

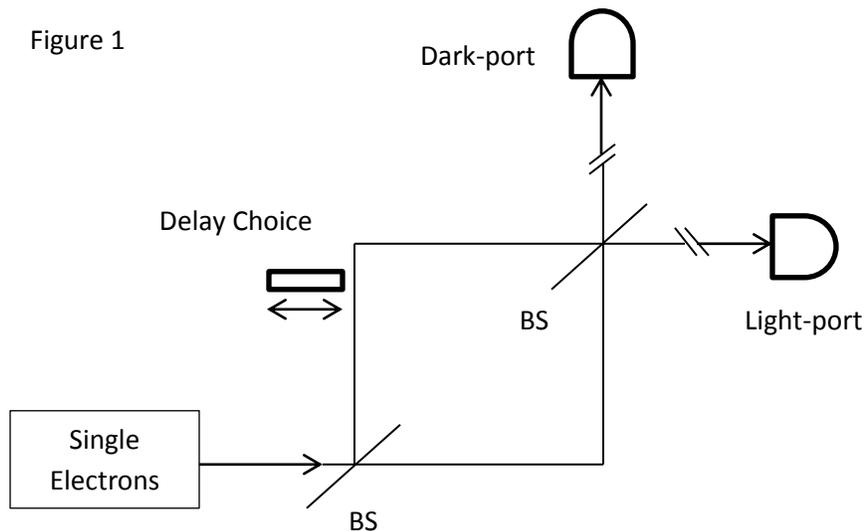
## Potential Energy Shift in Single Electron Spatial Interference

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*I propose an idealized electron interference experiment with a light-port/dark-port configuration and consider the possible outcomes for the average energy of electric potential that is present in a statistically relevant set of single electron measurements. The changes in energy of electric potential of the electron during its evolution will cause a shift in the kinetic energy or momentum of the electron, and this calculated shift may be interpreted differently for the case of interference and non-interference. I also propose an idealized single electron spatial interference effect and the associated changes in states of electric potential.*

### The Idealized Light-port/Dark-port Single Electron Interferometer

We consider the preparation of a single electron interference effect which uses an electron beam splitting and recombination apparatus and two detectors. The ideal optical interferometer of this form is that which would be referred to as the Mack-Zehnder interferometer in light optics. In electronics this idealized form of interferometry is the Marton interferometer. We do not profess to utilize this exact form in our thought experiment which we propose here, but rather to simply state the form of such an interferometer that would suit our thought experiment. This includes that the interferometer have two possible paths that the electrons may take after the electron beam is split, and that the electron beams be coherently recombined at equal path lengths in such a manner that the recombination has two possible outputs where single electron interference with path difference may be observed (light-port and dark-port). In addition we require that there be a choice of the preparation that may be made to cause a delay in one path which destroys interference at the outputs. This may be achieved by inserting a delay in one path which will create a difference in the path lengths of the two arms of the interferometer such that the difference is greater than the uncertainty of the electron's position in the direction of propagation. In other words, the choice is to either have interference or to not have interference.



We illustrate this preparation in Figure 1 where it is seen that the input source of electrons emits a beam of single electrons which are incident upon the beam splitting apparatus (BS). The two output beams are redirected to meet at a second beam splitting apparatus which recombines them. The two outputs of this second beam splitter are both coherent recombinations of the original beam. In this configuration we would expect interference to be observable in the detections of the two outputs, so we refer to these outputs as the light-port and dark-port detections. The only other apparatus is the mechanism for the delay choice, a mechanism which the experimenter may insert into one arm of the interferometer. If the choice is to insert the delay, then there is no interference observed and the two output ports have equal probabilities of detection. If the choice is to not insert the delay, then there is interference observed and all the electrons are detected at the light port and no electrons are observed at the dark port. For the purposes of the thought experiment we assume that the path lengths from the second BS to the detectors is very large, specifically large enough to ensure that the energy of electric potential between any two electrons positioned at the two detectors have a negligible energy of electric potential between them in comparison to the energy of electric potential that each may have with its nearest neighbours in its beam. In other words, the distance between detectors is far greater than the average distance of separation of the electrons within the beam.

The mathematical analysis that we provide for this thought experiment will provide us with the energy of electric potential of a distribution of electrons in the final output configuration for the case of interference and non-interference. The energy of electric potential for a distribution of charged particles can be calculated by simply summing the energy of electric potential between all the point charges. For two point charges,  $q_1$  and  $q_2$ , separated by a distance  $r_{12}$ , the electric potential at the position of the first charge due to the second,  $V_1$ , is given as

$$V_1 = \frac{q_2}{4\pi\epsilon_0 r_{12}}$$

The energy of electric potential,  $E_{12}$ , for the two charges is given as

$$E_{12} = q_1 V_1 = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

But this is for the simple case of two electrons. The total charge for an electron beam is roughly distributed over one dimension (that of the beam's propagation) so we may sum all the terms of the potential that a given electron in the beam is subject to. For  $N$  electrons distributed over a beam with an average spacing of  $x$  between nearest neighbouring electrons, the potential that the  $n^{\text{th}}$  electron is subject to,  $V_n$ , is given as the sum

$$V_n = \frac{e}{4\pi\epsilon_0} \left( \sum_{i=1}^{n-1} \frac{1}{|n-i|x} + \sum_{i=n+1}^N \frac{1}{|n-i|x} \right)$$

where  $e$  is the electron charge and the first sum goes from values of  $i$  between 1 and  $n-1$  and the second sum goes from values of  $i$  between  $n+1$  and  $N$ . In other words we have  $i \neq n$ . So we may re-express this summation as

$$V_n = \frac{e}{4\pi\epsilon_0 x} \left( \sum_{i=1, i \neq n}^N \frac{1}{|n-i|} \right)$$

We may now express the total energy of electric potential for the distribution of  $N$  electrons in a beam,  $E_N$ , as

$$E_N = \frac{1}{2} \sum_{n=1}^N eV_n$$

$$E_N = \frac{e^2}{8\pi\epsilon_0 x} \left( \sum_{n=1}^N \left( \sum_{i=1, i \neq n}^N \frac{1}{|n-i|} \right) \right)$$

This expression has a double sum in brackets which has a solution that is a Bessel based function. This solution may be computed with a decimal approximation for any given value of  $N$ . The table below gives the approximate decimal value of the double sum for different values of  $N$ .

N	Sum
5	12.8
10	38.6
25	140.8
50	349.9
100	837.5
125	1102.4
150	1377.4
200	1951.2
250	2550.3
300	3169.6
350	3805.6

By plotting the above data we find a non-linear increase in the sum with  $N$ . The plot does become linear at high values of  $N$ , so we simply use the larger data points ( $N > 125$ ) and fitting the linear curve to an equation we get

$$\text{Sum} \approx 12N - 467$$

So the sum has an intercept of 467 and slope of 12 when plotted linearly with  $N$  (for the bottom 5 data points). In our thought experiment we assume that there is a statistically relevant set of electrons in the distribution, so we use this data with high  $N$ , which means that we may simply state our approximation of the sum as  $\text{Sum} \approx 10N$ . We also use the following approximation

$$\frac{e^2}{8\pi\epsilon_0 x} = \frac{10^{-9}}{x} (eV \cdot m)$$

So the approximate energy of electric potential,  $E_N$ , for a distribution of  $N$  electrons in a beam with average electron spacing  $x$  is given as (for very large  $N > 500$ )

$$E_N = \frac{10^{-8}N}{x} (eV \cdot m)$$

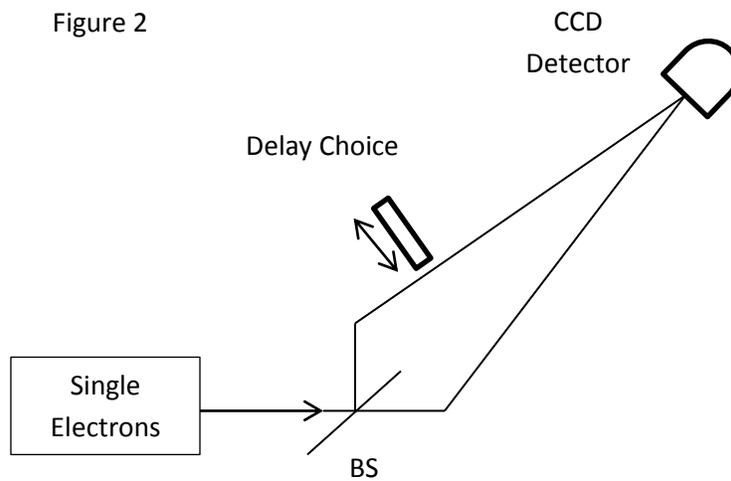
This means that the average energy of electric potential per electron in the beam is  $\approx(10^{-8}/x)eV \cdot m$ , which may be adjusted with the final substitution of the average electron spacing as being on the order of one centimeter,  $x=10^{-2}m$ , so that the average potential energy per electron in the beam is  $\approx 10^{-6}eV$ .

This of course is only a statement of an approximate potential energy per electron, and only for the specific case where the electrons are spaced by 1cm in the beam. So let's assume this to be an indication of the *order of magnitude of the average potential energy of the electron* in our thought experiment. Surely we should be able to measure this order of magnitude in the case of an electron interference effect where the total energy of the electron is on the order of electron volts? This would be easy enough to resolve. So where would this energy show up in measurement? Well, we need to consider two cases, one where the interferometer produces interference (all electrons emitted in one beam with an average electron spacing of  $x$ ) and the other where there is no interference (electrons emitted in two beams each with an average electron spacing of  $2x$ ). If the electrons exit the interferometer in two beams, as in the case of non-interference, then we would expect that the average potential energy per electron is halved, so the kinetic energy of these electrons is equally increased or blue shifted (by an amount that is on the order of  $\approx \frac{1}{2}10^{-6}eV$ ). Or are they? Who are we to suppose that "non-interfering electrons are blue shifted" when we could easily also speculate the exact opposite that "interfering electrons are red shifted"? Well, we should mention that the observations of non-interference, having equal detections of electrons in both output ports, are also achievable with interference. If we use a delay mechanism in one arm of the detector that only causes a  $\lambda/4$  phase delay between the two beams then we would not expect the two output ports to exhibit constructive interference or destructive interference but rather half way in between, with equal numbers of electrons detected at the two output ports. This would imply that the presence of interference or non-interference has no relevance to the argument, and only the output configuration will determine the final state of potential and kinetic energy of the electrons. But before we accept this argument we consider the case of full spatial interference with single electrons.

### **The Idealized Single Electron Spatial Interferometer**

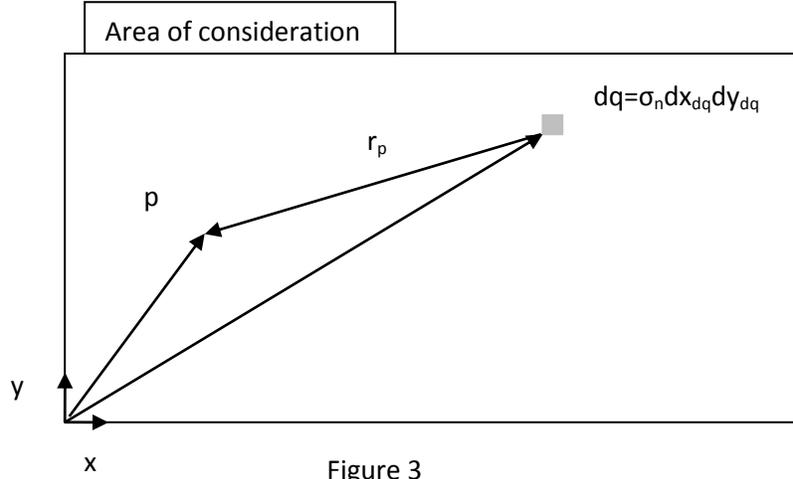
In Figure 2 we illustrate a similar experimental arrangement to our idealized thought experiment, except in this setup we use a source of single electrons that are beam split and then made incident upon a CCD detector screen that measures a spatial interference effect with a fringe pattern. The electron source beam is split with a beam splitting (BS) apparatus and the two output beams are reflected by mirrors to be incident on a distant CCD screen at a small angle to one another in an overlapping fashion. The outputs of the beam splitting apparatus are coherent and the two path lengths from the BS to the detector are identical. With this preparation we would expect to see interference fringes in the detection probability across the screen. The delay mechanism (which increases the length of one path

by an amount greater than the uncertainty of the electron's position in the direction of its propagation) can be inserted in one arm which has the effect of destroying interference. If the delay is inserted then we observe a detection probability at the CCD screen of a continuous Gaussian distribution across the screen that is similar to the cross section of the electron beam (which is identical to the detection probability that would be observed if we had detected the electrons immediately after emission). If we do not insert the delay mechanism then we would observe a detection probability at the CCD screen which has the continuous Gaussian distribution across the screen that is similar to the cross section of the electron beam but with a fringed pattern.



So we consider the state of the electron's potential and kinetic energy in these two possible distributions. In the non-interference case we would expect that the electrons, which are only observed in a configuration that is identical to that of their emitted state (a Gaussian blob), do not experience a shift in their state of energy. With the non-interference case the electrons are emitted in a certain energy state in a single beam and their final state has a position distribution that covers one beam cross section, so there is no possibility that the final state has a different energy of electric potential. In the interference case the final state of the electrons has a position distribution over one beam cross section, but the distribution is that of a fringed pattern. This definitely changes the state of energy of electric potential. However, the calculation of this change is an involved calculation to say the least. (Note that in this thought experiment, unlike the last thought experiment, we do not consider the electric potential *along* the beam but rather *across* the beam.)

To start, we begin by assuming that the electron (or a statistically relevant set of electrons) is distributed over the cross section of the beam, so in order to calculate the potential energy of the two dimensional charge distribution we must integrate  $dq/r$  over the cross section of the beam to first find the potential at a point in the distribution, and then integrate this potential over the cross section to find the energy of electric potential. So we illustrate our calculation with Figure 3 where we consider an area of the cross section of the beam which is not necessarily the full cross section but rather a portion there of.



We start with the case of non-interference where the surface charge density,  $\sigma_n$ , may be estimated as that of a constant continuous even distribution of charge over the area of the screen. The potential at a point  $p$  inside the charge distribution,  $V_p$ , is given by

$$V_p = \frac{1}{4\pi\epsilon_o} \int \frac{1}{r_p} dq$$

where the integral is evaluated over the cross sectional area of the beam so we may substitute the charge element,  $dq$ , for an area element with  $dq = \sigma_n dx_{dq} dy_{dq}$ , where  $x_{dq}$  and  $y_{dq}$  define the position of  $dq$  on the  $x$  and  $y$  axis respectively. The length  $r_p$  is the distance between the point  $p$  and the charge element  $dq$ , which means that  $r_p$  can be expressed as the square root of a polynomial of  $x$  and  $y$ ,

$$r_p = \sqrt{(x_{dq} - x_p)^2 + (y_{dq} - y_p)^2}$$

where  $x_p$  and  $y_p$  define the position of  $p$  on the  $x$  and  $y$  axis respectively. This now means that the integral for  $V_p$  would be very difficult to calculate, even for the case of non-interference with a continuous distribution. The integral may be stated as,

$$V_p(x_p, y_p) = \frac{\sigma_n}{4\pi\epsilon_o} \int \frac{dx_{dq} dy_{dq}}{r_p}$$

However, assuming we arrive at a closed form of  $V_p$  as a function of  $x_p$  and  $y_p$  then we may calculate the potential energy of the whole charge distribution,  $E_n$ , which is evaluated with a second surface integral,

$$E_n = \frac{1}{2} \int \sigma_n V_p(x_p, y_p) dA$$

where this integral is evaluated over the same surface as the last. The area element,  $dA$ , may be substituted with  $dx_p dy_p$ . The combined expression would have the form,

$$E_n = \frac{(\sigma_n)^2}{8\pi\epsilon_0} \int \left\{ \int \frac{dx_{dq} dy_{dq}}{r_p} \right\} dx_p dy_p$$

If we could find a closed expression for this double integral, we would only have finished the easy part. After this we need to perform the whole calculation over again except for the case of an ordered distribution of charges that have a sinusoidal dependence on screen position as the case for interference. In general this changes the calculation by introducing a sinusoidal dependence of the charge distribution,  $\sigma_i$ , on the screen position in one direction,  $x$ , which is easily approximated as

$$\sigma_i(x) = \frac{\sigma_n}{2} \left( 1 + \sin \left\{ \frac{2\pi x}{z} \right\} \right)$$

where the  $x$  in this case applies to both integrals as  $x_{dq}$  and  $x_p$ , and  $z$  is the fringe spacing for the electron distribution. Substituting this into the expression for energy we may calculate the total potential energy of the distribution for the case of interference,  $E_i$ ,

$$E_i = \frac{(\sigma_n)^2}{32\pi\epsilon_0} \int \int \frac{\left( 1 + \sin \left\{ \frac{2\pi x_{dq}}{z} \right\} \right) dx_{dq} dy_{dq}}{r_p} \left( 1 + \sin \left\{ \frac{2\pi x_p}{z} \right\} \right) dx_p dy_p$$

With this now calculated, we may proceed to take the difference in energy of the two cases (interference and non-interference) where we assume the interference case to be the larger of the two energies,

$$\Delta E = E_i - E_n$$

$$\Delta E = \frac{(\sigma_n)^2}{32\pi\epsilon_0} \int \int \frac{\left( 1 + \sin \left\{ \frac{2\pi x_{dq}}{z} \right\} \right) dx_{dq} dy_{dq}}{r_p} \left( 1 + \sin \left\{ \frac{2\pi x_p}{z} \right\} \right) dx_p dy_p$$

$$- \frac{(\sigma_n)^2}{8\pi\epsilon_0} \int \left\{ \int \frac{dx_{dq} dy_{dq}}{r_p} \right\} dx_p dy_p$$

$$\Delta E = \frac{(\sigma_n)^2}{32\pi\epsilon_0} \int \int \frac{\left( \sin \left\{ \frac{2\pi x_{dq}}{z} \right\} \sin \left\{ \frac{2\pi x_p}{z} \right\} + \sin \left\{ \frac{2\pi x_{dq}}{z} \right\} + \sin \left\{ \frac{2\pi x_p}{z} \right\} - 3 \right) dx_{dq} dy_{dq}}{r_p} dx_p dy_p$$

It is argued here that this quantity, the difference in potential energy between states of interference and non-interference, is a non-zero positively valued quantity. That is to say, we assume both  $E_i$  and  $E_n$  are non-zero positively valued energies and that  $E_i > E_n$  so that  $\Delta E = E_i - E_n > 0$ . Surely, the energy of electric potential of the average electron (or a statistically relevant set) is greater in the configuration of interference than that of non-interference. In the case of interference, the end configuration of the electrons is a fringed Gaussian blob. In the case of non-interference, the end configuration of electrons is not fringed but rather it is only a Gaussian blob. There is necessarily a difference in the energy of electric potential for the two possible configurations, and it must be the case that the potential energy of the interference configuration (fringes) is the greater of the two energies. Although, if we attempt to

calculate the above integral by assigning limits of integration and adding a Gaussian factor to the charge density  $\sigma_n$ , then we would have a calculation that is beyond the scope of this paper. For the purposes of this paper, we need only conclude that  $\Delta E$  is non-zero and positively valued and that it may be calculated.

This brings us back to our earlier argument about the first thought experiment of Figure 1, is there a blue shift of the electrons that do not exhibit interference or is there a red shift of the electrons that do exhibit interference? Well, we may now look at the results of our analysis of the second thought experiment of Figure 2, that there seems to be a difference in the potential energy of the electron configuration for interference and non-interference, with the interference configuration having the higher potential energy. Even if we allow interference in the second thought experiment and we insert a  $\lambda/4$  phase delay in one arm of the interferometer, then we will get the same output configuration except that the fringes will be moved in one direction across the screen. So adding a phase delay in one arm of the interferometer will not change the output configuration as it did in the first thought experiment, with an associated change in energy of electric potential. In the second thought experiment the interference case is always the configuration with the higher energy of electric potential per electron. The second thought experiment seems to indicate that the interference case always has a higher state of energy of electric potential, and that the kinetic energy or momentum of the electrons is then red shifted to accommodate for this. But the first thought experiment seems to imply that the presence of interference or non-interference has no relevance to the argument, and only the output configuration will determine the final state of potential and kinetic energy of the electrons. To further discuss this matter we categorize our argument in two distinct interpretations on the matter;

**Interpretation 1:** Energy is conserved due to physical law, and for this reason we know that any experiment that has a change in the state of electric potential energy of a system in going from its input state to its output state must have an equal and opposite change in the kinetic energy of the final state, which means there must be a shift in the momentum distribution because kinetic energy is in proportion with  $\mathbf{p}\cdot\mathbf{p}$ . *The state of energy is always calculated in accordance with the total preparation.*

**Interpretation 2:** In interference effects, the position of the system is actualized upon measurement collapse, and there is an energy of electric potential that is associated with the final position distribution. However, the momentum distribution of the electrons is **always** calculated by assuming the case of non-interference, even when we get interference. *The calculation of energy of electric potential is mutually exclusive to the calculation of interference.*

For the most part, physicists would advocate something like interpretation 1 with the assumption that conservation of energy accounts for any calculated change in the energy of the output configuration. In this case, we would calculate the final energy of electric potential for any experiment based solely on the configuration of the output. If the final energy of electric potential is greater than that of the initial, then there is a red shift in the state of momentum. If the final energy of electric potential is less than that of the initial, then there is a blue shift in the state of momentum.

So what about the second interpretation? First of all, one of the assumptions of all interference effects for individual systems is that the effect itself, interference, and its associated theoretical account have absolutely no dependence whatsoever on the interactions that take place *between* individual systems. It is often noted that the great Paul Dirac once quoted in his textbook on quantum mechanics something to the effect that in interference *each individual system only interacts with itself*. One of the underlying assumptions to the theory of interference is that the individual systems do not interact with one another, which is why it is preferential to be dealing with non-interacting bosonic systems. In the case that the system of interference is electronic or some other fermionic system then we must make assumptions concerning the maximum occupation number of the beam so that there is no interactions taking place. But we should add one point of significance to this argument, that all interference effects may indeed be collected *one individual system at a time*, and that by adding the results together at the end of a statistically relevant set of trials we will have fulfilled the necessary observations of an interference effect in the exact same way that we would if we had collected all the trials in one grand experiment with a beam of higher intensity.

In this picture, where we may collect our electron interference effect one electron at a time, we never have any interaction between any two electrons so we definitely will not have any energy of electric potential and no subsequent shift in the momentum. Or do we? Should we take into consideration the time evolution of the single electron as it is separated into many distinct paths leading up to measurement (in which case our analysis stands), or should we only take into account the final measurements? Well, whenever we perform theoretical calculation in quantum mechanics we do so assuming a statistically relevant set or a distribution, regardless of the fact that the observations can be collected one by one, regardless of the fact that we only ever observe individual systems in quantum mechanics. The individual system may be the definitive *measurement* of quantum mechanics but it is certainly not indicative of the *theory* of quantum mechanics.

Taking these arguments into consideration, I have come to the conclusion that Interpretation 2 has merit. I believe that the calculation of energy of electric potential and the calculation of interference are *mutually exclusive calculations*. First we calculate the energy of electric potential based upon the output configuration in the case of non-interference, and then we deduce the momentum distribution (with shift) from this, and then we consider the additional energy of electric potential based upon the final output configuration for the final state of the given preparation.

#### Case 1: A Simple Beam Splitter

If we were to put an electron beam (with many distantly spaced electrons) through a beam splitter and allowed the two outputs of electrons to become infinitely spaced upon measurement, then we would calculate a blue shift in the momentum distribution due to the loss of electric potential.

#### Case 2: Experiment 1 with Non-interference

If we perform thought experiment 1 with non-interference then we have a similar output configuration, and we would calculate the same blue shift and the same momentum distribution.

Case 3: Experiment 1 with Interference

If we perform thought experiment 1 with interference then we have a different output configuration (a single beam), but we would have *the same momentum distribution* **and** we would get the *additional energy of electric potential* due to the output being actualized upon measurement in a single beam.

Case 4: Experiment 1 with Interference and a  $\lambda/4$  Delay

If we perform thought experiment 1 with interference and with a  $\lambda/4$  delay in one arm of the interferometer then we have the original output configuration (two equal beams), and we would have the same momentum distribution and we have no additional energy of electric potential because the output is split in two beams. In other words the additional energy of electric potential only shows up in the case of constructive and destructive interference.

Case 5: Experiment 2 with Non-interference

If we perform thought experiment 2 with non-interference then we have an output configuration that is identical to spontaneous emission (a Gaussian blob) and we consider the energy and momentum distributions of the electrons to be identical to that of their emitted state.

Case 6: Experiment 2 with Interference

If we perform thought experiment 2 with interference then we have an output configuration that is a fringed Gaussian blob, so the energy and momentum distributions of the electrons is identical to that of their emitted state **and** there is an additional energy of electric potential ( $\Delta E$ ) associated with this configuration.

Case 7: Experiment 2 with Interference and a  $\lambda/4$  Delay

If we perform thought experiment 2 with interference and with a  $\lambda/4$  delay in one arm of the interferometer then we have an output configuration that is a fringed Gaussian blob (except it is shifted by half a fringe spacing), so the energy and momentum distributions of the electrons is identical to that of their emitted state and there is an additional energy of electric potential ( $\Delta E$ ) associated with this configuration. We tabulate all the possibilities below.

Experiment	Insert in the interferometer?	Interference?	Output Configuration?	Additional Potential Energy?
Just Beam Splitter	None	No	2 Grey Ports	0
Expt. 1	None	Yes	Dark & Light Port	$\approx 10^{-6}$ eV per $e^-$
Expt. 1	$\Delta x$ Delay	No	2 Grey Ports	0
Expt. 1	$\lambda/4$ Delay	Yes	2 Grey Ports	0
Expt. 2	None	Yes	Fringes	$\Delta E$
Expt. 2	$\Delta x$ Delay	No	No Fringes	0
Expt. 2	$\lambda/4$ Delay	Yes	Moved Fringes	$\Delta E$

In the table we have all the possibilities expressed for the two experiments. We note that the first and third rows, for a beam splitter and experiment 1 with non-interference respectively, we have the same entries in all columns. There is no interference and the output configuration is a pair of grey ports. In the second row we have experiment 1 with interference so we indicate the presence of a dark/light output configuration and the additional energy. In row five and six we have experiment 2 with and without interference respectively, so we indicate the additional energy for the interference preparation only.

The interesting entries are in row four and seven where we have experiment 1 and 2 respectively, both with the insertion of a  $\lambda/4$  phase delay in one arm of the interferometer. Each has interference but only experiment 1 indicates an additional energy. This is because in experiment 2 there is always a measurement collection occurring for all possible combinations of phase from the two beams, where in experiment 1 we only have one phase combination being measured. In experiment 2 the insertion of the  $\lambda/4$  phase delay causes the interference pattern to shift in a manner that the center of the pattern now displays the "grey region" of the interference pattern (the same position where the constructive maxima would have been if there were no delay apparatus). In experiment 1 the insertion of the  $\lambda/4$  phase delay causes the interference pattern to produce two grey ports. In this sense, the two grey ports of experiment 1 are displaying the same output as the pixels in the center of the screen in experiment 2. We note that the grey region of pixels in experiment 2 are in no way contributing to the additional energy of  $\Delta E$  because they are identical to that of spontaneous emission, a grey Gaussian blob. This justifies our conclusion of row four that experiment 1 will not give an additional energy of electric potential, as would be predicted by its output configuration, even though there is interference.

No References, but I have discussed similar subject matter to this in a web page:

<http://violationprotocol.webs.com/redistribution-protocol>