On Bell's Inequality

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We show that when spin eigenfunctions are not fully orthonormal, Bell's inequality does allow local hidden variables. In the limit where spin eigenfunctions are Dirac orthonormal, we recover a significant extremal case. The new calculation gives a possible accounting for $\alpha_{\text{MCM}} - \alpha_{\text{QED}}$.

As it has been understood, Bell's inequality rules out the new variable proposed in the MCM. No analytic form has been found for the eigenfunctions of the spin operator but they are assumed to be orthonormal. In this short paper we examine the case when spin eigenfunctions are not orthonormal [1]. Derivation of Bell's inequality often starts with a statement of the average value of the product of the spins when the detectors are aligned along spatial unit vectors $\vec{a}$ and $\vec{b}$ and θ is the angle between them [2].

$$P(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b} = -\cos(\theta) \tag{1}$$

This is derived by taking the expectation value of the product of two spins in a singlet state. Moving directly to the end of that calculation we find the following.

$$\begin{aligned} P(\vec{a},\vec{b}) \;&=\; \frac{\sin(\theta)}{\sqrt{2}}\langle 0\,0|1\;-1\rangle - \\ &-\;\cos(\theta)\langle 0\,0|0\,0\rangle + \frac{\sin(\theta)}{\sqrt{2}}\langle 0\,0|1\,1\rangle \end{aligned} \tag{2}$$

When spin states are orthogonal, equation (2) reduces to equation (1). When they are not orthogonal, the $\sin(\theta)$ terms do not go to zero. Let the magnetic quantum number distinguish $\delta_{\pm}$.

$$P(\vec{a},\vec{b}) = \delta_{-} - \vec{a}\cdot\vec{b} + \delta_{+} \tag{3}$$

Bell's inequality is derived from the difference between $P(\vec{a},\vec{b})$ and $P(\vec{a},\vec{c})$. Using the normal prescription [2] that $-\vec{a}\cdot\vec{b} = A(\vec{a})A(\vec{b})$ and moving to the hidden variable formalism, we may write the following.

$$\begin{aligned} P(\vec{a},\vec{b}) \;&-P(\vec{a},\vec{c}) = \\ =\;& \int [1 - A(\vec{b},\lambda)A(\vec{c},\lambda)]A(\vec{a},\lambda)A(\vec{b},\lambda)\rho(\lambda)d\lambda + \\ +\;& \int (\delta_{-}^{ab} - \delta_{-}^{ac})\rho(\lambda)d\lambda + \int (\delta_{+}^{ab} - \delta_{+}^{ac})\rho(\lambda)d\lambda \end{aligned} \tag{4}$$

The system in question decays to two particles so it is not possible to directly test the the theory's prediction for three different detector alignments $\{\vec{a},\vec{b},\vec{c}\}$. The experimenter would have to perform a test in one apparatus configuration $\{\vec{a},\vec{b}\}$, then reconfigure the table for $\{\vec{a},\vec{c}\}$ and take more data at some later time. In the process of reconfiguring, the observer moves to a different level of $\aleph$ so $\delta^{ab} \neq \delta^{ac}$. The delta resultant from the earlier measurement is infinitely smaller than the later one and can safely be ignored.

When the delta is taken to be the Dirac delta, we find the extremal case in which local hidden variables are always allowed.

$$|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \leq 3 + P(\vec{b},\vec{c}) \tag{5}$$

Now consider the case when $\delta_{\pm}$ are integrated according to the prescription in reference [1].

$$|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \leq 1 + P(\vec{b},\vec{c}) + 2\pi + (\Phi\pi)^3 \tag{6}$$

This leads to a possible accounting for the small discrepancy between the expected value for the fine structure constant and the empirically determined one α_{QED}.

$$\alpha_{\text{MCM}} - \alpha_{\text{QED}} = P(\vec{b},\vec{c}) + 1 \tag{7}$$

The difference is less than one so it is possible the +1 factor needs to be dropped. Another possibility is that the requirement for a probability not between zero and one is a manifestation of non-unitarity. Interesting is that $\alpha_{\text{MCM}} - \alpha_{\text{QED}} \approx \varphi$.

The two axes $\vec{b}$ and $\vec{c}$ are not necesarily related to local orientation. They could define the angle of intersection between the worldlines that made this universe come into existence. This description of α_{MCM} both allows and tightly constrains a varying fine structure constant. Small fluctuations in the historical value for α_{QED} may be caused in part by orbital and other wobbles. Such cases are readily optimized against empirical studies of the shocking anomalies in the CMB [3].

[1] J. Tooker, viXra:1312.0168 (2013)

[2] D.J. Griffiths, *Introduction to Quantum Mechanics*, 2 Ed., Ch. 12 (2004)

[3] R.E. Schild and C.H. Gibson, arXiv/astro-ph: 0802.3229 (2008)