

Predictions on Observations and Measurements of the Deflection of Light on the Occasion of 2015 Solar Eclipse According to the Theory of Reference Frames

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Abstract

This article results from a Ludwig Combrinck' s communication, in ResearchGate, on his experimental project of measuring the deflection of light, coming from the stellar field, in the sun gravitational field on the occasion of the solar eclipse in 2015 March. A brief exchange of ideas has convinced me on cogency of his project and I have decided to verify qualitative and quantitative deductions deriving from the application of the Theory of Reference Frames to the deflection of light in the sun gravitational field.

1. Introduction

in 1919, three years after the publication of A. Einstein's "Fundamentals of the Theory of General Relativity" in which the great scientist calculated the theoretical value (equal to $\Delta=1.75''$) of deflection of light that passed close to the sun, the astronomer A. Eddington performed the first measurement of the deflection of light passing close to the sun surface, caused by the sun gravitation, in two different points of the earth (Guinea and Brazil). The two measurements were $\Delta=1.61''$ (Guinea) and $\Delta=1.98''$ (Brazil); they are quite different from Einstein's theoretical value ($\Delta=1.75''$) with an error in the first case of 8% and in the second case of 13%. Also almost all subsequent measurements performed on the occasion of solar eclipses, including the measurement performed with new technologies in 1973 in Africa by the study group of the University of Texas, proved still a significant deviation from the theoretical value. Besides it is common knowledge that the measured value of deflection of light that passes close to the sun surface suffers from the "atmosphere effect" which certainly, for optical reasons, affects the measured value. In this historical context Ludwig Combrinck decided to perform the measurement of the light deflection in the sun gravitational field on the occasion of the solar eclipse in the March 2015, at notable distance from the sun surface in a range from 2 to 3 solar radii. The expected result is a hyperbolic trend. The result predicted in the Theory of Reference Frames is that the light deflection due to the gravitational field doesn't have a hyperbolic trend.

2. Ludwig Combrinck's Experimental Project for the measurement of the deflection of light.

In a comment in ResearchGate, L. Combrinck defined his project with these words:

" Firstly the difference in the 1919 to 1973 experiments and mine lies to a large extent in the technology that will be used, secondly in the observing strategy and thirdly in the analysis techniques. So, instead of using photographic plates and a plate measuring machine, I will be using a CMOS camera, image stacking and specialised software for the analysis. The observing strategy is different in that instead of having a wide field of view, covering a star field of several degrees including the eclipsed Sun and its corona, the captured star field will be from 2 radii to 3 radii, and a block of about 15 x 7 arcminutes. The idea is then to obtain hundreds or thousands of stars in the CMOS images. Previous experiments required bright stars ($M < 8$) to be on the photographic plate, less bright stars were obliterated by the bright coronal glare. My images will therefore not be of the Sun and Moon, with some stars around it from which to determine light bending, but will be from starfields in an annulus of 2R to 3R around the Sun. This will hopefully reduce coronal glare and allow many more fainter stars to be captured".

Then Combrinck presented a graph of light deflection deduced from the mathematical model demonstrated by Einstein in General Relativity^[1] (fig.1).

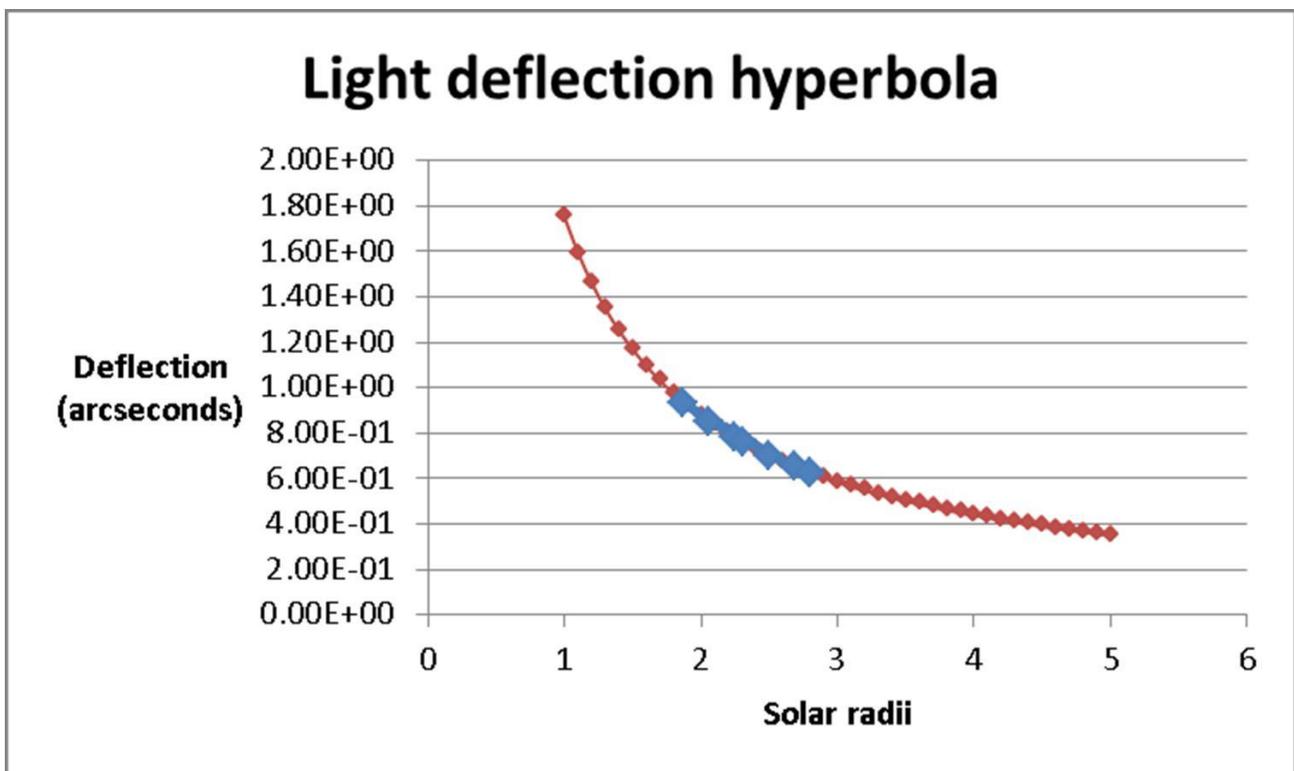


Fig.1 Light deflection in the range 1-5 solar radii (Combrinck's graph according to GR).

Obviously the graph has a hyperbolic trend. In fact in GR Einstein demonstrated the following mathematical expression for light deflection C

$$C = \frac{4GM}{c^2 r} \quad (1)$$

where $G=6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$ is the gravitational constant, c is the physical speed of light, M is mass of celestial body, r is the distance of light ray from the barycentre of mass M . In particular a ray of light passing close to the sun ($M=1.989 \times 10^{30} \text{ kg}$), at a distance of one solar radius $r=r_s = 1R$ ($r_s=697 \times 10^3 \text{ km}$) experiences a deflection of about $1.75''$ in concordance with the (1) and with the fig.1.

3. Light Deflection in the Gravitational Field according to the Theory of Reference Frames

In the Theory of Reference Frames^[2] (TR) the light deflection in the gravitational field of a celestial body presents two components^[3]:

- a. an optical component Δ_o due to the effect of double refraction of light produced by atmosphere of celestial body,
- b. a gravitational component Δ_g due just to the gravitational field generated by mass M .

3a. The optical deflection is caused by the atmosphere and therefore it is present generally only in the range $(1-2)R$. Considering the fig.2, it is possible to calculate the linear deflection h

$$h = (Q - q)\cos\alpha \approx 2R(\text{tg}\alpha - \text{tg}\alpha') \quad (2)$$

Because in the event of the sun $\text{tg}\alpha \approx R/D_{TS} = 4.65 \times 10^{-3}$, where $D_{TS} \approx 150 \times 10^6 \text{ km}$ is the average distance between the sun and the earth, and $R \approx 697 \times 10^3 \text{ km}$ is the sun radius, we have $\cos\alpha \approx 1$.

The angles α and α' are smallest and have the same order of magnitude. Besides they occasion both different measured values and the statistical error that is present in the different measurements performed with regard to the light deflection, because of variable and random physical conditions of the atmosphere of celestial body, as in the event of the sun. Consequently we can assume $m = \text{tg}\alpha - \text{tg}\alpha' \approx 10^{-3}$, and the angular deflection (or optical deflection) Δ_o is given by

$$\Delta_o \approx \frac{h}{\sqrt{(D_{TS} + R)^2 + R^2}} \approx \frac{h}{D_{TS}} = \frac{2R10^{-3}}{D_{TS}} \quad (3)$$

(1-2)R. Repeating calculations at a range of $1.5R$, we have a linear deflection h not much smaller in concordance with physical conditions of the sun atmosphere, and therefore we can assume with good approximation an angular deflection $\Delta_o \approx 1.80''$.

At a range of $2R$, certainly we have $\Delta_o = 0$.

We call this optical deflection of light as "Hannon effect" in remembrance of the independent scientist (Robert J. Hannon) who during our exchange of ideas via email encouraged me often to pursue my researches in the order of General Relativity.

3b. The gravitational deflection of light is due properly to the action of the gravitational field generated by celestial mass.

In order to calculate the gravitational deflection we consider light is a photon beam.

Photons are energy quanta ($E=hf$), with frequency in the light band, that travel at the physical speed of light c and have an equivalent electrodynamic mass given by $m_f = hf/c^2$.

We have demonstrated^[4] that a photon, with typical frequency $f=0.5 \times 10^{15}$ Hz, has an equivalent mass $m_f = 0.369 \times 10^{-35}$ kg and it suffers, on the surface of the earth at sea level, a gravitational force equal to $F_g = 3.63 \times 10^{-35}$ N that is a smallest value. On the surface of the sun, repeating the calculation for the same photon we find that the gravitational force is equal to $F_{gs} = 1.00 \times 10^{-33}$ N and therefore it is greater than the gravitational force on the surface of the earth.

Under the action of the gravitational field generated by the celestial body every photon suffers a deflection with respect to its straight path and that deflection can be calculated considering the law of the gravitational motion^{[2][3]} photon is subjected to (fig.3)

$$v \frac{dv}{dr} = - \frac{GM}{r^2} \quad (5)$$

In our case v is the physical speed c of both, light and photons, that can change with the refractive index of medium, for which we can write

$$c \frac{dc}{dr} = - \frac{GM}{r^2} \quad (6)$$

We know that $d \sin \alpha / \sin \alpha_n = dc / c_o$, where $\alpha_n = n \alpha_o$, $n=1, 2, 3 \dots$, $\alpha_o \approx R/D_{TS} = 4.65 \times 10^{-3}$ for the sun, c_o is the physical speed of light in a vacuum. Because $c \approx c_o$, we can write

$$\int d \sin \alpha = \sin \alpha_n \left[- \frac{GM}{c_o^2} \int \frac{dr}{r^2} \right] \quad (7)$$

Let's suppose that Δ_n' is the light deflection, represented by the first member of the (7), from infinite distance to the distance r_n which is the distance that we consider from the centre of celestial mass. Positions of stars and directions of light in fig.3 are symbolic. Because deflection angles are smallest, with good approximation we have

$$\Delta_n' = \frac{GM}{c_0^2 r_n} \sin \alpha_n \quad (8)$$

According to also the deflection from distance r_n to negative infinite distance, which for symmetry reasons is equal to Δ_n' , the complete deflection of light is given by

$$\Delta_n = 2\Delta_n' = \frac{2GM}{c_0^2 r_n} \sin \alpha_n \quad (9)$$

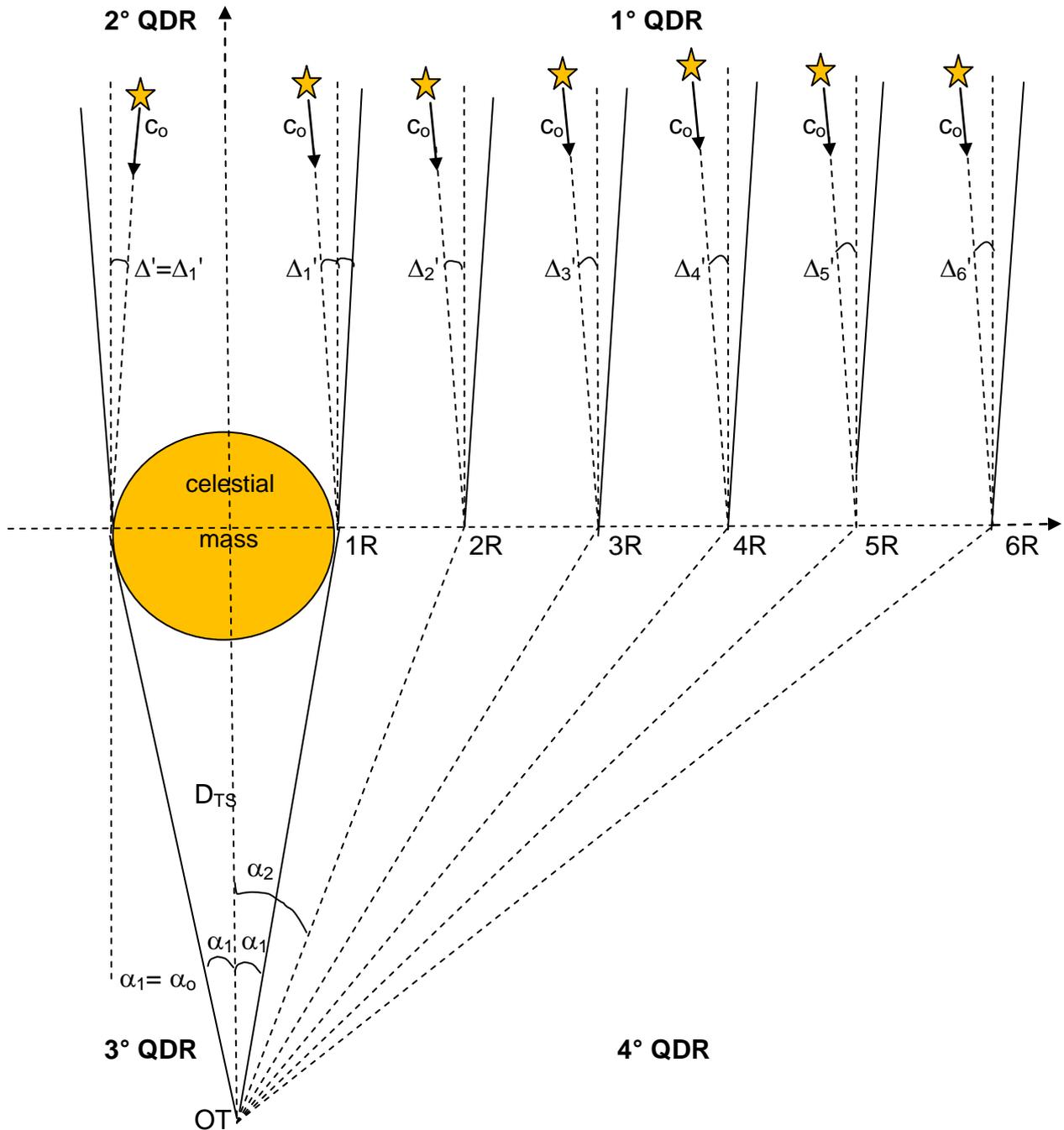


Fig.3 Representation of the light deflection due to the gravitational field for several multiples of the solar radius in the first quadrant (QDR) and in the second quadrant of the stellar field.

Assuming for the distance r_n distances equal to the number of radii of celestial body $r_n = nR$, where $n=1, 2, \dots$, we have

$$\Delta_n = 2\Delta_n' = \frac{2GM}{c_o^2 R} \frac{\sin\alpha_n}{n} \quad (10)$$

Because $\alpha_o \approx 4.65 \times 10^{-3}$ in the event of the sun, for $n=1, 2, \dots, 6$, is

$$\begin{aligned} \sin\alpha_1 &= 0.00465 \\ \sin\alpha_2 &= 0.00920 \\ \sin\alpha_3 &= 0.00140 \\ \sin\alpha_4 &= 0.01860 \\ \sin\alpha_5 &= 0.02330 \\ \sin\alpha_6 &= 0.02790 \end{aligned} \quad (11)$$

For the sun the values of gravitational deflection of light are

$$\begin{aligned} \Delta_1 &= 0.0197 \times 10^{-6} \text{ rad} = 0.0041'' \\ \Delta_2 &= 0.0197 \times 10^{-6} \text{ rad} = 0.0041'' \\ \Delta_3 &= 0.0197 \times 10^{-6} \text{ rad} = 0.0041'' \\ \Delta_4 &= 0.0197 \times 10^{-6} \text{ rad} = 0.0041'' \\ \Delta_5 &= 0,0197 \times 10^{-6} \text{ rad} = 0.0041'' \\ \Delta_6 &= 0,0197 \times 10^{-6} \text{ rad} = 0.0041'' \end{aligned} \quad (12)$$

The gravitational deflection is therefore the same for all distances from the sun, at least to about 6 solar radii. We call "Einstein second effect" the gravitational deflection of light in honour of the scientist who first understood its existence.

The graph of fig.4 represents both deflections of light, the optical deflection Δ_o and the gravitational deflection Δ_g , inside range from $1R$ to $6R$ where R is the solar radius. In the graph blue points indicate calculated values of the optical deflection (Hannon effect) and green points indicate calculated values of the gravitational deflection (Einstein second effect).

We can affirm that near the sun surface (about 1 solar radius) the optical deflection is far and away greater than the gravitational deflection for which values of deflection measured in all preceding experiments on the occasion of solar eclipses, as from 1919, are in actuality measurements of the optical deflection. At this distance ($1R$) the gravitational deflection is altogether negligible to about $2R$. Approximately at this distance ($2R$) the optical deflection suffers a sharp zeroing and the gravitational deflection becomes the only measurable existing effect. Calculations show that the gravitational deflection is smallest with respect to the optical deflection.

Calculations prove also that the gravitational deflection of $0.0041''$ is valid as far as about $20R$ (solar radii). For greater distances than $20R$ the gravitational deflection begins to decrease with a law defined by the term $\sin(n\alpha_o)/n$.

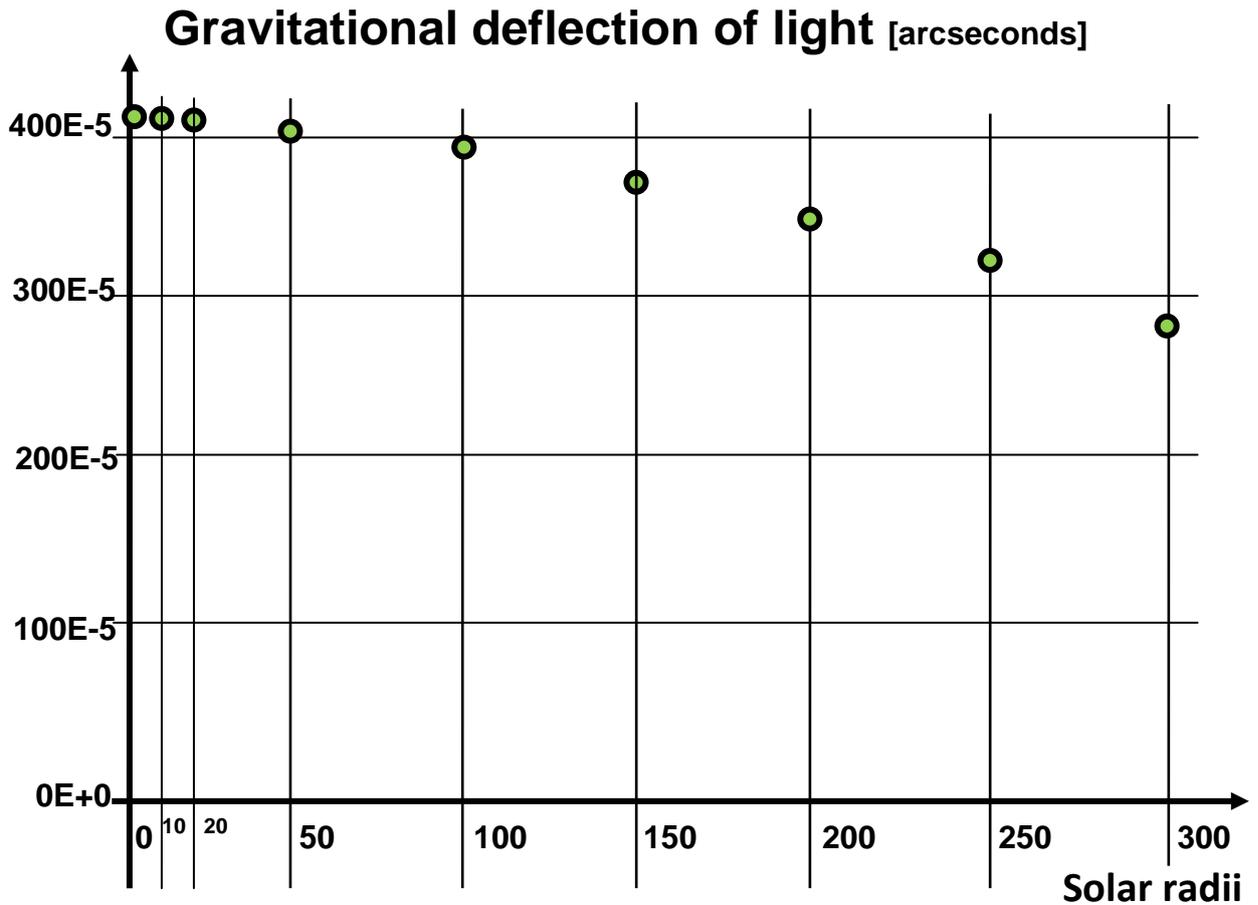


Fig.5 Gravitational deflection of light in the first quadrant of the stellar field with range to 300R.

We can consider also the stellar field with range to 1350R, in the first quadrant (fig.6). Calculations give the following values of deflection

$\Delta_{350} = 0.00249''$	$\Delta_{900} = -0,000838''$	
$\Delta_{400} = 0.00210''$	$\Delta_{950} = -0,000879''$	
$\Delta_{450} = 0,00168''$	$\Delta_{1000} = -0,000871''$	
$\Delta_{500} = 0.00129''$	$\Delta_{1050} = -0,000819''$	
$\Delta_{550} = 0,000873''$	$\Delta_{1100} = -0,000730''$	
$\Delta_{600} = 0,000501''$	$\Delta_{1150} = -0,000611''$	(14)
$\Delta_{650} = 0,000154''$	$\Delta_{1200} = -0,000471''$	
$\Delta_{700} = -0.000054''$	$\Delta_{1250} = -0,000317''$	
$\Delta_{750} = -0,000395''$	$\Delta_{1300} = -0,000158''$	
$\Delta_{800} = -0,000597''$	$\Delta_{1350} = -0,000037''$	
$\Delta_{850} = -0,000745''$	$\Delta_{1400} = 0,000141''$	

We observe the deflection becomes negative, always in consequence of the trigonometric term $\sin(n\alpha_0)/n$, for instance in the range about (675-1350)R. A negative deflection indicates an inversion of star position.

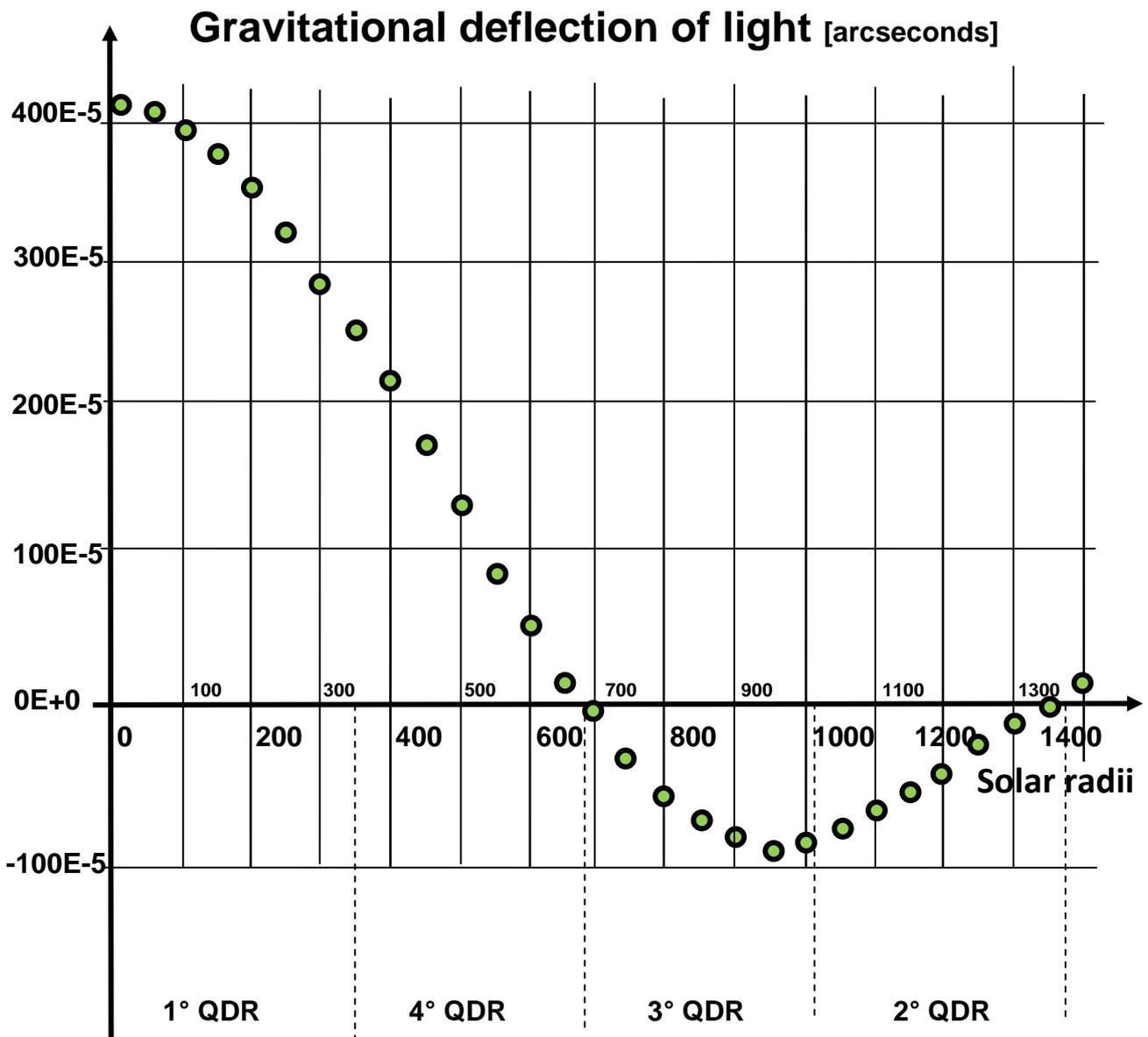


Fig.6 Gravitational deflection of light in the stellar field with range to 1350R.

Nature and laws of physics are wonderful. The universe that we observe from our reference of the earth is the real universe to about 15R, changed similarly in all directions only by a smallest deflection of light produced by the gravitational force of celestial mass. For greater distances than 15R we have calculated a smallest light deflection that follows a trigonometric trend ($\sin(n\alpha_0)/n$) and not a hyperbolic trend.

References

- [1] A. Einstein, Fundamentals of the Theory of General Relativity, Annales of Physics, 1916
- [2] D. Sasso, Physico-Mahematical Fundamentals of the Theory of Reference Frames, viXra.org, 2013, id: 1309.0009
- [3] D. Sasso, Not Linear Element, Cosmological Redshift and Deflection of Light in the Gravitational Field, viXra.org, 2011, id: 1111.0036
- [4] D.Sasso, On Different Meanings of Mass in Physical Systems, viXra.org, 2014, id: 1401.0047