CARDINALS AND BIJECTION OF L AND P AND NP

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1. Abstract

This article describes about that L is not P and P is not NP by using bijection reduction between each problems.

Deterministic Turing Machine (DTM) can easily change bijection DTM. Therefore, if L is P then bijection LDTM can reduce CIRCUIT-VALUE to UNDIRECTED-PATH. But CIRCUIT-VALUE have same cardinals' co-problem and UNDIRECTED-PATH have little cardinals' co-problem. Therefore, bijection LDTM cannot reduce CIRCUIT-VALUE to co-UNDIRECTED-PATH and L is not P.

And using L is not P, we can prove P is not NP. All P problem have equivalent reversible function and DTM can reduce from NP-Complete problem to another NP-Complete problem by using this reversible function. If P is NP, equivalent Logarithm space reductcion exists. But that means L is P and contradict L is not P. Therefore, P is not NP.

2. Preparation

In this article, we will use words and theorems of References [1, 2, 3] in this paper.

About problem and turing machine types, we use description as follows;

Definition 1. We will use the term "Input" as data that Turing Machine compute, "Output" as result that Turing Machine compute. "Problem" as set of all input that same Turing Machine can compute same output. "L" as L problem set, "P" as P problem set, "$P-Complete$" as P-Complete problem set, "$NP-Complete$" as NP-Complete problem set, "FL" as Logarithm space function problem set, "FP" as Polynomial time function problem set. "DTM" as Deterministic Turing Machine set. "$LDTM$" as Turing Machine set that compute L and FL, "$pDTM$" as Turing Machine set that compute P and FP. "$RpDTM$" as Reversible $pDTM$.

Define concrete problem as follows;

Definition 2. We will use the term "$CIRCUIT-VALUE$" as a P-Complete problem of computing circuit output value. To simplify, we add YES gate that output input value (reverse NOT gate). "$UNDIRECTED-PATH$" as a L-Complete problem of finding undirected graph path between assigned two vertexes. To simplify, we describe each edge exist or not. That is, each vertex are array of edges existence that listed another vertexes.

Define problems cardinals within finite.

Definition 3. We will use the term "Problem cardinals" and "$|P|_n$" as a cardinals of problem that input length is n.

3. L is not P

Prove $L \neq P$ by using cardinals difference of between $CIRCUIT-VALUE$ and $\overline{UNDIRECTED-PATH}$. $CIRCUIT-VALUE$ have high resolution and $\left|CIRCUIT-VALUE\right|_n = \left|\overline{CIRCUIT-VALUE}\right|_n$. And all $P,Q \in P\text{−}Complete$ have bijection LDTM reduction. But $UNDIRECTED-PATH$ have low resolution and $\left|UNDIRECTED-PATH\right|_n \gg \left|\overline{UNDIRECTED-PATH}\right|_n$. Therefore $CIRCUIT-VALUE$ cannot reduce to $\overline{UNDIRECTED-PATH}$.

Theorem 4. *$CIRCUIT-VALUE$ cardinals is same as $\overline{CIRCUIT-VALUE}$ cardinals. That is,*
$\left|CIRCUIT-VALUE\right|_n = \left|\overline{CIRCUIT-VALUE}\right|_n$

Proof. This is trivial because all $p \in CIRCUIT-VALUE$ have dual circuit $q \in \overline{CIRCUIT-VALUE}$. □

Theorem 5. *$UNDIRECTED-PATH$ cardinals is more than $\overline{UNDIRECTED-PATH}$ cardinals. That is,*
$\left|UNDIRECTED-PATH\right|_n > \left|\overline{UNDIRECTED-PATH}\right|_n$
and
$$\frac{\left|UNDIRECTED-PATH\right|_n}{\left|\overline{UNDIRECTED-PATH}\right|_n} = O\left(c^n\right)$$

Proof. Set finding path problem as graph p start vertex s to goal vertex t in k vertices. All $p \in UNDIRECTED-PATH$ if p have edge that link s to t, but some $p \in UNDIRECTED-PATH$ if p have no edge that link s to t. Number of graph that have edge that link s to t is equal that have no edge that link s to t. Therefore,
$\left|UNDIRECTED-PATH\right|_n > \left|\overline{UNDIRECTED-PATH}\right|_n$

Some graph that have no link s to t is in $UNDIRECTED-PATH$ to get around another vertex. Number of such graph is amount $O\left(c^n\right)$ because atmost $O\left(c\right)$ edges are fixed and another edges are acceptable that edge exist or not. Therefore, $\left|UNDIRECTED-PATH\right|$ is very larger than $\left|\overline{UNDIRECTED-PATH}\right|$
$$\frac{\left|UNDIRECTED-PATH\right|_n}{\left|\overline{UNDIRECTED-PATH}\right|_n} = O\left(c^n\right)$$
□

Theorem 6. *$P,Q \in P-Complete$ have bijection reduction. That is,*
$\forall P,Q \in P\left(\exists h \in LDTM\left(h^{-1}\left(P\right) = Q\right)\right)$

Proof. This is trivial by using The Cantor-Bernstein-Schroeder theorem[4].

Define injection
$f : P \to Q$, $g : Q \to p$
then bijection h is
$$h(x) = \begin{cases} g^{-1}(x) & \text{if x in target of } g \\ f(x) & \text{else} \end{cases}$$

LDTM can compute h because DTM can easily reverse to NTM and if g^{-1} compute nondeterministic transition then h compute f. Therefore, this theorem was shown. □

Theorem 7. $L \neq P$

Proof. We prove it using reduction to absurdity. We assume that $L = P$. Therefore, bijective LDTM can reduce $CIRCUIT - VALUE$ to $\overline{UNDIRECTED - PATH}$.

But mentioned above 45,$\left|CIRCUIT - VALUE\right|_n = \left|\overline{CIRCUIT - VALUE}\right|_n$ and $\dfrac{\left|UNDIRECTED - PATH\right|_n}{\left|\overline{UNDIRECTED - PATH}\right|_n} = O\left(c^n\right)$. Therefore $\left|\overline{UNDIRECTED - PATH}\right|_n$ is too small to reduce bijective to $\left|CIRCUIT - VALUE\right|_n$. Therefore LDTM cannot compute this reduction and contradict $L = P$.

Therefore, this theorem was shown than reduction to absurdity. □

4. P is not NP

Prove $P \neq NP$ by using $L \neq P$.

Theorem 8. $P \neq NP$

Proof. We prove it using reduction to absurdity. We assume that $P = NP$, therefore all $p, q \in NP - Complete$ have $f \in LDTM$ that reduce p to q.

$\forall p, q \in NP - Complete \exists f \in LDTM \left(f\left(p\right) = q\right)$

If $p \in NP - Complete$ and $g \in RpDTM$ then

$p \leq_p g\left(p\right)$

and

$g\left(p\right) \leq_p g^{-1}\left(g\left(p\right)\right) = p \in NP \rightarrow g\left(p\right) \in NP$

Therefore

$g\left(p\right) \in NP - Complete$

That is,

$\forall p \in NP - Complete \forall g \in RpDTM \exists f \in LDTM \left(f\left(p\right) = g\left(p\right)\right)$

But mentioned above7, $RpDTM \neq LDTM$ and contradict it.

Therefore, this theorem was shown than reduction to absurdity. □

References

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