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Intuitionistic Fuzzy Ideals Topological Spaces

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Abstract

In this paper we introduce the notion of intuitionistic fuzzy ideals which is considered as a generalization of fuzzy ideals studies in [1, 2, 3, 11], the important intuitionistic fuzzy ideal has been given. The concept of intuitionistic fuzzy local function is also introduced here by utilizing the ε - neighborhood structure for an intuitionistic fuzzy topological space. These concepts are discussed with a view to find new intuitionistic fuzzy topology from the original one in [10, 12].The basic structure, especially a basis for such generated intuitionistic fuzzy topologies and several relations between different intuitionistic fuzzy ideals and intuitionistic fuzzy topologies are also studied here. Finally, several properties of all investigated new notions are discussed.

Keywords: intuitionistic fuzzy ideals; intuitionistic fuzzy local function

1. Introduction

The concept of fuzzy sets and fuzzy set operations was first introduced by Zadeh [12]. Accordingly, fuzzy topological spaces were introduced by Chang [8]. Several researches were the generalizations of the notion of fuzzy set. The idea of intuitionistic fuzzy set (IFS, for short) was first published by Atanassov [4,5,6]. Subsequently, Coker and Saadati [8, 10] defined the notion of intuitionistic fuzzy topology and studied the basic concept of intuitionistic fuzzy point [10]. Our aim in this paper is to extend those ideas of general topology in intuitionistic fuzzy topological space (IFTS, in short). In section 3, we define intuitionistic fuzzy ideal for a set. Here we generalize the concept of fuzzy ideal topological concepts, first initiated by Sarker [10] in the case of intuitionistic fuzzy sets. In section 4, we introduce the notion of the intuitionistic fuzzy local function corresponding to IFTS.

Recently we have deduced some characterization theorems for such concepts exactly analogous to general topology and succeeded in finding out the generated new intuitionistic fuzzy topologies for any IFTS.

2. Preliminaries

Deifintion.2.1.[10]. A nonempty collection of fuzzy sets ℓ of a set X is called fuzzy ideal on X iff i) A $\in \ell$ and B $\subseteq$ A $\Rightarrow$ A $\in \ell$ (heredity),
(ii) A $\in \ell$ and B $\in \ell$ $\Rightarrow$ A $\vee$ B$\in \ell$ (finite additivity).

We shall present the fundamental definitions given by Atanassov:

Definition .2.2. [5, 6, 7]. Let X is a nonempty fixed set. An intuitionistic fuzzy set (IFS for short) A is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x) : x \in X \rangle\}$where the function $\mu_A : X \to [0,1]$and$\nu_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non- membership (namely $\nu_A(x)$)of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$.

Remark. 2.1. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A, \nu_A \rangle$ for the IFS $A = \{\langle x, \mu_A(x), \nu_A(x) : x \in X \rangle\}$.

Definition 2.3.[9]. $O_\sim = \{\langle x, 0, 1 : x \in X \rangle\}$ and $1_\sim = \{\langle x, 1, 0 \rangle : x \in X\}$

Definition 2.4.[9]. An intuitionistic fuzzy topology (IFT for short) on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:
(T_1) $O_\sim, 1_\sim \in \tau$,
(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(T_3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X .

Definition 2.5.[8,10]. The complement $C(A)$ of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X .

Definition 2.6.[9]. Let (X, τ) IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be IFS in X. Then the fuzzy interior and fuzzy closure of A are defined by:
$cl(A) = \cap\{K : K \textit{ is} \text{ an IFCS in X and } A \subseteq K\}$,

$\text{int}(A) = \bigcup\{G : G \text{ is an IFOS in X and G} \subseteq \text{A}\}$.

3- Basic properties of Intuitionistic Fuzzy Ideals.

Definition .3.1. Let X is non-empty set and L a non–empty family of IFSs. We will call L is a intuitionistic fuzzy ideal(IFL for short)on X if
$A \in L$ and $B \subseteq A \Rightarrow B \in L$ [heredity],
$A \in L$ and $B \in \text{L} \Rightarrow A \vee B \in \text{L}$ [Finite additivity].

A fuzzy ideal L is called a σ-fuzzy ideal if $\{A_j\}_{j \in N} \leq L$, implies $\underset{j \in J}{\vee} A_j \in L$ (countable additivity).

The smallest and largest intuitionistic fuzzy ideals on a non -empty set X are $\{0_\sim\}$ and IFSs on X. Also, $F.\text{L}_\text{f}$, F.L_c are denoting the intuitionistic fuzzy ideals (IFLS for short) of fuzzy subsets having finite and countable support of X respectively. Moreover, if A is a nonempty IFS in X, then $\{B \in IFS : B \subseteq A\}$ is an IFL on X. This is called the principal IFL of all IFSs of denoted by IFL$\langle A \rangle$.

Remark 3.1.
i) If $1_\sim = \{\langle x,1,0 \rangle : x \in X\} \notin$ L, then L is called intuitionistic fuzzy proper ideal.
ii) If $1_\sim \in$ L, then L is called intuitionistic fuzzy improper ideal.
iii) $O_\sim = \{\langle x,1,0 \rangle : x \in X\} \in$L.

Example.3.1. Any fuzzy ideal ℓ on X in the sense of Sarker is obviously and IFL in the form $L = \{A : A = \langle x, \mu_A, \nu_A \rangle \in \ell\}$.

Example.3.2. Let $A = \langle x,0.2,0.6 \rangle$, $B = \langle x,0.5,0.8 \rangle$, and $D = \langle x,0.5,0.6 \rangle$, then the family $L = \{O_\sim, A, B, D\}$ of IFSs is an IFL on X.

Example.3.3. Let $X = \{a,b,c,d,e\}$ and $A = \langle x, \mu_A, \nu_A \rangle$ given by :

X	$\mu_A(x)$	$\nu_A(x)$
a	0.6	0.3
b	0.5	0.3
c	0.4	0.4
d	0.3	0.5
e	0.3	0.6

Then the family $L = \{O_\sim, A\}$ is an IFL on X.

Definition .3.2. For every two IFSs A and B. Then A = B [mod L] if A Δ B ∈ L, Δ is a symmetric differences of two IFSs.

Definition. 3.3.. Let L_1 and L_2 are two IFLs on X. Then L_2 is said to be finer than L_1 or L_1 is coarser than L_2 if $L_1 \leq L_2$. If also $L_1 \neq L_2$. Then L_2 is said to be strictly finer than L_1 or L_1 is strictly coarser than L_2.

Two IFLs said to be comparable, if one is finer than the other. The set of all IFLs on X is ordered by the relation L_1 is coarser than L_2 this relation is induced the inclusion in IFSs.

The next Proposition is considered as one of the useful result in this sequel, whose proof is clear.

Proposition.3.1. Let $\{L_j : j \in J\}$ be any non - empty family of intuitionistic fuzzy ideals on a set X. Then $\bigcap_{j\in J} L_j$ and $\bigcup_{j\in J} L_j$ are intuitionistic fuzzy ideal on X, where $\bigcap_{j\in J} L_j = \langle \wedge\mu_{L_j}, \vee\mu_{L_j} \rangle$ and $\wedge\mu_{L_j}(x) = \inf\{\mu_{A_i}(x) : i \in J, x \in X\}$ $\vee\nu_{L_j}(x) = \sup\{\nu_{A_i}(x) : i \in J, x \in X\}$

In fact L is the smallest upper bound of the set of the L_j in the ordered set of all intuitionistic fuzzy ideals on X.

Remark.3.2. The intuitionistic fuzzy ideal by the single intuitionistic fuzzy set $O_\sim = \{\langle x,1,0\rangle : x \in X\}$ is the smallest element of the ordered set of all intuitionistic fuzzy ideals on X.

Proposition.3.3. A IFS A in intuitionistic fuzzy ideal L on X is a base of L iff every member of L contained in A.

Proof. (Necessity) Suppose A is a base of L. Then clearly every member of L contained in A.

(Sufficiency) Suppose the necessary condition holds. Then the set of intuitionistic fuzzy subset in X contained in A coincides with L by the Definition 1.3.

Proposition.3.4. For an intuitionistic fuzzy ideal L_1 with base A, is finer than a fuzzy ideal L_2 with base B iff every member of B contained in A.

Proof. Immediate consequence of Definitions

Corollary.3.1. Two intuitionistic fuzzy ideals bases A, B, on X are equivalent iff every member of A, contained in B and via versa.

Theorem.3.1. Let $\eta = \{\mu_j : j \in J\}$ be a non empty collection of intuitionistic fuzzy subsets of X. Then there exists a intuitionistic fuzzy ideal L $(\eta) = \{A \in \text{IFSs}: A \subseteq \vee$

A_j} on X for some finite collection {A_j : j = 1,2, ..., n $\subseteq \eta$}.

Proof: Clear.

Remark.3.3
ii) The intuitionistic fuzzy ideal L (η) defined above is said to be generated by η and η is called subbase of L (η).

Corollary.3.2. Let L_1 be an intuitionistic fuzzy ideal on X and A $\in$ IFSs, then there is a intuitionistic fuzzy ideal L_2 which is finer than L_1 and such that A $\in L_2$ if and only if A $\vee$ B $\in L_2$ for each B$\in L_1$.

Theorem.3.2. If an IFS $L = \{O_\sim, \langle \mu_A, \nu_A \rangle\}$ is an intuitionistic fuzzy ideal on X , then so is
□ L=$\left\{O_\sim, \left\langle \mu_A, \bar{\mu}_A \right\rangle\right\}$ is an intuitionistic fuzzy ideal on X.

Proof. Clear

Theorem.3.3. An IFS $L = \{O_\sim, \langle \mu_A, \nu_A \rangle\}$ is an intuitionistic fuzzy ideal on X if and only if the fuzzy sets μ_A, and $C(\nu_A)$ are fuzzy ideals on X.

Proof. Let $L = \{O_\sim, \langle \mu_A, \nu_A \rangle\}$ be an IFL of X, $A = \langle x, \mu_A, \nu_A \rangle$, Ten clearly μ_A is a fuzzy ideal on X. Then $C(\nu_A(x)) = 1 - \nu_A(x) = \max\left\{\left(\bar{\nu}_A(x), 0\right)\right\} = \min\left\{1, 1 - \nu_A(x)\right\}$ if $\nu_{C(A)}(x) = O_x$ then is the smallest fuzzy ideal or $\bar{\nu}_A(x) = 1_x$ then is the largest fuzzy ideal on X.

Corollary.3.3. an IFS $L = \{O_\sim, \langle \mu_A, \nu_A \rangle\}$ is an intuitionistic fuzzy ideal on X if and only if □L=$\left\{O_\sim, \left\langle \mu_A, \bar{\mu}_A \right\rangle\right\}$ and $\Diamond L = \left\{O_\sim, \left\langle \bar{\nu}_A, \nu_A \right\rangle\right\}$ are intuitionistic fuzzy ideals on X.

Proof. Clear from the definition 1.3.

Example.3.4. Let X a non empty set and IFL on X given by: $L = \{O_\sim, \langle 0.3, 0.6 \rangle, \langle 0.3, 0.5 \rangle \langle 0.2, 0.5 \rangle\}$.Then □L=$\{O_\sim, \langle 0.3, 0.7 \rangle, \langle 0.2, 0.8 \rangle\}$ and $\Diamond$

$L = \{O_{\sim}, \langle 0.4, 0.6 \rangle, \langle 0.5, 0.5 \rangle\}$ and □L ⊆ ◊L.

Theorem.3.4. Let $A = \langle x, \mu_A, \nu_A \rangle \in L_1$ and $B = \langle x, \mu_B, \nu_B \rangle \in L_2$, where L_1 and L_2 are intuitionistic fuzzy ideals on the set X. then the intuitionistic fuzzy set $A * B = \langle \mu_{A*B}(x), \nu_{A*B}(x) \rangle \in L_1 \vee L_2$ on X. and $\mu_{A*B}(x) = \vee\{\mu_A(x) \wedge \mu_B(x) : x \in X\}$, and $\nu_{A*B}(x) = \wedge\{\nu_A(x) \vee \nu_B(x) : x \in X\}$.

Definition.3.4. For a IFTS (X, τ), A ∈ IFSs. Then A is called

i) Intuitionistic fuzzy dense if cl (A) = $1_{\sim}$.

ii) Intuitionistic fuzzy nowhere dense subset if Int (cl(A))= $O_{\sim}$.

iii) Intuitionistic fuzzy codense subset if Int (A) = $O_{\sim}$.

v) Intuitionistic fuzzy countable subset if it is a finite or has the some cardinal number.

iv) Intuitionistic fuzzy meager set if it is a intuitionistic fuzzy countable union of intuitionistic fuzzy nowhere dense sets.

The following important Examples of intuitionistic fuzzy ideals on IFTS (X.τ).

Example.3.5. For a IFTS (X, τ) and L_n= {A ∈ IFSs : Int (cl(A)) = $O_{\sim}$ } is the collection of intuitionistic fuzzy nowhere dense subsets of X. It is a simple task to show that L_n is intuitionistic fuzzy ideal on X.

Example.3.6 For an IFTS (X,τ) and L_m = {A ∈ IFSs: A is a countable union of intuitionistic fuzzy nowhere dense sets} the collection of intuitionistic fuzzy meager sets on X. one can deduce that L_m is intuitionistic fuzzy σ - ideal on X.

Example.3.7. For an IFTS (X,τ) with intuitionistic fuzzy ideal L. then
$< L \cap \tau^c > = \{A \in$ IFSs: there exists $B \in L \cap \tau^c$ such that $A \subseteq B\}$ is a intuitionistic fuzzy ideal on X.

Example.3.8. Let f: (X,τ_1) (Y, τ_2) ~~be~~ a function, and L, J are two intuitionistic fuzzy ideals on X and Y respectively. Then

i) f (L) = {f (A): A ∈ L} is an intuitionistic fuzzy ideal.

ii) If f is injection. Then f^{-1} (J) is intuitionistic fuzzy ideal on X.

4.Intuitionistic Fuzzy local Functions and $*-$IFTS

Definition.4.1. Let (X, τ) be an intuitionistic fuzzy topological spaces (IFTS for short) and L be intuitionistic fuzzy ideal (IFL, for short) on X. Let A be any IFS of X. Then the intuitionistic fuzzy local function $A^*(L,\tau)$ of A is the union of all intuitionistic fuzzy points (IFP, for short) $C(\alpha,\beta)$ such that if $U \in N(C(\alpha,\beta))$ and $A^*(L,\tau) = \vee\{C(\alpha,\beta) \in X : A \wedge U \notin L \text{ for every } U \in N(C(\alpha,\beta))\}$.

$A^*(L,\tau)$ is called an intuitionistic fuzzy local function of A with respect to τ and L which it will be denoted by $A^*(L,\tau)$, or simply $A^*(L)$.

Example .4.1. One may easily verify that.

If L= $\{0_\sim\}$, then $A^*(L,\tau) = cl(A)$, for any intuitionistic fuzzy set $A \in IFSs$ on X.

If $L = \{\text{all IFSs on X}\}$ then $A^*(L,\tau) = 0_\sim$, for any $A \in IFSs$ on X .

Theorem.4.1. Let (X,τ) be a IFTS and L_1, L_2 be two intuitionistic fuzzy ideals on X. Then for any intuitionistic fuzzy sets A, B of X. then the following statements are verified

$A \subseteq B \Rightarrow A^*(L,\tau) \subseteq B^*(L,\tau)$,

$L_1 \subseteq L_2 \Rightarrow A^*(L_2,\tau) \subseteq A^*(L_1,\tau)$.

$A^* = cl(A^*) \subseteq cl(A)$.

$A^{**} \subseteq A^*$.

$(A \vee B)^* = A^* \vee B^*$.,

$(A \wedge B)^*(L) \leq A^*(L) \wedge B^*(L)$.

$\ell \in L \Rightarrow (A \vee \ell)^* = A^*$.

$A^*(L,\tau)$ is fuzzy closed set .

Proof.

Since $A \subseteq B$, let $p = C(\alpha,\beta) \in A^*(L_1)$ then $A \wedge U \notin L$ for every $U \in N(p)$. By hypothesis we get $B \wedge U \notin L$, then $p = C(\alpha,\beta) \in B^*(L_1)$.

Clearly. $L_1 \subseteq L_2$ implies $A^*(L_2,\tau) \subseteq A^*(L_1,\tau)$ as there may be other IFSs which belong to L_2 so that for IFP $p = C(\alpha,\beta) \in A^*$ but $C(\alpha,\beta)$ may not be contained in $A^*(L_2)$.

Since $\{O_\sim\} \subseteq L$ for any IFL on X, therefore by (ii) and Example 1.4, $A^*(L) \subseteq A^*(\{O_\sim\}) = cl(A)$ for any IFS A on X. Suppose $p_1 = C_1(\alpha,\beta) \in cl(A^*(L_1))$. So for every $U \in N(p_1)$, $A^* \wedge U \neq O_\sim$, there exists $p_2 = C_2(\alpha,\beta) \in A^*(L_1) \wedge U)$ such that fore every V $\varepsilon - nbd$ of $p_2 \in N(p_2)$, $A \wedge U \notin L$. Since $U \wedge V \in N(p_2)$ then $A \wedge (U \cap V) \notin L$ which leads to $A \wedge U \notin L$, for every $U \in N(C(\alpha,\beta))$

therefore $p_1 = C(\alpha,\beta) \in (A^*(L))$ and so $cl(A^*) \leq A^*$. while, the other inclusion follows directly. Hence $A^* = cl(A^*)$ .But the inequality $A^* \leq cl(A^*)$.

The inclusion $A^* \vee B^* \leq (A \vee B)^*$ follows directly by (i). To show the other implication, let $p = C(\alpha,\beta) \in (A \vee B)^*$ then for every $U \in N(p)$, $(A \vee B) \wedge U \notin L, i.e, (A \wedge U) \vee (B \wedge U) \notin L$. then, we have two cases

$A \wedge U \notin L$ and $B \wedge U \in L$ or the converse, this means that exist $U_1, U_2 \in N(C(\alpha,\beta)$ such that $A \wedge U_1 \notin L$, $B \wedge U_1 \notin L$, $A \wedge U_2 \notin L$ and $B \wedge U_2 \notin L$. Then $A \wedge (U_1 \wedge U_2) \in L$ and $B \wedge (U_1 \wedge U_2) \in L$ this gives $(A \vee B) \wedge (U_1 \wedge U_2) \in L$, $U_1 \wedge U_2 \in N(C(\alpha,\beta)$ which contradicts the hypothesis. Hence the equality holds in various cases .

By (iii), we have $A^{*^*} = cl(A^*)^* \leq cl(A^*) = A^*$

Let (X,τ) be a IFTS and L be IFL on X . Let us define the intuitionistic fuzzy closure operator $cl^*(A) = A \cup A^*$ for any IFS A of X. Clearly, let $cl^*(A)$ is a intuitionistic fuzzy operator. Let $\tau^*(L)$ be IFT generated by cl^* .i.e $\tau^*(L) = \{A : cl^*(A^c) = A^c\}$. Now $L = \{O_\sim\} \Rightarrow cl^*(A) = A \cup A^* = A \cup cl(A)$ for every intuitionistic fuzzy set A. So, $\tau^*(\{O_\sim\}) = \tau$. Again $L = \{all \text{ IFSs on X}\} \Rightarrow$ $cl^*(A) = A$, because $A^* = O_\sim$, for every intuitionistic fuzzy set A so $\tau^*(L)$ is the intuitionistic fuzzy discrete topology on X. So we can conclude by Theorem 4.1.(ii). $\tau^*(\{O_\sim\}) = \tau^*(L)$ i.e. $\tau \subseteq \tau^*$, for any intuitionistic fuzzy ideal L_1 on X. In particular, we have for two intuitionistic fuzzy ideals L_1, and L_2 on X, $L_1 \subseteq L_2 \Rightarrow \tau^*(L_1) \subseteq \tau^*(L_2)$.

Theorem.4.2. Let τ_1, τ_2 be two intuitionistic fuzzy topologies on X. Then for any intuitionistic fuzzy ideal L on X, $\tau_1 \leq \tau_2$ implies

$A^*(L,\tau_2) \subseteq A^*(L,\tau_1)$, for every $A \in L$.

${\tau^*}_1 \subseteq {\tau^*}_2$

Proof. Since every τ_1-$\varepsilon - nbd$ of any IFP $C(\alpha,\beta)$ is also a τ_2-$\varepsilon - nbd$ of $C(\alpha,\beta)$ .therefore, $A^*(L,\tau_2) \subseteq A^*(L,\tau_1)$ as there may be other of $C(\alpha,\beta)$ where the condition for $C(\alpha,\beta) \in A^*(\tau_2, L)$ may not hold true, although $C(\alpha,\beta) \in A^*(\tau_1, L)$ Clearly ${\tau^*}_1 \subseteq {\tau^*}_2$ as $A^*(L,\tau_2) \subseteq A^*(L,\tau_1)$.

A basis $\beta(L,\tau)$ for $\tau^*(L)$ can be described as follows:

$\beta(L,\tau) = \{A - B : A \in \tau, B \in L\}$ Then we have the following theorem

Theorem. 4.3. $\beta(L,\tau) = \{A - B : A \in \tau, B \in L\}$ forms a basis for the generated IFT-τ^* of the IFT (X,τ) with intuitionistic fuzzy ideal L on X.

Proof. Straight forward.

The relationship between τ and τ^*(L) established throughout the following result which have an immediately proof.

Theorem. 4.4. Let τ_1, τ_2 be two intuitionistic fuzzy topologies on X. Then for any intuitionistic fuzzy ideal L on X, $\tau_1 \subseteq \tau_2$ implies $\tau^*{}_1 \subseteq \tau^*{}_2$.

Theorem. 4.5. Let (X,τ) be a IFTS and L_1, L_2 be two intuitionistic fuzzy ideals on X . Then for any intuitionistic fuzzy set A in X, we have

$$A^*(L_1 \vee L_2, \tau) = A^*(L_1, \tau^*(L_1)) \wedge A^*(L_2, \tau^*(L_2))$$

$$\tau^*(L_1 \vee L_2) = (\tau^*(L_1))^*(L_2) \wedge (\tau^*(L_2))^*(L_1)$$

Proof. Let $p = C(\alpha,\beta) \notin (L_1 \vee L_2, \tau)$, this means that there exists $U_p \in N(P)$ such that $A \wedge U_p \in (L_1 \vee L_2)$i.e. There exists $\ell_1 \in L_1$ and $\ell_2 \in L_2$ such that $A \wedge U_p \in (\ell_1 \vee \ell_2)$ because of the heredity of L_1, and assuming $\ell_1 \wedge \ell_2 = O_\sim$.Thus we have $(A \wedge U_p) - \ell_1 = \ell_2$ and $(A \wedge U_p) - \ell_2 = \ell_1$ therefore $(U_p - \ell_1) \wedge A = \ell_2 \in L_2$ and $(U_p - \ell_2) \wedge A = \ell_1 \in L_1$.Hence $p = C(\alpha,\beta) \notin A^*(L_2, \tau^*(L_1))$ or $p = C(\alpha,\beta) \notin A^*(L_1, \tau^*(L_2))$ because p must belong to either ℓ_1 or ℓ_2 but not to both. This gives $A^*(L_1 \vee L_2, \tau) \geq A^*(L_1, \tau^*(L_1)) \wedge A^*(L_2, \tau^*(L_2))$ .To show the second inclusion, let us assume $p = C(\alpha,\beta) \notin A^*(L_1, \tau^*(L_2))$. This implies that there exist $U_p \in N(P)$ and $\ell_2 \in L_2$ such that $(U_p - \ell_2) \wedge A \in L_1$. By the heredity of L_2, if we assume that $\ell_2 \leq A$ and define $\ell_1 = (U_p - \ell_2) \wedge A$. Then we have $A \wedge U_p \in (\ell_1 \vee \ell_2) \in L_1 \vee L_2$. Thus, $A^*(L_1 \vee L_2, \tau) \leq A^*(L_1, \tau^*(L_1)) \wedge A^*(L_2, \tau^*(L_2))$ and similarly, we can get $A^*(L_1 \vee L_2, \tau) \leq A^*(L_2, \tau^*(L_1))$. This gives the other inclusion, which complete the proof.

Corollary. 4.1. Let (X,τ) be a IFTS with intuitionistic fuzzy ideal L on X. Then

$A^*(L,\tau) = A^*(L,\tau^*)$ and $\tau^*(L) = (\tau^*(L))^*(L)$.

$\tau^*(L_1 \vee L_2) = \left(\tau^*(L_1)\right) \vee \left(\tau^*(L_2)\right)$

Proof. Follows by applying the previous statement.

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