The Gravitational origin of Velocity Time Dilation

A generalization of the Lorentz Factor for comparable masses

Arindam Sinha[*†]

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Abstract

This paper posits that experiments have conclusively proven that velocities need to be measured from the Center of Gravity of the local gravitational system for computation of velocity time dilation (actual differential clock rates between bodies). Extending this understanding, a more general form of the velocity time dilation metric (i.e. the Lorentz factor) is derived. This allows prediction of velocity time dilation between bodies of any mass ratio, including comparable masses such as the Earth-Moon system. This is not possible using the Lorentz factor in its current form. The generalized form of the Lorentz factor remains consistent with results of all experiments conducted.

1 Introduction

Special Theory of Relativity (SR)[1] was conceived by Einstein on the basis of *kinematics*. One of the consequences has been that velocity time dilation is an 'all-or-none' phenomenon between bodies, since mass plays no role. The metric used, Lorentz factor ($\gamma = 1/\sqrt{1 - v^2/c^2}$), only has the parameters v the relative velocity of the bodies, and c the speed of light. It therefore appears that only one of the two bodies can undergo a clock slowdown, by an amount computed based on their relative velocities.

Experiments over the years have indicated that in real-life situations dynamics plays its part in velocity time dilation, and clock rate difference between two bodies cannot be predicted without taking into account the gravitational background, implicitly or explicitly. Both bodies involved may undergo different amounts of measurable velocity time dilation, as would be the case in the Earth-Moon system or binary stars.

[*]E-mail: arindam_sinha1@yahoo.com

[Note: The term "time dilation" in this paper denotes "differential aging", or experimentally verified clock rate difference between two bodies, not including any reciprocal time dilation 'as seen' by the two bodies because of Doppler effects.]

For example, in Earth based experiments, small moving objects have shown clock slowdown effects, with Earth implicitly acting as the 'second body'. Given the many orders of magnitude difference in masses, the small objects have practically had all the velocity, and the 'all-or-none' computation of velocity time dilation using the Lorentz factor has worked (though the mass ratio has decided which clock is time dilated in experiments).

Even clocks on Earth's surface slow down because of the Earth's rotation velocity, compared to clocks fixed to the sidereal (non-rotating) axis around which Earth rotates, as shown by the Hafele-Keating[2, 3] experiment. This sidereal axis, called the Earth Centered Inertial Frame (ECIF), is the frame from which all velocities are measured in practice for computing velocity time dilation for Earth-based experiments. This is the same velocity as used in momentum conservation and satellite orbital velocity calculations.

The inference we can draw from results of experiments like Hafele-Keating, GPS satellite time dilation[4] and Bailey et. al. muon lifetime extension[5] experiments is that time dilation needs to be computed based on velocities from the center of gravity of the local gravitational system, rather than any relative velocities between objects.

In this paper we examine how the Lorentz factor may be generalized, based on lessons from these experiments, to provide more intuitive predictions on velocity time dilation in a broad range of situations without an 'all or none' approach, and without invoking General Relativity (GR).[6]

2 The Lorentz Factor in experiments

At present, computations based on the Lorentz factor allows prediction of velocity time dilation (differential clock rates) only in two mass ratio situations:

a. **Disparity between masses of two bodies is many orders of magnitude:** For example, the clock of a small object moving near Earth slows down as predicted by SR and computed by the Lorentz factor. Since the mass of the small object is negligible compared to Earth, the latter may be considered stationary. In an analogy to the well-known *twin paradox*, the small objects may be considered the traveling twin.

b. **Two bodies have identical mass:** For example, binary stars of equal masses would not have any clock rate difference between themselves because of the symmetry of the situation. In terms of the *twin paradox*, the stars may be considered to be twins traveling at equal velocities but in different directions.

Since mass does not feature in the Lorentz factor, it cannot predict velocity time dilation between masses that are comparable but unequal, such as the Earth and the Moon (or unequal binary stars). However, leveraging the lessons from experiments, we can generalize the Lorentz factor to predict velocity time dilation in such situations by taking the mass ratio into account.

The experiments we will consider are the *Hafele-Keating* experiment, *GPS satellite time dilation* and *Bailey et. al.* muon lifetime extension experiment.

The common thread among all these experiments is that 'small objects' (compared to Earth) are considered to have the complete velocity, and their clocks have been proven to slow down as computed using the Lorentz factor. If we consider the Sun-Earth situation though, it is Earth clocks which would slow down compared to the Sun because of velocity time dilation, as the Earth would have all the velocity.

It is clear that velocities used to compute differential clock rates are measured from a sidereal (non-rotating) frame through the Center of Gravity (CG) of the local gravitational system. In these experiments, the CG practically coincides with the CG of Earth because of the small moving masses involved. Therefore, the velocities are measured from the ECIF.

The following points from the experiments help exemplify this:

- In **Hafele-Keating**, clocks on eastward bound planes were found to be slower than Earth stationary clocks, which were in turn slower than clocks on westward bound planes. In this case, the westward bound planes were actually traveling *slower* than the Earth stationary clocks because of Earth's rotational velocity in the ECIF.
- **GPS** satellites' clocks run slower (ignoring gravitational time dilation) compared to Earth-based clocks, but do not show any clock drift among different satellites, in spite of moving in six different planes.
- In the **Bailey** experiment, it is of course the muon whose 'clock' slows down because of its velocity in the lab or Earth frame.

How would the clock rate difference change with the comparative mass ratio in a two body situation?

Existing SR does not provide a solution for this. Nor is there an exact solution in GR for such a scenario. The Schwarzschild metric [7, 8] may be used in a complex iterative way between two bodies to provide an approximate answer. There is no real solution in GR for a system of more than two bodies.

We derive below a more general form of the Lorentz factor which will help us predict velocity time dilation in comparable mass situations like the Earth-Moon system or even for multi-body gravitational systems. The approximate answer from GR ultimately converges to this generalized Lorentz factor. Moreover, the interpretation of the GR answer supports the more direct logic of our derivation.

3 Generalized Lorentz Factor formula

Let M and m be two masses in circular orbit around their common center of gravity (barycenter) with orbital velocities v_M and v_m respectively. The momentum conservation equation is:

$$Mv_M = mv_m \tag{1}$$

If v is the *relative velocity* of the two objects around a sidereal axis through the center of gravity, we can write each velocity in terms of v:

$$v = v_M + v_m \tag{2}$$

$$v_M = \left(\frac{m}{M+m}\right) v \tag{3}$$

$$v_m = \left(\frac{M}{M+m}\right) v \tag{4}$$

The time dilation factors for M and m may then be modified as:

$$\gamma_M = \frac{1}{\sqrt{1 - \frac{{v_M}^2}{c^2}}} = \frac{1}{\sqrt{1 - \left(\frac{m}{M+m}\right)^2 \times \frac{v^2}{c^2}}} \tag{5}$$

$$\gamma_m = \frac{1}{\sqrt{1 - \frac{{v_m}^2}{c^2}}} = \frac{1}{\sqrt{1 - \left(\frac{M}{M+m}\right)^2 \times \frac{v^2}{c^2}}} \tag{6}$$

This is the *generalized form of the Lorentz factor*, including the mass ratio. This allows us to predict clock rate differences between bodies even if their masses are not many orders of magnitude different (or exactly equal).

Note that if Earth is M in the experiments referred to earlier, and the moving objects (plane, satellite, muon) are m, we have $m \ll M$, such that $v_M \cong 0$. In these cases, the time dilation factor of m reduces in the limit to the current Lorentz factor $\gamma_m = 1/\sqrt{1 - v^2/c^2}$, and that of Earth is $\gamma_M \cong 1$ (no time dilation), as seen in the experiments. *Thus, the generalized Lorentz factor remains consistent with existing experiments.*

We may write ${v_m}^2$ and ${v_M}^2$ in terms of M, m, G (Gravitational constant) and R (distance between M and m) as:

$$ {v_M}^2 = \frac{Gm^2}{R\,(M+m)} \tag{7} $$

$$ {v_m}^2 = \frac{GM^2}{R\,(M+m)} \tag{8} $$

The generalized Lorentz factors in (5) and (6) may then also be written as:

$$ \gamma_M = \frac{1}{\sqrt{1-\frac{{v_M}^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{Gm^2}{R(M+m)c^2}}} \tag{9} $$

$$ \gamma_m = \frac{1}{\sqrt{1-\frac{{v_m}^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{GM^2}{R(M+m)c^2}}} \tag{10} $$

Therefore, we can compute the velocity time dilations of both bodies from their masses and the distance between them.

Let us apply these formulations to different situations and see how this helps explain naturally the velocity time dilations that may be expected in each case (without any confusions as to which twin is moving, in an analogy to the twin paradox).

4 Application to different situations

We will consider the different situations below using a real example in each case.

4.1 M is much larger than m ($M \gg m$)

Taking M as the Earth, this applies to all the three experiments mentioned, with small objects m moving near Earth. As explained above, the time dilations will be computed using $\gamma_m = 1/\sqrt{1-v^2/c^2}$.

- In **Hafele-Keating**, the clocks on the planes and Earth-stationary clocks all have time dilations given by $\gamma_{m_i} = 1/\sqrt{1-{v_i}^2/c^2}$, where v_i stands for velocities of the clocks (m_i) in the sidereal inertial frame at the CG of Earth. The difference of these computed clock rates have been verified in the experiment.
- For **GPS**, the small rotational velocity of Earth may be ignored. Clocks on Earth will not have any velocity time dilation (i.e. will run at the same rate as a 'coordinate

clock' at rest), while the time dilation of the satellites will be given by the formula $\gamma_m = 1/\sqrt{1 - v^2/c^2}$. Here, v is the rotational velocity of satellites around Earth's CG, and is the same for all satellites, even though traveling in six different planes.

- Muons in the **Bailey et. al.** experiment have near light-speed velocities, and any velocity of Earth is immaterial. We obtain the time dilation (lifetime extension) factor using the metric $\gamma_m = 1/\sqrt{1 - v^2/c^2}$, where v is the velocity of the muons in the muon ring.

4.2 M is much smaller than m ($M \ll m$)

Again taking M as the Earth, we may consider the Sun as m. In this case, it is Earth which gets the complete time dilation because of its velocity around the Sun, given the Sun's overwhelmingly larger mass. Clocks on Earth will run slower compared to a coordinate clock by a factor of $\gamma_M = 1/\sqrt{1 - v^2/c^2}$, where v is the orbital velocity of Earth in the sidereal frame through the CG of Sun (around which the Sun rotates).

4.3 M is the same as m ($M = m$)

Binary stars of equal mass exemplify this situation. From the symmetry of the situation, we do not expect any clock rate difference between the two stars, but their clocks both run equally slower than coordinate clocks. The time dilation factor of either will be given by:

$$\gamma_M = \gamma_m = \frac{1}{\sqrt{1 - \left(\frac{M}{M+M}\right)^2 \times \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{v^2}{4c^2}}} \tag{11}$$

where v is the *relative velocity* between the stars (double the orbital velocity).

Note that there is no way of arriving at this value using the current Lorentz factor. While the current Lorentz factor can show that they will not have any differential clock rates between themselves because of identical velocities, the correct difference of their clock rates from a coordinate clock cannot be computed.

4.4 M is somewhat greater than m ($M > m$)

The Earth-Moon system is an example of this (as are unequal sized binaries). Since the mass of the Moon ($m = 7.35 \times 10^{22} kg$) is not negligible, the CG of the system cannot be considered to coincide with the CG of Earth ($M = 5.97 \times 10^{24} kg$). We have to compute their time dilation factors separately. Taking both as point masses, this may be done using known values $v_M = 12.6m/s$, $v_m = 1,023m/s$ and $c = 299,792,458m/s$ from (5) and (6). The difference of γ_M and γ_m will give the difference of clock rates between Earth and Moon clocks.

Given the small velocities involved, the clock slowdown for Earth (0.08 $nanoseconds/day$) and Moon (503 $nanoseconds/day$) compared to a coordinate clock are very small, but even the Earth's time dilation is not zero.

4.5 Multiple bodies with total mass M ($M = \Sigma m_i$)

When there are multiple bodies in a gravitationally bound system (e.g. the Solar System), the velocity time dilation of each body may be found using the equation $\gamma_{m_i} = 1/\sqrt{1 - {v_i}^2/c^2}$, where m_i is the mass of the i^{th} body, and v_i is its velocity with respect to the CG of the system.

5 Conclusions

Experimentally verified velocity time dilation depends on gravity, at least in the definition of the local inertial frame. Velocities of objects are determined from this local CG inertial frame for computing time dilation. The arbitrary designation of 'stationary' and 'moving' states in the twin paradox, implicit in all resolutions of twin paradox within the SR framework to explain differential clock rates, is unambiguously resolved in real life situations by the local gravitational inertial frame.

We derived a generalized Lorentz factor in this paper based on the lessons from actual experiments such that velocity time dilations even between comparable masses may be predicted. This generalized Lorentz factor is valid for existing experiments as well.

An important question that arises from these considerations is whether velocity time dilation is in fact a gravitational phenomenon.*

*(This is discussed in detail in another paper by the author on the overall theory including gravitational time dilation: Relativity and the Gravitational Potential of the Universe)

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