \text { Values [1]: } \\ \mu=m_{p} / m_{e} \end{gathered}$ | $\begin{gathered} \text { c } \\ (\mathrm{m} / \mathrm{sec}) \end{gathered}$ | $\begin{gathered} \text { Compton } \lambda_{\mathrm{p}} \\ * 10^{-15} \mathrm{~m} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{m}_{\mathrm{p}} \\ * 10^{-27} \mathrm{~kg} \\ \hline \end{gathered}$ | $\begin{gathered} \mathbf{G} \\ * 10^{-} \\ \hline \end{gathered}$ | $\begin{gathered} \mathbf{G} \\ \mathrm{kg}^{-1} \mathrm{~m}^{3} \mathrm{~s}^{-2} \\ \hline \end{gathered}$ |
| 1969 | 137.03602(21) | 1836.1090(110) | 299792500 | 1.3214409(90) | 1.672614(11) | 6.6732 (31) | 6.67402(92) |
| 19 | 137.036040(110) | 1836.15152(70) | 299792458 | 1.3214099(22) | $1.6726485(86)$ | 6.6720(41) | 6.67373(46) |
| 1986 | 137.0359895(61) | 1836.152701(37) | 299792458 | 1.32141002 (12) | $1.6726231(10)$ | $6.67259(85)$ | 6.673832(46) |
| 1998 | 137.03599976(50) | 1836.1526675(39) | 299792458 | 1.321409847(10) | $1.67262158(13)$ | 6.673(10) | 6.6738367(57) |
| 2002 | 137.03599911(46) | 1836.15267261(85) | 299792458 | 1.3214098555(88) | 1.67262171(29) | 6.6742(10) | 6.673836(16) |
| 2006 | 137.035999679(94) | 1836.15267247(80) | 299792458 | 1.3214098446(19) | $1.672621637(83)$ | $6.67428(67)$ | $6.6738365(34)$ |
| 2010 | 137.035999074(45) | 1836.15267245(75) | 299792458 | $1.32140985623(94)$ | $1.672621777(74)$ | 6.67384(80) | 6.6738360(30) |

Table 1 shows that the value of G determined by the formula in year 1973 achieved the accuracy from year 2010 in [1]. The value of G determined by the formula for year 2010 has two significant digits more that the CODATA value.


Figure 1

## Universal gravitational constant - G in the 1969-2010 period CODATA values [1] and values achieved by formula (8)

Figure 1 visually presents the advantage of determining the value of $G$ by applying the formula in relation to the CODATA method.

## Conclusion

The article shows the predictive power of the formula (8) for determining the value of the universal gravitational constant $G$ by applying physical constants whose experimental determination gives the values much more accurate than the experimentally obtained G.

In the formula (9), the values are:

$$
\begin{align*}
& R_{u}=\lambda_{p} * \sqrt{2 \pi * 2^{(c y-p)}}=1.2916530 \mathrm{E}+26 \mathrm{~m}  \tag{10}\\
& M_{u}=m_{p} * 2^{p}=1.73944912 \mathrm{E}+53 \mathrm{~kg} \tag{11}
\end{align*}
$$

$\mathbf{R}_{\mathbf{u}}$ is radius of universe and $\mathbf{M}_{\mathbf{u}}$ is mass of universe.
Then, from (9), (10) and (11):

$$
\begin{equation*}
G=c^{2} M_{u}^{-1} * R_{u}=M_{u}^{-1} * R_{u}^{3} * T_{u}^{-2} \tag{12}
\end{equation*}
$$

which is the basic and simple formula presenting the essence of the universal gravitational constant. There is also a possibility to determine G even more accurately through other constants or even exactly by redefining the International System of Units.

## References:

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