Exotic R4 and E8 Physics

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E8 Physics (see viXra 1312.0036 and 1310.0182) is based on E8 which lives in the Clifford Algebra Cl(16) and models fermions by using spinor structures. As spinor structures are fundamentally related to Exotic Spheres, E8 Physics fits nicely with Exotic Sphere structures.

Exotic R4 is also relevant to physics as Carl Brans, Torsten Asselmeyer-Maluga, and their co-workers have shown in increasing detail. This paper is an attempt to describe how the physics of Exotic R4 might be related to E8 Physics.

As to fermions: Exotic R4 gives a mass term $\mu \cdot vol(S^1 \times S^3)$ for fermions with the constant $\mu = \bar{\mu}$ representing the curvature of $S^1 \times S^3$ but Torsten Asselmeyer-Maluga et al say "... at the moment we have no idea how to generate realistic masses from this idea ...". E8 Physics shows how to "generate realistic masses" using geometric volumes in a way that may be equivalent to the Exotic R4 approach (see viXra 1311.0088 which unites Schwinger's Source Theory, the geometry of L. K. Hua, and the work of Armand Wyler).

As to gauge bosons: Exotic R4 gets the Standard Model U(1)xSU(2)xSU(3) from the structure of connecting tubes that have "similarity with ... brane theory: n parallel branes ... described by ... U(n) gauge theory". E8 Physics when formulated as 26D String Theory with Strings as World-Lines (see viXra 1210.0072) also gets gauge bosons for the Standard Model and MacDowell-Mansouri Conformal Gravity in terms of brane-to-brane connecting links.

As to a hyperfinite von Neumann factor Algebraic Quantum Field Theory (AQFT):
Exotic R4 constructs a hyperfinite von Neumann factor algebra by foliations of S3 of an Exotic R4 Akbulut Cork with Casson handle but lacks an explicit AQFT.
E8 Physics has such an AQFT from a generalization of the II1 factor, constructed according to Periodicity of Real Clifford Algebras by completing the union of all tensor products of the Clifford Algebra Cl(16) that contains E8 = D8 + half-spinor of D8.
Exotic R4 and E8 Physics may provide complementary information for a realistic AQFT.

Here on the following pages are some details about Exotic Structures.

Corresponding details about E8 Physics are given in the viXra references set out above.

Thanks to Daniel Rocha for suggesting that I study the interesting Exotic R4 work of Carl Brans, Torsten Asselmeyer-Maluga, and their co-workers.

There are **two kinds of exotic differentiable structures**:

The first kind is **Exotic Spheres**,
such as Smooth=Differentiable manifolds that are Combinatorial=Piecewise Linear equivalent to the n-sphere Sn. Some examples:

S1 - 1
S2 - 1
S3 - 1 (but S3 is a subset of any exotic R4)
S4 - 1
S5 - 1
S6 - 1
S7 - 28
S8 - 2
S9 - 8
S10 - 6
S11 - 992
S12 - 1
S13 - 3
S14 - 2
S15 - 16,256
S16 - 2

Note that

S7 = Spin(8) / Spin(7) and Spin(8) half-spinors are 8-dim and 28 = 8/\8 = 8x7/2

S11 = Spin(12) / Spin(11) and Spin 12 half-spinors are 32-dim and 992 = 32x31

S15 = Spin(16) / Spin(15) and Spin 16 half-spinors are 128-dim and 16,256 = 128x127

It turns out that Exotic Sphere Structures are directly due to Spinor Structures
and
are accounted for in E8 Physics by the Clifford Algebra foundation of E8 Physics.

The second kind is **Exotic R4.**

Exotic R4 corresponds to 3 fundamental components of E8 Physics:
1 - fermionic fields 2 - bosonic fields 3 - hyperfinite factor AQFT

Torsten Asselmeyer-Maluga and Helge Rose in arXiv 1006.2230 said:
"... start... with a smooth 4-manifold M
admitting an exotic smoothness structure MK ... constructed by using knot surgery. ...
consider... the Einstein-Hilbert action on MK and the decomposition

$$M = (M \setminus N(T^2)) \cup_{T^2} (S^1 \times (S^3 \setminus N(K)))$$

... with $N(T^2) = D^2 \times T^2$ ...

Because of the diffeomorphism invariance of the action, one can split the Einstein-Hilbert action like

$$S_{EH}(M_K) = S_{EH}(M \setminus N(T^2)) + \int_{S^1 \times (S^3 \setminus N(K))} R_K \sqrt{g_K}\, d^4x$$

...

$$S_{exotic} = \mu \cdot vol(S^1 \times S^3) - \int_{S^1 \times \partial(N(K))} H_\theta \sqrt{h}\, d\theta d^2x - \lambda \cdot vol(D^2)$$

with the constant $\mu = \bar{\mu}$ representing the curvature of $S^1 \times S^3$ which is identical to the curvature of S^3. with H_θ as *mean curvature* of $\partial(N(K))$
The manifold $S^1 \times \partial N(K)$ is a knotted 3-torus $T^3(K) = K \times S^1 \times S^1$.

... exotic smoothness generates fermionic and bosonic fields ...

... define an action over the knot complement to identify two contributions:
knotted tori and connecting tubes between two tori.

1. a knotted solid torus can be described by a spinor
so that the mean curvature is the Dirac action of this spinor. ...

2. connecting tube ... as cobordism between two tori ... we obtained the Yang-Mills action ... The three possible types of torus bundles were identified with three interactions to get the gauge group

$$U(1) \times SU(2) \times SU(3)$$

The action $\int_{S^1 \times \partial N(K_-)} H_{\partial N(K)} \sqrt{g}\, d\theta d^2x$ is completely determined by the knotted torus $\partial N(K) = K \times S^1$ and its mean curvature $H_{\partial N(K)}$.

This knotted torus is an immersion of a torus $S^1 \times S^1$ into $\mathbb{R}^3$.

Spin representation of a surface gives back an expression for $H_{\partial N(K)}$ and the Dirac equation as geometric condition on the immersion of the surface.

The action can be interpreted as Dirac action of a spinor field.

$$\int_M (R + \bar{\Phi} D^M \Phi) \sqrt{g} d^4x$$

... ...

The action is the usual Einstein-Hilbert action for a Dirac field Φ as source.

What about the mass term? In our scheme there is one possible way to do it: using the constant length $|\Phi|^2 = const.$ of the spinor, we can introduce the scalar curvature R_Γ of an additional 3-manifold Γ with constant curvature coupled to the spinor. Then we obtain

$$\int_M \bar{\Phi}(D^M - m)\Phi \sqrt{g}\, d^4x$$

with $m = -R_\Gamma$ and $\Gamma \subset M$. But we already have a natural choice for this manifold, the 3-sphere $\Gamma = S^3$ as the embedding space for the knotted torus $\partial N(K) = K \times S^1$. Then the knotted 3-torus $T^3(K) = K \times S^1 \times S^1$ is given by an embedding of the 3-torus T^3 into $S^1 \times S^3$. Therefore as a conjecture the term $\mu \cdot vol(S^1 \times S^3)$ can be interpreted as mass term for the fermions.

Especially we obtain

$$\int_M m\bar{\Phi}\Phi \sqrt{g}\, d^4x = \mu \cdot vol(S^1 \times S^3)$$

having the correct sign in the action But at the moment we have no idea how to generate realistic masses from this idea.

consider the integral $$\int_{(S^3\backslash N(K))} R_{(3)}\sqrt{h}N\, d^3x$$

Let $C(K) = S^3 \setminus N(K)$ be the knot complement for the knot K and assume for K a sum

$$K = K_1 \# K_2$$

of prime knots K_1, K_2. Then the knot complements admits a splitting

$$C(K) = C(K_1) \cup_{T^2} T(K_1, K_2) \cup_{T^2} C(K_2)\,.$$

We call $T(K_1, K_2)$ the *connecting tube* between the knot complements $C(K_1)$ and $C(K_2)$. The connecting tube $T(K_1, K_2)$ has a boundary consisting of three disjoint tori $\partial T(K_1, K_2) = T_1^2 \sqcup T_2^2 \sqcup T_3^2$ (we ignore the orientation) where one of these tori T_3^2 is the boundary $\partial C(K) = T_3^2$ of $C(K)$. If we ignore this boundary (by closing it with a solid torus $T(K_1, K_2) \cup_{T_3^2} (D^2 \times S^1)$) then we have a trivial torus bundle $T^2 \times [0, 1]$ between T_1^2 and T_2^2.

...

we obtain for the action $S_{EH}(S^1 \times T(K_1, K_2))$

$$\int_{S^1 \times T(K_1, K_2)} R_K \sqrt{g_K}\, d^4x = L_{S^1} \cdot L \cdot CS(T(K_1, K_2), A)$$

with respect to the (Levi-Civita) connection A and the length L and the Chern-Simons action $CS(T(K_1, K_2), A)$. For the 3-manifold $T(K_1, K_2)$, there is a 4-manifold M_T with $\partial M_T = T(K_1, K_2)$ (take for instance $M_T = T(K_1, K_2) \times [0, 1) \subset T(K_1, K_2) \times S^1$). By using the Stokes theorem we obtain

$$S_{EH}(M_T) = \int_{M_T} tr(F \wedge F)$$

with the curvature $F = DA$

...

We contract $T(K_1, K_2)$ to thin tubes connecting the thick parts. Conversely one also finds a scaling so that the thin part becomes large (but the thick part has the same size). Thus we can interpret the curvature $\bar{F}$ of the thin part as field located between the thick part. The thick part can be interpreted as fermions the action integral of the bosons can be written as

$$\int_{M \setminus vol(fermion)} tr(\bar{F} \wedge *\bar{F})$$

considered over $M \setminus vol(fermions)$

...

In the action we have two constant terms $\mu \cdot vol(S^1 \times S^3)$ and $\lambda \cdot vol(D^2)$
The first constant was interpreted as a mass term of the fermion.
the second constant $\lambda \cdot vol(D^2)$ as the *cosmological constant* Λ

$$\Lambda = \frac{\lambda \cdot vol(D^2)}{vol(M)} .$$

showing a combined Dirac-gauge-field coupled to the Einstein-Hilbert action

$$S(M) = \int_M \left(R - \Lambda + \sum_n (\bar{\Phi}(D^M - m)\Phi)_n \right) \sqrt{g}\, d^4x + \int_M tr(\bar{F} \wedge *\bar{F})$$

... the connecting tube T(K1,K2) is ... a torus bundle ... which can always be decomposed into three elementary pieces ...

finite order (orders 2,3,4,6): the tangent bundle is 3-dimensional
... 2 isotopy classes (= no/even twist or odd twist) ...
no twist must be the photon ... even twist or odd twist ... should be ... the Z0 boson ...

Dehn-twist (left/right twist): the tangent bundle is a sum of a 2-dim and a 1-dim bundle
... 2 isotopy classes (= left or right Dehn twists) ...
are the W+/- bosons ...

Anosov: the tangent bundle is a sum of three 1-dim bundles.
... 8 isotopy classes (= ... orientations of the three line bundles ..)
correspond to the 8 gluons ...

... We remark the similarity with ... brane theory:
n parallel branes ... are described by an U(n) gauge theory ...".

3 - Exotic smoothness generates hyperfinite factor AQFT

John Baez in his week 175 said:

"... a von Neumann algebra is a ... *-algebra of operators that is closed in the weak topology. Every von Neumann algebra can be built from ... "simple" ones as ... a "direct integral" ... People call simple von Neumann algebras "factors" ...

The first step in classifying factors was done by von Neumann and Murray, who divided them into types I, II, and III. ...

We say a factor is type I if it admits a nonzero trace for which the trace of a projection lies in the set {0,1,2,...,+∞}. We say it's type I_n if we can normalize the trace so we get the values {0,1,...,n}. Otherwise, we say it's type I_∞, and we can normalize the trace to get all the values {0,1,2,...,+∞}.

It turn out that every type I_n factor is isomorphic to the algebra of n x n matrices. ...

a factor is type II_1 if it admits a trace whose values on projections are all the numbers in the unit interval [0,1]. We say it is type II_∞ if it admits a trace whose value on projections is everything in [0,+∞]. Playing with type II factors amounts to letting dimension be a continuous rather than discrete parameter!

...

to construct a type II_1 factor ... Start with the algebra of 1 x 1 matrices, and stuff it into the algebra of 2 x 2 matrices ... This doubles the trace, so define a new trace on the algebra of 2 x 2 matrices which is half the usual one. Now keep doing this, doubling the dimension each time, using the above formula to define a map from the 2^n x 2^n matrices into the 2^{n+1} x 2^{n+1} matrices, and normalizing the trace on each of these matrix algebras so that all the maps are trace-preserving. Then take the *union* of all these algebras... and finally, with a little work, complete this and get a von Neumann algebra ... this von Neumann algebra is a factor. It's pretty obvious that the trace of a projection can be any fraction in the interval [0,1] whose denominator is a power of two. But actually, *any* number from 0 to 1 is the trace of some projection in this algebra - so we've got ... a type II_1 factor. This isn't the only II_1 factor, but it's the only one that contains a sequence of finite-dimensional von Neumann algebras whose union is dense in the weak topology. A von Neumann algebra like that is called "hyperfinite", so this guy is called "the hyperfinite II_1 factor" ... the algebra of 2^n x 2^n matrices is a Clifford algebra, so the hyperfinite II_1 factor is a kind of infinite-dimensional Clifford algebra.

But the Clifford algebra of 2^n x 2^n matrices is ... another name for the algebra generated by creation and annihilation operators on the fermionic Fock space over C^{2n} ...

the hyperfinite II_1 factor is the smallest von Neumann algebra containing the creation and annihilation operators on a fermionic Fock space of countably infinite dimension.

... the hyperfinite II_1 factor is the right algebra of observables for a free quantum field theory with only fermions. ...

The most mysterious factors are those of type III ... pretty much all the usual field theories on Minkowski spacetime have type III factors as their algebras of "local observables" - observables that can be measured in a bounded open set. ...".

Torsten Asselmeyer-Maluga and Jerzy Krol in arXiv 1001.0882 said:
"... non-standard smooth R4 's exist as a 4-dimensional smooth manifolds ...
a small exotic structures of the R4 is determined by the so-called Akbulut cork ...
and its embedding given by an attached Casson handle.
The boundary of the cork is a homology 3-sphere containing a 3-sphere S3
such that the codimension-1 foliations are determined by the foliations of S3 ...
we ... relate the exotic R4 to ...the hyperfinite factor III1 von Neumann algebra ...
we obtain a foliation of the horocycle flow ... which determines the factor II∞
...
we are loking for a classical algebraic structure which would give the ... noncommutative algebra of observables as a result of quantization
...
The classical structure ... has the structure of a Poisson algebra ... idempotents were ... constructed as closed curves in the leaf of the foliation of S3 ...
a quantization procedure of the ... Poisson algebra ... is the skein algebra ... directly related to the factor III1 von Neumann algebra derived from the foliation of S3 ...
the skein algebra is ... the factor II1 algebra Morita equivalent to the factor II∞ which in turn determines the factor III1 of the foliation ...
the ... main building blocks of ... 4-exotic smooth structures ... i.e., Casson handles, determine the factor II1 algebras ...
a Casson handle is represented by a labeled finitely-branching tree Q ...
Every path in this tree represents one leave in the ...horocycle ... foliation of the S3 .
Two different paths in the tree represent two different leaves in the foliation.
Then we have to consider two paths in the tree Q ,
the reference path for the given leaf and a path for the another leaf of the foliation.
Thus, a pair of two paths corresponds to one element of the algebra ...
this algebra is given by ... Clifford algebra ... i.e. by the hyperfinite factor II1 algebra
...
We do not have explicit descriptions of a RAQFT ... algebraic relativistic QFT ... on an exotic R4 or even classical field theory on it since we do not have an exotic metric nor the global exotic smooth structures glued from local coordinate patches.
...
all we need is the knowledge about the existence of theories which have the quantum algebra of observables spanned on the factor III1 and the classical algebra spanned on a Poisson algebra
...
The 4-exotics approach is essentially 4-dimensional. The factor III1 von Neumann algebra is unique. When one wants to vary different exotic R4 's in this approach, the net of algebras suitably embedded into each other should be probably considered. ...".