

# Possible CGLE signatures in solar system: Spiral gravity from spherical kinetic dynamics<sup>1</sup>

The present article discusses how some known phenomena in solar system, including the Lense-Thirring effect of anomalous precession, could be described using spherical kinetic dynamics approach. Other implications include a plausible revised version of the Bohr-Sommerfeld quantization equation described by Rubčić & Rubčić. Our proposition in this paper can be summarized as follows: by introducing time-incremental to the ordinary celestial quantization method (Nottale *et al.*), we can expect to observe signatures of CGLE (complex Ginzburg-Landau equation) in Solar system. Possible verification may include the use of Earth-based satellites, which go beyond traditional GTR tests such as precession of the first planet. Further observation to verify or refute this conjecture is recommended, plausibly using LAGEOS-type satellites.

*Keywords:* Lense-Thirring effect, Bohr-Sommerfeld quantization, quantized vortices, celestial quantization, LAGEOS satellite, boson condensation, signature of CGLE in solar system, spiral gravity

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## Introduction

It is known that the use of Bohr radius formula to predict celestial quantization, based on Bohr-Sommerfeld quantization rules [2][3], has led to numerous verified observations [1]. While this kind of approach is not widely accepted yet, this could be related to wave mechanics equation to describe large-scale structure of the Universe [4], and also a recent suggestion to reconsider Sommerfeld's conjectures in Quantum Mechanics [5]. Some implications of this quantum-like approach include exoplanet prediction, which becomes a rapidly developing subject in recent years [6][7].

Rubić & Rubić's approach [2] is particularly interesting in this regard, because they begin with a conjecture that Planck mass ( $m_p = \sqrt{\hbar c / 2\pi G}$ ) is the basic entity of Nature, which apparently corresponds to Winterberg's assertion that Planckian aether is comprised superfluid of phonon-roton pairs [8]. In each of these pairs, superfluid vortices can form with circulation quantized according to  $\oint v_{+-} dx = n\hbar / m_p$ . This condition implies the Helmholtz vortex theorem,  $d/dt \oint v_{+-} dx = 0$ . This relationship seems conceivable, at least from the viewpoint of likely neat linkage between cosmology phenomena and various low-temperature condensed matter physics [9][10][11]. In effect, celestial objects at various scales could also be regarded as spinning Bose-Einstein condensate; which method has been used for neutron stars [32].

Despite these aforementioned advantages of using quantum mechanical viewpoint to describe astrophysical phenomena, it is also known that all of the existing celestial quantization methods [1][2][3] thus far have similarity that they assume a circular motion, while the actual celestial orbits (and also molecular orbits) are elliptical.

Historically, this was the basis of Sommerfeld's argument in contrast to Bohr's model, which also first suggested that any excess gravitational-type force would induce a precessed orbit. Similar argument is used here as the starting premise of the present article, albeit for brevity we will not introduce elliptical effect yet [12].

Using a known *spherical kinetic dynamics* approach, some known interesting phenomena are explained, including the receding Moon, the receding Earth from the Sun, and also anomalous precession of the first planet (*Lense-Thirring effect*). Despite some recent attempts to rule out the gravitational quadrupole moment ( $J_2$ ) contribution to this effect [13][14][15][16][17], it seems that the role of spherical kinetic dynamics [12] to describe the origin of Lense-Thirring effect has not been taken into consideration thus far, at least to this author's knowledge.

After deriving prediction for these known observed phenomena, this article will also present a revised version of quantization equation of L. Nottale [1] in order to take into consideration this spherical kinetic dynamics effect. Some implications are discussed, including possible time-incremental modification of ordinary Bohr-type quantization for solar system, which can take the form of spiral gravity. In turn, this 'spiralling gravity' phenomena can be considered as signatures of CGLE (complex Ginzburg Landau equation) in solar system.

Our paper starts from simple hypothesis that smaller celestial objects acquire its (spinning) energy from the larger systems. That is, Earth spinning motion gets its energy from the Sun. In turn, Solar system gets its spinning energy from its Galaxy center. One can say that this is just an astrophysics implications of turbulence dynamics (see Gibson et al. [22][23]), where *energy cascades from the larger scales down to the smaller scales*.

If this proposition described here corresponds to the facts, then one can say that it is possible to ‘re-derive’ General Relativity phenomena from the viewpoint of Bohr-Sommerfeld quantization and spherical kinetic dynamics. Possible verification of this proposition may include the use of Earth-based satellites, which go beyond traditional GTR-tests such as precession of the first planet. Further observation to verify or refute this conjecture is recommended, plausibly using LAGEOS-type satellites ( see Ciufolini and others [14]-[16]).

## Spherical kinetic dynamics: Earth bulging from Earth geodynamics

Analysis of spinning dynamics of solid sphere with mass M (see Appendix I) yields:

$$\Delta M / \Delta t = -\dot{\omega} \cdot MR^2 \omega / (5 \cdot c_s^2) \quad (1)$$

where  $c_s$  represents the sound velocity obeying [10b; p.4]:

$$c_s^2(n) = (n/m)(d^2 \epsilon / dn^2) \quad (2)$$

For  $\dot{\omega} = 0$  the equation (1) shall equal to zero, therefore this equation (1) essentially says that a linear change of angular velocity observed at the *surface* of the spinning mass corresponds to mass flux, albeit this effect is almost negligible in daily experience. But for celestial mechanics, this effect could be measurable.

If, for instance, we use the observed anomalous deceleration rate [30] of angular velocity of the Earth as noted by Kip Thorne [19]:

$$|\omega| / |\dot{\omega}| = 6 \times 10^{11} \text{ years} \quad (3)$$

And using values as described in Table 1 for other parameters:

Table 1. Parameter values to compute kinetic expansion of the Earth

| Parameter  | Value                 | Unit  |
|------------|-----------------------|-------|
| $R_e$      | $6.38 \times 10^6$    | M     |
| $M_e$      | $5.98 \times 10^{24}$ | Kg    |
| $T_e$      | $2.07 \times 10^6$    | Sec   |
| $\omega_e$ | $3.04 \times 10^{-6}$ | rad/s |
| $c_s$      | 0.14112               | m/s   |

It is perhaps worth noting that the only free parameter here is  $c_s = 0.14112$  m/sec. This value is approximately within the range of Barcelo *et al.*'s estimate of sound velocity (at the order of cm/sec) for gravitational Bose-Einstein condensate [11], provided the Earth could be regarded as a spinning Bose-Einstein condensate. Alternatively, the sound velocity could be calculated using equation (ii) in Appendix I, but this obviously introduces another kind of uncertainty in the form of determining temperature (T) inside the center of the Earth; therefore this method is not used here.

Then by inserting these values from equation (3) and Table 1 into equation (1) yields:

$$\Delta M / \Delta t \approx 3.76 \times 10^{16} \text{ kg / year} \quad (4)$$

Perhaps this effect could be related to a recent Earth bulging data, which phenomenon lacks a coherent explanation thus far [36].

Now we want to know how this mass accumulation affects the Earth surface and also its rotational period. Assuming a solid sphere, we start with a known equation [34]:

$$M = 4\pi \cdot \rho_{\text{sphere}} \cdot r^3 / 3 \quad (5)$$

where  $\rho_{\text{sphere}}$  is the average density of the 'equivalent' solid sphere. For Earth data (Table 1), we get  $\rho_{\text{sphere}} = 5.50 \times 10^6$  gr/m<sup>3</sup>. Using the same method with equation (8f), which will be discussed subsequently, equation (5) could be rewritten as:

$$M + \Delta M / \Delta t = 4\pi \cdot \rho_{sphere} \cdot (r + \Delta r / \Delta t)^3 / 3 \quad (6)$$

or

$$\Delta r / \Delta t = \sqrt[3]{(M + \Delta M / \Delta t) \cdot 3 / (4\pi \cdot \rho_{sphere})} - r \quad (6a)$$

From equation (7) we get  $dr/dt=13.36$  mm/year for Earth.

It would be worth here to compare this result with the known *Expanding Earth hypothesis* by Pannella, Carey, Vogel, Shields and others, who suggested that the Earth was only 60% of its present size in the Jurassic [49]. There is also a recent suggestion that Earth has experienced a slow down in spin rate during the past  $9 \times 10^8$  years.<sup>2</sup> To get a numerical estimate of Earth's radial increase each year, we quote here from Smoot [49]:

“In order for this to happen, the lunar tides would have to slow down, which would affect the length of the lunar month. ... an Earth year of 447 days at 1.9 Ga decreasing to an Earth year of 383 days at 290 Ma to 365 days at this time. However, the Devonian coral rings show that the *day is increasing by 24 seconds every million years*, which would allow for an expansion rate of about 0.5% for the past 4.5 Ga, all other factors being equal.”

This observation seems to be in agreement with known ‘facts’ from *geochronometry* [50]:

“It thus appears that the length of the day has been increasing throughout geological time and that the number of days in the year has been decreasing. At, the beginning of the Cambrian the length of the day would have been 21 h.”

Now using this value of  $\Delta T=24$  sec/million years,  $T=23.9$  hours, and rotational velocity  $v = 2\pi R / T$ , and assuming that rotational

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<sup>2</sup> <http://image.gsfc.nasa.gov/poetry/ask/a11765.html>

velocity is the same throughout, then we could write in the same way with equation (6):

$$T.(1 + \Delta T/T) = 2\pi.R(1 + \Delta R/R)/v \quad (7)$$

Inserting these values into equation (7) including Earth radius value from Table 1, we get  $\Delta R=1.7766$  mm/year for Earth, which is surprisingly of the same order of magnitude with the result from equation (6). Of course, some difference could be expected because this approximation was obtained from Devonian coral rings observation, which could contain some biases.[49]

In the subsequent sections we will discuss an alternative method to measure this effect more precisely. It is worth to note here that this result does not necessarily mean to support all arguments related to Expanding Earth hypothesis by Panella-Carey-Vogel-Shields, despite its calculated result can be quite similar, because nowhere they have considered quantization of motion [49].

## **Derivation of extended celestial quantization and prediction of the receding Moon**

Now let suppose that this predicted value (4) is fully conserved to become inertial mass, and then we could rewrite Nottale's method of celestial quantization [1]. Alternatively, we could begin with the known Bohr-Sommerfeld quantization rule [3]:

$$\oint p_j . dq_j = n_j . 2\pi . e^2 / (\alpha_e . c) \quad (8a)$$

Then, supposing that the following substitution is plausible [3]:

$$e^2 / \alpha_e \rightarrow GMm / \alpha_g \quad (8b)$$

where  $e, \alpha_e, \alpha_g$  represents electron charge, Sommerfeld's fine structure constant, and gravitational-analogue of fine structure constant,

respectively. This corresponds to Nottale's basic equations  $v_n = \alpha_g \cdot c / n = v_o / n$  and  $v_o = 144 \text{ km/sec}$  [1]. And by introducing the gravitational potential energy [12]:

$$\Phi(r, \vartheta) = -GM / r \cdot [1 - J_2 \cdot (a/r)^2 \cdot (3 \cos^2 \vartheta - 1) / 2] \quad (8c)$$

where  $\vartheta$  is the polar angle (collatude) in spherical coordinate,  $M$  the total mass, and  $a$  the equatorial radius of the solid.

Neglecting *higher order* effects of the gravitational quadrupole moment  $J_2$  [13][14][15][16][17], then we get the known Newtonian gravitational potential:

$$\Phi = -GM / r \quad (8d)$$

Then it follows that the semi-major axes of the celestial orbits are given by [1][3]:

$$r_n = GMn^2 / v_o^2 \quad (8e)$$

where  $n=1,2,\dots$  is the principal quantum number.

It could be shown, that equation (8a) also corresponds to the conjecture of *quantization of circulation* [4b], which may correspond to the observation of quantized vortices dynamics, in particular in condensed matter physics (superfluidity etc.) [8][9][10][11] Therefore one can say that Bohr-Sommerfeld quantization has neat link with quantized vortice dynamics, just like Thompson's vortex hypothesis (before Rutherford). [51] In other words, our proposition for using Bohr-Sommerfeld quantization to describe celestial orbits may be just another implications of recent development in superfluid analogy in astrophysics, by Volovik *et al.*

By re-expressing equation (8e) for mass flux effect (5) by defining  $M_{n+1} = M_n + \Delta M_n / \Delta t_n$ , then the total equation of motion becomes:

$$(M + \Delta M / \Delta t) = (r + \Delta r / \Delta t) \cdot v_o^2 / (G \cdot n^2) \quad (8f)$$

For  $\Delta \rightarrow 0$ , equation (8f) can be rewritten as:

$$dM / dt - \chi \cdot dr / dt + M - r \cdot \chi = 0 \quad (8g)$$

where

$$\chi = v_0^2 / (G.n^2) \quad (8h)$$

Now inserting (5a) into equation (8g), and dividing both sides by  $\chi$ , yields:

$$dr / dt - M / \chi + r + \dot{\omega}.MR^2\omega / (\chi 5.c_s^2) = 0 \quad (8i)$$

This equation (8i) can be rewritten in the form:

$$\dot{r} + r + \varphi = 0 \quad (8j)$$

by denoting  $\dot{r} = dr / dt$  and

$$\varphi = -M / \chi . [1 - \dot{\omega}.R^2\omega / (5.c_s^2)] \quad (8k)$$

if we suppose a linear deceleration at the surface of the spinning mass. This proposition corresponds to the Expanding Earth hypothesis, because [49]:

“In order for expansion to occur, the moment of inertia constraints must be overcome. An expanding Earth would necessarily rotate more slowly than a smaller diameter planet so that angular momentum would be conserved.”

Equation (8j) and (8k) is obviously a first-order linear ODE equation [26], which admits exponential solution. In effect, this implies that the revised equation for celestial quantization [1][2] takes the form of *spiral motion*. This could also be interpreted as a plausible solution of diffusion equation in *dissipative* medium [33], which perhaps may also correspond to the origin of spiral galaxies formation [28]. And if this corresponds to the fact, then it could be expected that the spiral galaxies and other gravitational clustering phenomena [22b] could also be modeled using the same quantization method [39], as described by Nottale [1] and Rubčić & Rubčić [2].

To this author's knowledge these equations (8j) and (8k) have not been presented before elsewhere, at least in the context of celestial

quantization. In the subsequent section we will discuss how this spiral path could be understood using Ginzburg-Landau equation.

Inserting result in equation (7) into (8e) by using  $n=3$  and  $v_0=23.71$  km/sec for the Moon [2] yields a receding orbit radius of the Moon as large as 0.0401 m/year, which is very near to the observed value  $\sim 0.04$  m/year [20]. The quantum number and specific velocity here are also free parameters, but they have less effect because these could be replaced by the actual Moon orbital velocity using  $v_n = v_0 / n$  [1].

While this kind of receding Moon observation could be described alternatively using oscillation of gravitational potential [30], it seems that the kinetic expansion explanation is more preferable particularly with regard to a known hypothesis of *continental drift* after A. Wegener [29][49]. Apparently, none of these effects could be explained using oscillation of gravitational field argument, because they are relentless effects.

## Effect of varying M, instead of varying G

In this regard, it is interesting to note that Sidharth has argued in favor of varying G [21]. From this starting point, he was able to explain –among other things-- anomalous precession (Lense-Thirring effect) of the first planet and also anomalous Pioneer acceleration, which will be discussed in the subsequent section. In principle, Sidharth's basic assertion is [21]:

$$G = G_{\otimes} \cdot (1 + t/t_{\otimes}) \quad (9)$$

It is worthnoting here that Barrow [40c] has also considered a somewhat similar argument in the context of varying constants:

$$G = G_{\otimes} \cdot t_{\otimes} / (t - c) \quad (9a)$$

However, in this article we will use (9) instead of (9a), partly because it will lead to more consistent predictions with observation data. Alternatively, we could also hypothesize using Maclaurin formula:

$$G = G_{\otimes} \cdot e^{t/t_{\otimes}} = G_{\otimes} \cdot (1 + t/t_{\otimes} + (t/t_{\otimes})^2/2! + (t/t_{\otimes})^3/3! + \dots) \quad (9b)$$

This expression is a bit more consistent with the exponential solution of equation (8j) and (8k). Therefore, from this viewpoint equation (9) could be viewed as first-order approximation of (9b), by neglecting second and higher orders in the series. It will be shown in subsequent sections, that equation (9) is more convenient for deriving predictions.

If we conjecture that instead of varying G, the spinning mass M varies, then it would result in the same effect as explained by Sidharth [21], because for Keplerian dynamics we could assert  $k=GM$ , where k represents the stiffness coefficient of the system. Accordingly, Gibson [22] has derived similar conjecture of *exponential* mass flux from Navier-Stokes gravitational equation, which can be rewritten in the form:

$$M = M_{\otimes} \cdot e^{t/t_{\otimes}} = M_{\otimes} \cdot (1 + t/t_{\otimes} + (t/t_{\otimes})^2/2! + (t/t_{\otimes})^3/3! + \dots) \quad (10)$$

provided we denote for consistency [22]:

$$t_{\otimes} = \tau_g / 2\pi \quad (10a)$$

Using the above argument of Maclaurin series, equation (10) could be rewritten in the similar form with (9) by neglecting higher order effects:

$$M = M_{\otimes} \cdot (1 + t/t_{\otimes}) \quad (11)$$

Now the essential question here is: which equation should be used, a varying G or varying M? A plausible reasoning could be given as follows: In a recent article Gibson & Schild [23] argue that their gravitational equation based on Navier-Stokes approach

results in better explanation than what is offered by Jeans instability, which yields equation (10). Furthermore, R.M. Kiehn has also shown that the Navier-Stokes equation corresponds exactly to Schrödinger equation [27].

In the meantime, Bertschinger [22b] has discussed a plausible extension of Euler equation and *Jeans instability* to describe gravitational clustering, which supports Gibson's arguments of invoking viscosity term and also turbulence phenomena [22c, 22d]. Therefore, from kinematical gravitational instability viewpoint, apparently equation (11) is more plausible than equation (9), albeit the result will be similar for most (Newtonian) gravitation problems.

From equation (11) we could write for M at time difference  $\Delta t = t_2 - t_1$ :

$$M_2 = M_{\otimes} \cdot (1 + t_2 / t_{\otimes}) \quad (12)$$

$$M_1 = M_{\otimes} \cdot (1 + t_1 / t_{\otimes}) \quad (13)$$

from which we get:

$$\Delta M = (M_{\otimes} / t_{\otimes}) \cdot (t_2 - t_1) \quad (14)$$

Inserting our definition  $\Delta t = t_2 - t_1$  yields:

$$\Delta M / \Delta t = (M_{\otimes} / t_{\otimes}) = k \quad (15)$$

For verification of this assertion, we could use equation (15) instead of (1) to predict mass flux of the Earth. Inserting the present mass of the Earth from Table 1 and a known estimate of Earth epoch of  $2.2 \times 10^9$  years, we get  $k = 0.272 \times 10^{16}$  kg/year, which is approximately at the same order of magnitude (ratio=13.83) with equation (4).

Inserting equation (15) into equation (1), we get:

$$M_{\otimes} / t_{\otimes} \approx -\dot{\omega} \cdot MR^2 \omega / (5 \cdot c^2) \quad (16)$$

which is the basic conjecture of the present article.

## Quantization of anomalous celestial precession

It is known that the Newtonian gravitation potential equation (8d) is only weak-field approximation, and that GTR makes a basic assertion that this equation is *exact*. And if gravitation could be related to boson condensation phenomena [9][10][11], then it seems worth to quote a remark by Consoli [9b; p.2]:

“for weak gravitational fields, the classical tests of general relativity would be fulfilled in any theory that incorporates the Equivalence Principle.”

And in the same paper he describes [9b; p.18]:

“Einstein had to start from the peculiar properties of Newtonian gravity to get the basic idea of transforming the classical effects of this type of interaction into a metric structure. For this reason, classical general relativity cannot be considered a dynamical explanation of the origin of gravitational forces.”

Furthermore, Consoli also argued that the classical GTR effects other than anomalous precession could be explained without introducing non-flat metric, as described by Schiff [9b; p.19], therefore it seems that the only remarkable observational vindication of GTR is anomalous precession of the first planet [37]. Therefore, it seems reasonable to expect that the anomalous precession effect could be predicted without invoking non-flat metric, which suggestion is particularly attributed to R. Feynman, who ‘*believed that the geometric interpretation of gravity beyond what is necessary for special relativity is not essential in physics*’ [9d]. It will be shown that a consistent approach with equation (10) will yield not only the anomalous celestial precession, but also a conjecture that such an anomalous precession is quantized.

By using the same method as described by Sidharth [21], except that we assert varying mass  $M$  instead of varying  $G$  – in accordance with Gibson’s solution [22]--, and denoting the average angular velocity of the planet by

$$\dot{\Omega} \equiv 2\pi / T \quad (17)$$

and period  $T$ , according to Kepler’s Third Law:

$$T = 2\pi \cdot a^{3/2} / \sqrt{GM} \quad (18)$$

Then from equation (10), (17), (18) we get:

$$\dot{\Omega} - \dot{\Omega}_o = -\dot{\Omega}_o \cdot t / t_{\otimes} \quad (19)$$

Integrating equation (19) yields:

$$\varpi(t) = \Omega - \Omega_o = -(\pi / T) \cdot t^2 / t_{\otimes} \quad (20)$$

which is average precession at time ‘ $t$ ’. Therefore the anomalous precession corresponds to the epoch of the corresponding system. For Mercury, with  $T=0.25$  year, equation (20) yields the average precession per year at time ‘ $t$ ’:

$$\varpi(t)_{Mercury} = \Omega - \Omega_o = -4\pi \cdot t^2 / t_{\otimes} \quad (21)$$

Using again  $t_{\otimes} = 2 \times 10^{10}$  year as the epoch of the solar system and integrating for years  $n=1 \dots 100$ , equation (21) will result in total anomalous precession in a century:

$$\varpi(n) = \sum_{n=1}^{n=100} \varpi(n) = 43.86'' \text{ per century} \quad (22)$$

It would be more interesting in this regard if we also get prediction of this effect for other planets using the same method (20), and then compare the results with GTR-prediction (using Lense-Thirring effect). Table 2 presents the result, in contrast with observation by Hall and also prediction by Newcomb, which are supposed to be the same [25].

**Table 2. Comparison of prediction and observed anomalous precession**

| Celestial Object | Period, T | $\omega_{prediction}$ | Hall/<br>Newcomb | Diff. | GTR/<br>Thirring | Diff. |
|------------------|-----------|-----------------------|------------------|-------|------------------|-------|
|                  | (year)    | (arcsec/cy)           | (arcsec/cy)      | (%)   | (arcsec/cy)      | (%)   |
| Mercury          | 0.25      | 43.86                 | 43.00            | 2.03  | 42.99            | -0.05 |
| Venus            | 0.57      | 19.24                 | 16.80            | 14.54 | 0.8              | -95.2 |
| Earth            | 1.00      | 10.96                 | 10.40            | 5.46  | 3.84             | -63.1 |
| Mars             | 1.88      | 5.83                  | 5.50             | 6.02  | 1.36             | -76.0 |
| Jupiter          | 4346.5    | $2.52 \times 10^{-3}$ |                  |       |                  |       |
| Saturn           | 10774.9   | $1.02 \times 10^{-3}$ |                  |       |                  |       |
| Uranus           | 30681.0   | $3.57 \times 10^{-4}$ |                  |       |                  |       |
| Neptune          | 60193.2   | $1.82 \times 10^{-4}$ |                  |       |                  |       |
| Pluto            | 90472.4   | $1.21 \times 10^{-4}$ |                  |       |                  |       |

It is obvious from Table 2 above that the result of equation (20) appears near to GTR's prediction and observation by Hall for the first planet, but there is substantial difference between GTR and observation for other planets particularly Venus. In the mean time, average percentage of error from prediction using equation (20) and observation (Hall) is 7.01%. The numerical prediction for Jovian planets is negligible; though perhaps they could be observed provided there will be more sensitive observation methods in the near future.

It is perhaps also worthnoting here, that if we use the expression of quantization of period [3]:

$$T = 2\pi.GM.n^3 / v_0^3 \quad (23)$$

where  $v_0 = \alpha_g.c = 144km/s$  in accordance with Nottale [1]. Inserting this equation (23) into (20), yields:

$$\varpi(t)_{precess} = \Omega - \Omega_0 = -(v_0^3 / 2GMn^3).t^2 / t_{\otimes} \quad (24)$$

or

$$T_{precess} = 2\pi / \varpi(t)_{precess} = -4\pi.t_{\otimes} GMn^3 / v_o^3 t^2 \quad (24a)$$

These equations (24) and (24a) imply that the anomalous precession of Lense-Thirring type should also be *quantized*. Apparently no such an assertion has been made before in the literature.

It would be interesting therefore to verify this assertion for giant planets and exoplanets, but this is beyond the scope of the present article.

## A plausible test using LAGEOS-type satellites

In this regard, one of the most obvious methods to observe those effects of varying spinning mass M as described above is using LAGEOS-type satellites, which have already been used to verify Lense-Thirring effect of Earth. What is presented here is merely an approximation, neglecting higher order effects [12][16][31].

Using equation (8c) we could find the rotational effect to satellite orbiting the Earth. Supposed we want to measure the precessional period of the inclined orbit period. Then the best way to measure quadrupole moment ( $J_2$ ) effect would be to measure the  $\vartheta$  component of the gravity force (8c):

$$g = 1/r.\partial V / \partial \vartheta = -3GM.a^2 J_2.\sin \vartheta.\cos \vartheta / r^4 \quad (25)$$

This component of force will apply a *torque* to the orbital angular momentum and it should be averaged over the orbit. This yields a known equation, which is often used in satellite observation:

$$\omega_p / \omega_s = -3a^2 J_2.\cos i / 2r^2 \quad (26)$$

where  $i$  is the inclination of the satellite orbit with respect to the equatorial plane,  $a$  is Earth radius,  $r$  is orbit radius of the satellite,  $\omega_s$  is the orbit frequency of the satellite, and  $\omega_p$  is the precession frequency of the orbit plane in inertial space. Now using LAGEOS satellite data [31] as presented in Table 3:

**Table 3. LAGEOS satellite parameters**

| Parameter           | Value                  | Unit  |
|---------------------|------------------------|-------|
| $R_{\text{LAGEOS}}$ | $12.265 \times 10^6$   | M     |
| $i_{\text{LAGEOS}}$ | 109.8                  | °     |
| $T_{\text{LAGEOS}}$ | 13673.4                | sec   |
| $\omega_s$          | $4.595 \times 10^{-4}$ | rad/s |
| $J_2$               | $1.08 \times 10^{-3}$  |       |

Inserting this data into equation (26) yields a known value:

$$\omega_p = 0.337561^\circ / \text{day} \quad (27)$$

which is near enough to the observed LAGEOS precession =  $0.343^\circ / \text{day}$ .

Now let suppose we want to get an estimate of the effect of Earth kinetic expansion to LAGEOS precession. Inserting  $(r+dr/dt)$  from equation (6) to compute back equation (26) yields:

$$\Delta\omega_p = \omega_{p,n+1} - \omega_{p,n} = 1.41 \times 10^{-9}^\circ / \text{day} = 2.558 \text{ arc sec} / \text{year} \quad (28)$$

Therefore, provided the aforementioned propositions correspond to the facts, it could be expected to find a secondary precession of LAGEOS-satellite around 2.558 arcsecond/year. To this author's knowledge this secondary effect has not been presented before elsewhere. And also thus far there is no coherent explanation of those aforementioned phenomena altogether, except perhaps in [21] and [30].

As an alternative to this method, it could be expected to observe Earth gravitational acceleration change due to its radius increment. By using equation (8d) and (5):

$$\ddot{r}(t) = GM / r^2 = 4\pi.G.\rho_{sphere}.r / 3 \quad (29)$$

From this equation, supposing there is linear radius increment, then we get an expression of the rate of change of the gravitational acceleration:

$$\ddot{r}(t) = \Delta\ddot{r} / \Delta t = 4\pi.G.\rho_{sphere}.(r + \Delta r / \Delta t) / 3 - \ddot{r}(t) \quad (30)$$

Therefore, it would be interesting to find observation data from LAGEOS to verify or refute this equation.

## **Ginzburg-Landau equation and solar system: possible signatures of spiral gravity**

The pattern formation is often described as result of diffusion reaction. And the most popular equation in these pattern-formation studies is CGLE (complex Ginzburg-Landau equation). These reaction-diffusion systems govern almost all phenomena in Nature from the smallest quantum entities to galaxies [40][41]. E. Goldfain has also considered CGLE with possible application in description of elementary particle masses [52].

In this regards, a considerable attempt has been made towards a better understanding of partial differential equations of parabolic type in *infinite space*. A typical equation is known as CGLE, which is commonly described as follows [42]:

$$\partial_t A = (1 - i\alpha)\Delta A + A - (1 + i\beta).A|A|^2 \quad (31)$$

The most interesting characteristics of CGLE is its superspiral solution [43], or ‘scroll waves’ pattern [44]. This equation could also lead to a kind of ‘dark soliton’, which is quite related to NLSE (nonlinear Schrödinger equation) [45].

A relative periodic orbit of the CGLE with drift  $(\varphi, S)$  and period T contains solutions that satisfy for all t [46]:

$$A(x, t) = e^{i\varphi} . A(x + S, t + T) \quad (32)$$

The corresponding solution of the system of ODEs derived from CGLE thus satisfies [46]:

$$a_m(t) = e^{i\varphi} . e^{imS} . a_m(t + T) \quad (33)$$

for all m and t. This equation could be reintroduced in the form [46]:

$$a_m(t) = e^{-t.L_g/T} . b(t/T) \quad (34)$$

Where b is periodic with the period one, and

$$L_g = \text{diag}(i\varphi + imS) \quad (34a)$$

Alternatively, solution of CGLE could be found in terms of MAW (modulated amplitude waves) with expression as follows [43]:

$$A(r, t) = a(z) . e^{i\phi(z)} . e^{i(qr - \omega t)} \quad (35)$$

Interestingly, this could be related to an extended solution of Bohr-radius-type equation of celestial quantization. In accordance with equation (8i)-(8j)-(8k), we could extend Bohr-radius type expression of quantized orbit of celestial bodies in solar system in the form of spiral motion. Therefore, it seems plausible to assert that the form of equation (34) and (35) appears very similar with equations (8i)-(8j)-(8k). This seems to suggest a possibility that CGLE could be related to quantization of celestial bodies, in lieu of describing this macroquantization using Schrödinger-Euler-Newton like Nottale’s Scale Relativity Theory [1]. In this regards, El Naschie has also noted the significance of spiral geometry to describe gravitation (sometimes called ‘spiral gravity’).

For observational verification, we could rewrite equation (8j) and (8k):

$$dr / dt = M / \chi \cdot [1 - \dot{\omega} \cdot R^2 \omega / (5 \cdot c_s^2)] - r \quad (36)$$

and inserting equation (15), we get:

$$dr / dt = M / \chi \cdot [1 + M_{\otimes} / (t_{\otimes} \cdot M)] - r \quad (37)$$

A plausible test of this conjecture could be made by inserting the result from equation (14) into equation (8e) and using  $M_{\otimes} = 1.98951 \times 10^{33} \text{ g}$  and  $t_{\otimes} = 2 \times 10^{10} \text{ year}$  as the epoch of the solar system [21], and specific velocity  $v_o = 144 \text{ km/sec}$  [1], then from equation (37) we get a receding Earth orbit radius from the Sun at the order of:

$$\Delta r_{Earth} / \Delta t = 6.03 \text{ m / year} \quad (38)$$

Interestingly, there is an article [24] hypothesizing that the Earth orbit is receding from the Sun at the order of 7.5 m/year, supposing Earth orbit radius has been expanding as large as  $93 \times 10^6$  miles since the beginning of the solar epoch. (Of course, it shall be noted that there is large uncertainty of the estimate of solar epoch, see for instance Gibson [22]).

Therefore, it is suggested here to verify this assumption of solar epoch using similar effect for other planets. For observation purposes, some estimate values were presented in Table 4 using the same approach with equation (37).

**Table 4. Prediction of planetary orbit radii (r) increment**

| Celestial object | Quantum number (n) | Orbit increment (m/yr) |
|------------------|--------------------|------------------------|
| Mercury          | 3                  | 2.17                   |
| Venus            | 4                  | 3.86                   |
| Earth            | 5                  | 6.03                   |
| Mars             | 6                  | 8.68                   |

## Concluding remarks

If physical theories could be regarded as continuing search to find systematic methods to reduce the entropy required to do calculation to minimum; then the fewer free parameters in a theory and the less computation cost required, the better is the method. Accordingly, in this article some twelve phenomena can be explained using only few free parameters, including:

- ❑ The Moon is receding from the Earth [20];
- ❑ Earth's angular velocity decrease (Kip Thorne, G. Smoot, J. Wells) [19];
- ❑ Planets are receding from the Sun [24];
- ❑ Lense-Thirring effect for inner planets, corresponding to Hall/Newcomb's observation;
- ❑ Celestial orbit prediction in solar system [1][2][3];
- ❑ Exoplanets orbit prediction [1][3];
- ❑ Pioneer-type anomalous acceleration [21];
- ❑ A plausible origin of increasing day length (24 second each million years);
- ❑ A plausible origin of continental drift effect [29];
- ❑ A plausible origin of spiral motion in spiral nebulae [22];
- ❑ Prediction of possible extra precession of LAGEOS satellite [31];
- ❑ Prediction of angular velocity decrease of other planets.

As a plausible observation test of the propositions described here, it is recommended to measure the following phenomena:

- ❑ Lense-Thirring effect of inner planets, compared to spherical kinetic dynamics prediction derived herein;
- ❑ Annual extra precession of Earth-orbiting LAGEOS-type satellites;
- ❑ Receding planets from the Sun;
- ❑ Receding satellites from their planets, similar to receding Moon from the Earth – all these celestial objects take the form of spiral motion;
- ❑ Angular velocity decrease of the planets;
- ❑ Angular velocity decrease of the Sun.

It appears that some existing spacecrafts are already available to do this kind of observation, for instance LAGEOS-type satellites [31]. Further refinement of the method as described here could be expected, including using *ellipsoidal kinetic dynamics* [12] or using analogy with neutron star dynamics [32]. Further extensions to cosmological scale could also be expected, for instance using some versions of Cartan-Newton theory [38]; or to find refinement in predictions related to varying constants.

All in all, the present article is not intended to rule out the existing methods in the literature to predict Lense-Thirring effect, but instead to argue that perhaps the notion of ‘*frame dragging*’ in GTR [14][16] could be explained in terms of *dynamical interpretation*, through invoking the spherical kinetic dynamics. In this context, the dragging effect is induced by the spinning spherical mass to its nearby celestial objects.

Provided all of these correspond to the observed facts, it seems plausible to suggest that it is possible to derive celestial quantization

in terms of (complex) Ginzburg-Landau equation, instead of the known Schrodinger-Euler-Newton like in Nottale's Scale Relativistic Theory [1]. Because CGLE is also commonly used in the context of Bose gas [43][48], then it seems also plausible to hypothesize that the subtle medium of subparticle structure may be described using Winterberg's superfluid phonon-roton model [8]. It is known that an essential feature of Winterberg's superfluid Planckian aether model is that the basic entity is comprised of pairs of Planck mass. Interestingly, similar hypothesis of Planck mass as the basic entity of Nature has also been suggested by Spaans, using topological arguments [47]. Other implications of this CGLE's superspiral quantization either in nuclei realm or cosmological prediction remain to be explored [48].

If this proposition described here corresponds to the facts, then one can say that it is possible to 're-derive' General Relativity phenomena from the viewpoint of Bohr-Sommerfeld quantization and spherical kinetic dynamics. Possible verification of this proposition may include the use of Earth-based satellites, which go beyond traditional GTR-tests such as precession of the first planet. Further observation to verify or refute this conjecture is recommended, plausibly using LAGEOS-type satellites

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## Appendix I: Derivation of equation (1)

We start with some basic equations that will be used throughout the present article. It is assumed that the solar nebula is disk-shaped and is in hydrostatic equilibrium in the vertical direction. Let suppose that the disk has approximately Keplerian rotation,  $\omega$ ; then the half-thickness of the disk is given by [4d; p.4-5]:

$$d = c_s / \omega \quad (\text{i})$$

and

$$c_s \approx \sqrt{kT / m} \quad (\text{ii})$$

where  $d$  and  $c_s$  represents half-thickness of the disk and sound velocity, respectively.

In order to find the spherical kinetic dynamics contribution to Lense-Thirring effect, we begin with the spinning dynamics of solid sphere with mass  $M$ . Using the known expression [12; p.6, p.8]:

$$E_{kinetic} = -I_{zz} \omega^2 / 2 \quad (\text{iii})$$

$$I_{sphere} = 2MR^2 / 5 \quad (\text{iv})$$

where  $I_{zz}$ ,  $\omega$ ,  $M$ ,  $R$  represents angular momentum, angular velocity, spinning mass of the spherical body, and radius of the spherical body, respectively. Inserting equation (iv) into (iii) yields:

$$E_{kinetic} = -MR^2\omega^2/5 \quad (v)$$

This known equation is normally interpreted as the amount of energy required by a spherical body to do its axial rotation. But if instead we conjecture that ‘galaxies get their angular momentum from the global rotation of the Universe due to the conservation of the angular momentum’ [34], and likewise the solar system rotates because of the corresponding galaxy rotates, then this equation implies that the rotation itself exhibits extra kinetic energy. Furthermore, it has been argued that the global rotation gives a natural explanation of the empirical relation between the angular momentum and mass of galaxies:  $J \approx \alpha M^{5/3}$  [34]. This conjecture seems to be quite relevant in the context of Cartan torsion description of the Universe [18][38]. For reference purpose, it is worthnoting in this regard that sometime ago R. Forward has used an argument of non-Newtonian gravitation force of this kind, though in the framework of GTR (*Amer.J.Phys.* **31** No. 3, 166, 1963).

Let suppose this kind of extra kinetic energy could be transformed into mass using a known expression in condensed-matter physics [10b; p.4], with exception that  $c_s$  is used here instead of  $v$  to represent the sound velocity:

$$E_{kinetic}(n, p) = c_s \cdot p = m_s \cdot c_s^2 \quad (vi)$$

where the sound velocity obeying [10b; p.4]:

$$c_s^2(n) = (n/m)(d^2 \epsilon / dn^2) \quad (vii)$$

Physical mechanism of this kind of mass-energy transformation is beyond the scope of the present article, albeit there are some recent

articles suggesting that such a condensed-matter radiation is permitted [35]. Now inserting this equation (vi) into (v), and by dividing both sides of equation (v) by  $\Delta t$ , then we get the incremental mass-energy equivalent relation of the spinning mass:

$$\Delta m_s / \Delta t = -\omega.(\Delta\omega / \Delta t).MR^2 / (5.c_s^2)$$

(viii)

By denoting  $\dot{\omega} = \Delta\omega / \Delta t$ , then this equation (viii) can be rewritten as:

$$\Delta m_s / \Delta t = -\omega.(\Delta\omega / \Delta t).MR^2 / (5.c_s^2) \quad (\text{ix})$$

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