The Schwarzschild metric adapted to radially symmetric anti-gravity

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A hypothesis of anti-gravity between matter and antimatter is presented that results in an alternative to the conventional Einstein field equations. Using the vacuum metric relationships identified by Schwarzschild in 1916, the radially symmetric vacuum anti-gravity metric is derived for the vacuum between a spherical core of either matter or anti-matter and an enclosing spherical shell of the other type of matter; anti-gravity prevents the shell from collapsing. Candidate black holes are hypothesized to consist of such a shell and core, and the gravitational behaviors of such a composite body are approximated. Observations of kinematics consistent with these behaviors would validate the existence of anti-gravity. (Version 3: Revised discussion before equation 16.) (Version 2: Revised 2nd paragraph section II; edits to equations 15a, 16, 18a.)

I. Introduction

Morrison considered the possibility that anti-gravity might exist between matter and anti-matter in 1958, and he was followed by others [1]. Investigations into the gravitational behaviors of antimatter are ongoing at CERN studying trapped cold anti-hydrogen [2, 3]. The simplest conceptualization of anti-gravity between matter and antimatter is characterized as each canceling the geometric metric distance changes of the other. The hypothesis of anti-gravity presented in section II is based upon this concept. The simplest static configuration that would demonstrate such a dynamic is a spherical shell of one enclosing a core of the other with the anti-gravity between the two supporting the spherical shell. The geometry of the vacuum space between the core and the shell can be described by a metric, and that metric is derived in section III. At some radius, between the shell and the core, the gravitational effects of respectively the shell and core on the vacuum metric distances are exactly mutually cancelling. Schwarzschild's original derivation of his widely known radially symmetric exterior vacuum metric is uniquely suited to deriving the metric of this radially symmetric vacuum anti-gravity metric [4], as I demonstrate herein. The existence of such a specific metric solution provides a reference example for the consideration of anti-gravity between matter and antimatter and whether it exists. In section IV, I discuss the detection of celestial bodies that might consist of a core and shell, with anti-gravity supporting the shell, and I expect those celestial bodies to be conventionally classified as candidate black holes. Candidate black holes are dark stars that are gravitationally entangled with visible stars and that are inferred to be black holes by conventional theory because their respective inferred masses are too great to be that of a dark neutron star [5].

II. The Hypothesis of Anti-Gravity

In hypothesizing that anti-gravity between matter and anti-matter exists, I am proposing an alternative geometric theory of gravity, a variant of conventional General Relativity theory. For both the proposed and conventional geometric theories, the gravitational evolutions of energy tensors are consistent with parallel transport of the tensors along the Levi-Civita metric connection of the evolving pseudo-Riemann geometry. Gravitational paths are geodesic paths of the geometry. The covariant derivatives of the tensors equal 0. All gravitational changes in the energy tensors can be attributed to the geometry.

The covariant derivatives of 0 for energy tensors of all forms of energy and the resulting covariant divergences of 0 of those aggregate energy tensors define energy and momentum conservation in non-Galilean coordinates for conventional theory and for the hypothesis of anti-gravity. I do not attempt to identify the energy of the gravitational field. I accept the consequences of the evolving space-time geometry, and I recognize and accept that energies and momentums do not exist as "extraneous" substances independent of that geometry of space-time [6]. For conventional theory, starting with Einstein's original proposal of the theory, there have been frequent attempts to identify a field energy and to use that energy to explain energy conservation within the gravitational dynamic [7].

For the hypothesis of anti-gravity, the only forms of energy that cause changes in the geometries of space-time are matter and antimatter, and the geometric effects of each cancel the geometric metric distance changes of the other. This contrasts with conventional theory in which all energies change the geometry of space-time, and there are no cancelations of gravitational effects. The geometry of the Universe without material energies, matter or antimatter, is consistent with a Minkowskian geometry: the geometry can be characterized by a Minkowskian metric. Particle/anti-particle creations and separations create gravitational effects. Particle/anti-particle annihilations eliminate the gravitational effects of the respective particle and anti-particle. Gravity with anti-gravity is a perfectly elastic dynamic: whatever is done by gravity to the geometry of space-time can be undone by anti-gravity.

For the hypothesis of anti-gravity, material energies and regions of space-time have gravitational polarities. Existing research does not consider the possibility of space-time polarities [1]. The gravitational polarities of material energies correspond to the distinction between matter and antimatter and to the effect respectively of matter or anti-matter on the geometry of a region. For a space-time region that matches the polarity of matter, the effects of matter on the geometry of the region are consistent with the conventional effects of positive masses, and the effects of antimatter on the geometry of the region are consistent with the conventional effects of negative masses. For a space-time region that matches the polarity of antimatter, the respective effects of matter and antimatter on the geometry of the region are opposite. The gravitational evolutions of all material energies, both matter and anti-matter, is consistent with a positive mass and a covariant derivative of 0; a negative mass is employed only to characterize the effect of a material energy on the geometry of space-time. (This restricted usage of negative mass and the use of space-time polarities distinguishes this hypothesis from the considerations of Bondi in 1957 [8].) Within the surface that forms a boundary between regions of space-time with opposite polarities, there are no gravitational changes in the metric distances of space-time; there, matter and antimatter each cancel the effects of the other.

Considering the boundaries between regions of opposite polarity throughout the Universe and independent of the regions that they separate, the boundaries are consistent with surfaces in a universal Minkowskian geometry; the boundaries maintain a globally flat Universe independent of the local expansions within the regions that they separate and independent of a critical energy density required by conventional theory. Cosmological expansion occurs only in the regions within the boundaries. Regions of like-polarity can accelerate towards one another and eventually merge, pushing through a region of opposite polarity that originally separated them. It is by such dynamics that anti-gravity segregates and respectively aggregates matter and antimatter.

The Einstein field equations describe that the Einstein tensor of the geometry of space-time is proportional to the energy tensor [9]. The covariant divergence of the Einstein tensor equals 0 [6]. For the hypothesis of anti-gravity and within a region of space-time of a particular polarity, the Einstein tensor is proportional to the material energy tensor with the sign of the mass consistent with the polarities of the region and the material energy: the effective mass is positive for matching polarities and negative for mismatching polarities. The geo-energy tensor is the material energy tensor with matter mass characterized as positive and antimatter mass characterized as negative. In the absence of transformations between material and non-material forms of energy, the geo-energy tensor has a covariant divergence of 0: the geo-energy tensor is the difference to two energy tensors that each has a covariant divergence of 0. Any transformation between material and non-material energies is characterized by quantum theory, and the effects of the transformation on the geo-energy tensor and the geometry of space-time are not smooth: the derivatives of the metric are not continuous. The Einstein tensor is undefined at the instant and location of material/non-material energy transformation. Individually, the respective covariant derivatives of the material and non-material energy tensors equal 0 as they approach and depart from the instant of energy transformation, and the aggregate material and non-material energy does not change at the instant of transformation. The Einstein and geo-energy tensors are well defined and proportional before and after the instant of transformation. The use of the geo-energy tensor is compatible with the architecture of the Einstein field equations; when and where the geometry is smooth, the proportionality between the Einstein and geo-energy tensors is maintained.

The polarities of space-time can be incorporated into the conventional derivation of the Einstein field equations using the Lagrangian of the Hilbert action [10, 11]. The action becomes:

$$S = \int((2k)^{-1}R+\mathcal{L}_m P)(-g)^{1/2}d^4x. \tag{1}$$

R is the Ricci Scalar; $\mathcal{L}_m$ is the geo-energy energy scalar; P is the polarity of space-time; and g is the determinant of the metric. The P is a wave function that in the limit of its series expansion equals 1 or -1 in the open intervals between the transition boundaries between space-time regions of opposite polarity. The variation of the integral of the action equals 0 for:

$$G_{uv} = k(T_{uv}P-2\mathcal{L}_m\delta P/\delta g^{uv}), \tag{2}$$
$$G_{uv} = R_{uv}-½g_{uv}R,$$
$$k = 8\pi Gc^{-4},$$
$$T_{uv} = -2(-g)^{-1/2}\delta((-g)^{1/2}\mathcal{L}_m)/\delta g^{uv} = g_{uv}\mathcal{L}_m-2\delta\mathcal{L}_m/\delta g^{uv}.$$

The first equation is the Einstein field equation generalized to accommodate anti-gravity using geo-energies and space-time polarities. The G_{uv} is the Einstein tensor; the R_{uv} is the Ricci tensor; the G is the gravitational constant; and the c is the coordinate speed of light in Special Relativity. The T_{uv} is the geo-energy tensor, not the conventional energy tensor. The polarities of the geo-energy and space-time are the same when the signs of P and the mass of T_{uv} are the same. The $-2\mathcal{L}_m\delta P/\delta g^{uv}$ term is new to the equations and results from the polarities of space-time.

As I demonstrate in the following section for the derived metric, the metric is continuous for some coordinates at the transition boundary between polarities and not continuous for others; for all coordinates, the metric is not smooth at the boundary; and non-zero Riemann and Weyl tensors are undefined at the boundary. As the metric approaches the boundary from either side, the Ricci and Einstein tensors must approach 0, consistent with a cancelation of all gravitational changes in metric distances at the boundary.

In order for a material particle to pass through the transition boundary, its effective geo-energy must be cancelled so that it can exist at the boundary where the Ricci tensor approaches 0 from both sides of the boundary; there can be no gravitational changes in the metric distances at the boundary. The $-2\mathscr{L}_m\delta P/\delta g^{uv}$ term must allow the net effective geo-energy tensor to become equal to 0 without the corresponding material geo-energy tensor becoming equal to 0; at the boundary, $T_{uv}P-2\mathscr{L}_m\delta P/\delta g^{uv}$ must equal 0, allowing the sign of the expression to flip at the boundary without a discontinuity. At the boundary, the geo-energy of the material energy is cancelled as it follows its time-like geodesic path. In a vacuum, with $\mathscr{L}_m$ equal to 0, the $2\mathscr{L}_m\delta P/\delta g^{uv}$ term equals 0. In order to maintain consistency with existing theory, sufficiently far from a transition boundary $2\mathscr{L}_m\delta P/\delta g^{uv}$ must equal 0. In contrast to material energies, non-material energies do not change the geometry of space-time; non-material energies pass through the boundary without complication.

For the hypothesis of anti-gravity and within a region of space-time of a particular single polarity, the observed behaviors are indistinguishable from conventional expectations. Most of the material energies in the Universe exist in regions of space-time for which the polarity of their geo-energies matches the polarities of space-time: this is the consequence of anti-gravity on the evolution of the Universe. Anti-gravity segregates and respectively aggregates matter and anti-matter, and the Universe expands with increasing respective aggregation. Approximately half of distant galaxies and galaxy clusters are composed predominantly of antimatter, in contradiction of conventional theory. Gravity with anti-gravity requires that the Universe contains equal quantities of matter and antimatter with anti-gravity segregating the two, solving the conventional contradiction that accepts both that the Universe is composed predominantly of matter and that particle and anti-particle creations and annihilations occur in matching pairs.

The hypothesis of anti-gravity is consistent with all empirical validations of conventional General Relativity. We have no ability to detect the geometric effects of either non-material energies or the meager amounts of antimatter that we can locally detect or accumulate. As Bondi explains, for geometric theories of gravity, Newton's 2nd Law that every action has and equal an opposite reaction does not apply [8]. Within a single polarity of space-time, he demonstrates that the conventional field equations predict that a negative mass repels a positive mass, while simultaneously the positive mass attracts the negative mass. In the laboratories at CERN scientists seek only to measure the matter attracting antimatter; and they are unable to measure whether the antimatter is repelling the matter [2, 3], which would indicate in conventional theory and the proposed theory that the gravitational effects of the anti-hydrogen were consistent with a negative mass. For the proposed theory and contrary to conventional theory, a star deflects the path of light, and the deflected light has no effect on the path of the star. The anti-neutrinos of supernova 1987A were of insufficient energy density to create space-time regions of opposite polarity; the geo-energies of the anti-neutrinos did not match the polarity of the space-time they traversed; they followed the same geodesics as the neutrinos; and both followed nearly the same geodesics as the photons from the supernova [12-15]. Most recently in 2013, Jentschura considered the gravitationally coupled Dirac equation for antimatter [16]. He considers the dynamics of the Dirac positron undergoing the geodesic accelerations of the Schwarzschild metric, and he concludes that its behavior due to gravity must be identical to that of the electron; however, he fails to allow the positron to flip the polarity of space-time.

The effective anti-gravity between bodies will be observed when a boundary between regions of space-time of opposite polarity is identified; it is across such boundaries that bodies accelerate away from each other and that space-time does not show evidence of cosmic expansion. For antimatter within a bubble of matching space-time gravitational polarity and observed from Earth in our galaxy, the antimatter would be de-accelerating as it enters our galaxy and accelerating as it exits our galaxy; and the antimatter must be of sufficient density to cancel all gravitational changes in metric distances at the boundary of the bubble. Existing literature does not consider such boundaries [1]. Such boundaries might exist amongst the galaxy cluster clumps and walls of our Universe [17]. While material bodies will accelerate away from the boundaries, the un-expanding space will diminish or eliminate any increasing separation between matter and antimatter galaxy clusters separated by such a boundary.

Antimatter, and more specifically the positron, was discovered by Anderson in 1932 [18], approximately 17 years after the proposal of General Relativity by Einstein in 1915 [9]. In the following year, 1933, Lemaître published "L'Univers en expansion," and the modern science of Cosmology was fully born without mention of antimatter [19, 20]. The discovery of antimatter was anticipated in 1928 by the Dirac equation that describes the relativistic quantum electron and for which the antimatter electron, the positron, has a negative rest energy and rest mass [21-23]. The electrical charges of matter and antimatter are opposite, and that has suggested that the gravitational interaction of matter-to-matter and antimatter-to-antimatter might be opposite that of matter-to-antimatter. However, a change in the essential relationships of General Relativity has never been proposed, and broadly considered, that would accommodate such dynamics and that would still be consistent with all empirical validations of conventional General Relativity. I am motivated to advance the hypothesis of anti-gravity because I find that such changes are possible: the revised theory provides simpler and more complete mechanisms to explain cosmological observations; validation of the theory appears to be possible; existing research does not exclude the possibility of the proposed theory; and ongoing research is unlikely to exclude it either. Existing research efforts have failed first to identify a paradigm for which antigravity could exist within a single geometry of space-time and then to identify tests that would validate that paradigm and, in doing so, invalidate the paradigm of conventional theory.

III. Derivation

Schwarzschild derived the exterior vacuum metric within months of Einstein's publication of his theory of General Relativity. In Einstein's original publication of that theory, he uses a simplified calculation of the Riemann and Ricci tensors for metrics with a determinant equal to $(-1)^{½}$, and although Schwarzschild starts with conventional radial coordinates, he adapts his derivation to coordinates that accommodate this restriction. He is able to solve incrementally the individual differential equations identified by Einstein and to identify the constants of integration that accumulate by that process. For the conventional Schwarzschild metric, the values of the constants of integration are consistent with a Minkowskian geometry at infinite radius and a discontinuity at the origin of the coordinates of the derivation. The discontinuity at that origin is at the Schwarzschild radius in reduced-circumference radius coordinates, and the static vacuum solution is not valid at that or any lesser reduced-circumference radii.

For the circumstances of anti-gravity, the metric is Minkowskian at a finite radius χ between the core and the shell, and the static vacuum metric is valid only for reduced-circumference radii greater than a limiting value. For radii less than χ, the gravitational effects of the core are consistent with its mass being characterized as positive, and for radii greater than χ, the polarity of space-time flips, and the gravitational effects of the core are consistent with its mass being characterized as negative.

Schwarzschild starts with a completely general radially symmetric metric using coordinates $\{t,r,\theta,\phi\}$ and of the form:

$$ds^2 = F\,dt^2-(G+H\,r^2)dr^2-G\,r^2(d\theta^2+\sin^2(\theta)d\phi^2). \tag{3}$$

In the absence of gravity and for suitable coordinates, H equals 0, and F and G equal 1. The existence of these coordinates within the derivation allows for characterizing the complete cancelation of gravitational effects at a specific radius with respect to anti-gravity.

Schwarzschild applies a coordinate transformation using $x_1=r^3/3$, $x_2=-\cos(\theta)$ and $x_3=\phi$ and re-organizes the metric into one with a determinant of $(-1)^{½}$:

$$ds^2 = f_4 dx_4{}^2-f_1 dx_1{}^2-f_2 dx_2{}^2/(1-x_2{}^2)-f_3 dx_3{}^2(1-x_2{}^2). \tag{4}$$

For the integration constants $\boldsymbol{\alpha}$, λ, $\boldsymbol{\sigma}$ and ρ and in order that the Ricci tensor equals 0, consistent with Einstein's expectations of a vacuum, Schwarzschild determined that the expressions for f_1, f_2, f_3 and f_4 must take the forms:

$$f_2(x_1) = \lambda(\rho+3x_1)^{2/3},$$
$$f_3(x_1) = f_2(x_1),$$
$$f_4(x_1) = \boldsymbol{\sigma}-\boldsymbol{\alpha}/(\lambda^2(\rho+3x_1)^{1/3}) \text{ and}$$
$$f_1(x_1) = 1/(f_2(x_1)^2\, f_4(x_1)). \quad (5)$$

For the boundary conditions of the conventional Schwarzschild metric, Schwarzschild chose values of the constants of $\lambda=1$, $\rho=\boldsymbol{\alpha}^3$, $\boldsymbol{\sigma}=1$ and, implicitly, $\boldsymbol{\alpha}=2Gmc^{-2}$ for c equal to 1. For the reduced-circumference radius R equal to $(\rho+3x_1)^{1/3}$ or, equivalently, $(\boldsymbol{\alpha}^3+r^3)^{1/3}$, the metric takes its conventional form:

$$ds^2 = (1-\boldsymbol{\alpha}/R)dt^2-(1-\boldsymbol{\alpha}/R)^{-1}dR^2-R^2(d\theta^2+\sin^2(\theta)d\phi^2. \quad (6)$$

For R less than $\boldsymbol{\alpha}$ and within the "Schwarzschild radius", r is negative and has no mapping to space-time without gravitational effects.

For the general case and in order that the Ricci tensor equals 0, $\boldsymbol{\sigma}$ must equal λ^{-3}. Otherwise, the following Ricci tensor components are non-zero:

$$R_{22} = (-1+\lambda^3\boldsymbol{\sigma})/(-1+x_2^2) \text{ and}$$
$$R_{33} = (-1+\lambda^3\boldsymbol{\sigma})/(-1+x_2^2). \quad (7)$$

Performing this substitution and converting back to radial coordinates $\{t,r,\theta,\phi\}$ the functions f_1, f_2, f_3 and f_4 become:

$$f_2(r) = \lambda(r^3+\rho)^{2/3},$$
$$f_3(r) = f_2(r),$$
$$f_4(r) = (1-\boldsymbol{\alpha}\lambda/(r^3+\rho)^{1/3})/\lambda^3 \text{ and}$$
$$f_1(r) = 1/(f_2(r)^2\, f_4(r)). \quad (8)$$

The metric in those coordinates is:

$$ds^2 = f_4(r)c^2dt^2-f_1(r)r^4dr^2-f_2(r)(d\theta^2+\sin^2(\theta)d\phi^2). \quad (9)$$

I have incorporated a constant factor of c^2 into the g_{tt} component to allow for scaling of the time coordinate.

In order that the metric is Minkowskian at the radius r equal to χ, λ must equal $\chi^2/(\chi^3+\rho)^{2/3}$, resulting in a value of χ^2 for $f_2(\chi)$. I introduce a simplifying substitution that ρ equals $\epsilon\chi^3$, and, then, λ equals $(1+\epsilon)^{-2/3}$. In order that $f_4(\chi)$ has a value of 1, consistent with a Minkowskian geometry, α must equal $\epsilon(2+\epsilon)\chi/(1+\epsilon)$. The functions f_1, f_2, f_3 and f_4 become:

$$f_2(r) = ((r^3+\epsilon\chi^3)/(1+\epsilon))^{2/3}, \tag{10}$$
$$f_3(r) = f_2(r),$$
$$f_4(r) = (1+\epsilon)^2-\epsilon(1+\epsilon)^{1/3}(2+\epsilon)\chi/(r^3+\epsilon\chi^3)^{1/3} \text{ and}$$
$$f_1(r) = 1/(f_2(r)^2\, f_4(r)).$$

At the radius χ, $f_2(\chi)$ and $f_3(\chi)$ equal χ^2, and $f_4(\chi)$ and $f_1(\chi)r^4$ equal 1, consistent with a Minkowskian geometry at that radius.

For reduced-circumference radius coordinates, R equals $f_2(r)^{½}$ or $((r^3+\epsilon\chi^3)/(1+\epsilon))^{1/3}$. Substituting $\lambda=\omega^2$ and accommodating the different values of dr and dR, the metric in such coordinates becomes:

$$ds^2 = f_4(R)c^2dt^2-f_1(R)(\omega^2(R/\omega)^{-4})^{-1}dR^2-f_2(R)(d\theta^2+\sin^2(\theta)d\phi^2) \tag{11}$$
or
$$ds^2 = f_4(R)c^2dt^2-f_1(R)(1+\epsilon)^2R^{-4}dR^2-f_2(R)(d\theta^2+\sin^2(\theta)d\phi^2),$$

and the functions f_1, f_2, f_3 and f_4 become:

$$f_2(R) = R^2, \tag{12}$$
$$f_3(R) = f_2(R),$$
$$f_4(R) = 1+\epsilon(2+\epsilon)(1-\chi/R) \text{ and}$$
$$f_1(R) = 1/(f_2(R)^2\, f_4(R)).$$

At radius χ, all of the metric components have the same value as when using the radial coordinate r except for g_{RR}. At that radius and for positive ϵ, g_{RR} is $(1+\epsilon)^2$ larger than g_{rr}, reflecting the differences in dR^2 and dr^2. It is only in the coordinates of the derivation, r and not R, that the metric is Minkowskian at radius χ. Valid values of R for the static vacuum solution are greater than the value of R for r equal to 0 or:

$$R > \chi(\epsilon/(1+\epsilon))^{1/3}. \tag{13}$$

Having resolved and simplified the formulation of the metric, it becomes necessary to relate the constant ϵ to a physical quantity that results in physical behaviors that are consistent with expectations and that might be validated. Substituting:

$$\epsilon \approx Gm_0/(c^2\chi) \text{ in } 1^{st} \text{ order approximation and} \tag{14a}$$
$$\epsilon = (1+2Gm_0/(c^2\chi))^{½}-1,$$

and the function $f_4(R)$ becomes:

$$f_4(R) = 1-2Gm_0/(c^2R)+2Gm_0/(c^2\chi). \tag{15a}$$

For c equal to 1, this $f_4(R)$ is the g_{tt} component of the metric. The G is the gravitational constant; the m_0 is the mass of the core; and the c is the coordinate speed of light in Special Relativity. The second term is the conventional term of the conventional Schwarzschild metric. The third term is a new term, and it reflects the gravitational effects of the spherical shell. As is the case for a Gaussian shell in electrostatic theory, the shell would not be expected to generate forces within its interior; the gravitational effects of the spherical shell in this metric include a change in the rate of time within its interior and a corresponding change in the energy of particles within the interior. (Consideration of the Riemann and Weyl tensors below will motivate a slightly different choice of value for ϵ that maintains unchanged first order approximations of $f_4(R)$.)

For static metrics with time-space cross-terms of 0, the energy of a test particle following time-like geodesics is proportional to $g_{tt}^{-½}$. The velocity of a test particle, characterized as a fraction of the varying coordinate speed of light, v_f, can be calculated by $\pm(1-(ds/g_{tt}^{½}dt)^2)^{½}$, and $(1-v_f^2)^{-½}$ equals $|g_{tt}^{½}dt/ds|$. For c equal to the coordinate speed of light of Special Relativity, m equal to the mass of a particle and relativistic momentum scalar p equal to $(mc)v_f(1-(v_f)^2)^{-½}$, $mc^2|g_{tt}^{½}dt/ds|$ equals $((mc^2)^2+(pc)^2)^{½}$, the total energy of Special Relativity. I use the expression $mc^2|g_{tt}^{½}dt/ds|$ to consider the energy variations of a test particle following a geodesic path, solving the geodesic equations for dt/ds. The expression describes that the energy of a particle varies with the ratio of the contribution of coordinate time to path proper time to the actual path proper time. For static metrics with time-space cross-terms of 0 and solving the geodesic equation for d^2t/ds^2, dt/ds is proportional to g_{tt}^{-1}, and the energy of the test particle is proportional to $mc^2g_{tt}^{-½}$. The constant of proportionality is $g_{tt}(\boldsymbol{\tau})^{½}(1-v_{\boldsymbol{\tau}}^2)^{-½}$ for $g_{tt}(\boldsymbol{\tau})$ equal to g_{tt} at a point $\boldsymbol{\tau}$ and $v_{\boldsymbol{\tau}}$ equal to the velocity of the particle as a fraction of the coordinate speed of light at that point. (I think of $mc^2g_{tt}(\boldsymbol{\tau})^{½}(1-v_{\boldsymbol{\tau}}^2)^{-½}$ as the effective underlying energy of a particle.) For the total expressed energy of $mc^2g_{tt}^{-½}g_{tt}(\boldsymbol{\tau})^{½}(1-v_{\boldsymbol{\tau}}^2)^{-½}$, any constant factor within g_{tt}, such as c^2, is canceled by the product of $g_{tt}^{-½}$ and $g_{tt}(\boldsymbol{\tau})^{½}$.

For c equal to 1 and in the first order approximations:

$$g_{tt}(R)^{-½} \approx 1+Gm_0/(c^2R)-Gm_0/(c^2\chi) \text{ and} \tag{16}$$
$$g_{tt}(\chi)^{-½} = 1.$$

The force on a test particle of either matter or antimatter, considered as a scalar quantity, is approximately $-Gm_0(m\, g_{tt}(\boldsymbol{\tau})^{½}(1-v_{\boldsymbol{\tau}}^2)^{-½})/R^2$ for m equal the positive mass of the test particle. For radii less than χ, the signs of ϵ and m_0 are positive, consistent with conventional theory. In order for anti-gravity to support the shell, the sign of m_0 in the metric for R greater than χ must be negative, resulting in a repulsive force.

I characterize the vacuum regions separated by the radius R equal to χ as having opposite polarities. If the core is of antimatter, the polarity of space-time for radii smaller than χ matches that of antimatter, and the effect of matter on the geometry of that region is to diminish gravitational effects. The polarity of space-time for radii larger than χ would match that of matter, and the effect of antimatter on the geometry of that region would be to diminish gravitational effects. For this static metric, the polarities of space-time are consistent with whichever polarity results in slower time and shorter metric path lengths for stationary paths.

If ϵ flips its sign at R equal to χ and ϵ is non-zero, the value of ϵ is undefined at that radius, and any quantity that is truly dependent on the value of ϵ at that radius is also undefined. The value of $g_{RR}(\chi)$, equal to $(1+\epsilon)^2$, is undefined, while the value of $g_{rr}(\chi)$, equal to 1, is defined. For coordinates using R, the metric is not continuous at R equal to χ. For coordinates using r, the metric is continuous at r equal to χ. For both coordinates, the metrics are not smooth because their derivatives are not continuous at R or r equal to χ.

For a vacuum, the Riemann tensor equals the Weyl tensor. For the metric, the sign of all the components of the Riemann and Weyl tensor flip if the sign of ϵ and m_0 flip. In reduced-circumference coordinates, the non-zero independent components of the co-variant Riemann tensor at χ are:

$$\begin{aligned}
&1212\text{:}\;\; \epsilon\, c^2(2+\epsilon)/\chi^2, \\
&1313\text{:}\;\; -\epsilon\, c^2(2+\epsilon)/(2(1+\epsilon)^2), \\
&1414\text{:}\;\; -\epsilon\, c^2(2+\epsilon)\sin^2(\theta)/(2(1+\epsilon)^2), \\
&2323\text{:}\;\; \epsilon(2+\epsilon)/2, \\
&2424\text{:}\;\; \epsilon(2+\epsilon)\sin^2(\theta)/2, \\
&3434\text{:}\;\; -\epsilon\, c^2(2+\epsilon)\chi^2\sin^2(\theta)/(1+\epsilon)^2.
\end{aligned} \tag{17}$$

The Riemann and Weyl tensors in reduced-circumference coordinates depend upon ϵ at radius χ, and for the circumstances of anti-gravity, these tensors are undefined at that radius χ.

In the radial coordinates of the derivation, r instead of R, the non-zero independent components of the co-variant Riemann tensor at radius χ are:

$$\begin{aligned}
&1212\text{:}\;\; c^2(1-(1+\epsilon)^{-2})/\chi^2, \\
&1313\text{:}\;\; -c^2(1-(1+\epsilon)^{-2})/2, \\
&1414\text{:}\;\; -c^2(1-(1+\epsilon)^{-2})\sin^2(\theta)/2, \\
&2323\text{:}\;\; (1-(1+\epsilon)^{-2})/2,
\end{aligned} \tag{18}$$

2424: $(1-(1+\epsilon)^{-2})\sin^2(\theta)/2$,
3434: $-\chi^2(1-(1+\epsilon)^{-2})\sin^2(\theta)$.

The only and common dependency of the tensor components on ϵ at radius χ is the factor $(1-(1+\epsilon)^{-2})$. For $r < \chi$, $\epsilon > 0$, and the factor is positive; and for $r > \chi$, $\epsilon < 0$, and the factor is negative. Approaching the radius χ from opposite sides and in these coordinates, the component values of the Riemann tensors approach unequal values that all respectively differ by a common constant factor less than 0; the Riemann and Weyl tensors in these coordinates and for anti-gravity are also undefined at the boundary at r equal to χ.

If ϵ equals $(1-2Gm_0/(c^2\chi))^{-½}-1$, instead of the $(1+2Gm_0/(c^2\chi))^{½}-1$ of equation (14a), then $(1-(1+\epsilon)^{-2})$ equals simply $2Gm_0/(c^2\chi)$. The first order approximations of these two possible expressions for ϵ are equal, and either might be the correct physical value. If this alternate value of ϵ is the correct physical value, then components of the Riemann and Weyl tensors approach values from opposite sides of the boundary that are the same but for opposite signs. I wonder whether and suspect that this is the general circumstance for metrics that are continuous across a stationary boundary that separates space-time regions of opposite gravitational polarities. Specifically for a vacuum, this would be the only tensor constraint that must be satisfied by the location of the boundary: the adapted Einstein field equations of section II do not explicitly constrain the location of the boundary in a vacuum. If this circumstance reflects a physical constraint, as I conjecture, then the constraint determines the exact value of ϵ for the adapted Schwarzschild metric:

$$\epsilon = (1-2Gm_0/(c^2\chi))^{-½}-1, \tag{18a}$$
$$f_4(R) = 1-(2Gm_0/(c^2R)-2Gm_0/(c^2\chi))/(1+2Gm_0/(c^2\chi)), \text{ and}$$
$$f_4(R) \approx 1-2Gm_0/(c^2R)+2Gm_0/(c^2\chi) \text{ in } 1^{st} \text{ order approximations.}$$

For the Newtonian approximation of anti-gravity, the gravitational potential energies generated by respectively matter and antimatter cancel each other. The net gravitational potential energy is the negative of the absolute value of the difference in the potential energies attributable respectively to matter and antimatter. The gravitational potential energy generated by the core at radius χ equals the gravitational potential energy generated throughout the interior of the shell for the radius of the shell:

$$Gm_0/\chi = Gm_x(R_x+h^2/R_x)^{-1} \tag{19}$$
$$R_x = \chi(m_x/m_0)-h^2/R_x$$

The m_x is the mass of the shell; R_x is the radius to the center of the shell with uniform mass distribution; and 2h is the thickness of the shell. The characterization of the potential energy generated by the shell is adapted from

Singh [24]. For h sufficiently smaller than R_x, the ratio of R_x to χ is approximately equal to the ratio of m_x to m_0. The net gravitational potential energy is approximately:

$$-|(Gm_0/R-Gm_x/R_x)| \text{ or} \tag{20}$$
$$-|(Gm_0/R-Gm_0/\chi)|.$$

The latter approximation matches the derived approximation.

For spherically symmetric anti-gravity as characterized by geometry or the Newtonian approximation, there are no actual net repulsive forces. Apparent repulsion arises from un-cancelled attractive forces; the core cancels the attractive forces from the other side of the shell that would otherwise have pulled the material of the shell to the center of the sphere, and all that remains are the locally attractive forces of the shell itself that are consistent with repulsion from the core.

IV. Detection

If the space-time of the observable Universe is only of the gravitational polarity of matter, it may be impossible to establish whether the hypothesis of anti-gravity or conventional theory is the more valid description of the physical realm. For the respective predictions of either theory, I can imagine no laboratory experiments that would validate the gravitational effects on the geometry of space-time predicted of non-material energies or of anti-matter energies, particularly for the quantities of low-energy anti-matter that can be detected or trapped. Within a single polarity of space-time, it is only those effects that distinguish conventional theory from the hypothesis of anti-gravity. The unambiguous validation of the hypothesis of anti-gravity and invalidation of conventional theory requires identifying where there exists a region of space-time of a gravitational polarity of antimatter.

One process that might validate the hypothesis of anti-gravity is to make conjectures as to where boundaries between space-time regions of opposite polarities might exist, and, then, to make and analyze observations that might support or reject those conjectures. How near to our galaxy might an antimatter galaxy exist on the far side of such a boundary? The location of such boundaries might be anywhere between two proximate celestial structures that do not exhibit classical gravitational dynamics between themselves: they do not accelerate towards one another, and they do not orbit each other. If such bodies or structures on opposite sides of a boundary move away from each other, the newly intervening space-time un-expands and diminishes the observable effects of that movement. Electromagnetic radiation crossing the un-expanding space-time near the boundary is blue-shifted by the geometric changes, in contrast to the redshifts of expanding space-time.

For an anti-matter particle generated in a collider by particle/anti-particle creation and that has sufficient energy density and momentum, it should flip the polarity of the space-time in which it exists and experience a reduction in momentum equivalent to climbing out of the “gravity well” of our region of gravitational polarity and, then, climbing back into the “anti-gravity well” of our region of space-time polarity. The anti-matter particle becomes encapsulated within its own space-time region of matching gravitational polarity. The change in momentum starts as the particle and the anti-particle separate, as the particle and the anti-particle no longer respectively cancel the gravitational effects of the other. The anti-particle itself is responsible for the changes in space-time geometry that change its momentum and, then, flip the gravitational polarity of space-time. If the original energy of the particle is insufficient for the energy reduction in full, the gravitational polarity of space-time does not flip.

The top anti-quark, the most massive elementary fermion, is a candidate to flip the gravitational polarity of our local space-time; however, for the top anti-quark, the ratio of its mass to the Planck length is approximately 8 orders of magnitude smaller than the ratio of Earth mass to Earth radius and approximately 12 orders of magnitude smaller than the ratio of estimated Milky Way mass to the radius of the Sun’s galactic orbit. In the Newtonian approximation of gravitational effects, the radius of the top anti-quark must be of radius much smaller than the Planck length to cancel our local gravitational changes in metric distances due to matter; however, both conventional General Relativity and the hypothesis of anti-gravity depart significantly from Newtonian relationships with respect to radii sufficiently near sufficiently dense masses, as evidenced by the calculation and usage of reduced-circumference radii in the previous section for the conventional and adapted Schwarzschild metrics. The anti-particle and the kinetic energy of that anti-particle that would flip the gravitational polarity of its local space-time in our proximity are undetermined.

I make the conjecture of a more pervasive manifestation of anti-gravity. I hypothesize that black holes do not form, from either matter or antimatter stars. For a matter star, a composite configuration forms consisting of a matter shell, an anti-matter core and a vacuum between the core and shell; for an antimatter star, the roles of matter and antimatter are swapped. For the static approximation, the metric derived in section III describes the geometry of the vacuum. The gravitational behaviors of such a celestial body would be observably different from a conventional body, and such observations would emphatically validate the hypothesis of anti-gravity.

The possibility of the validity of the conjecture becomes apparent by considering the energy densities of a collapsing star and by distinguishing rest energies from the energies that arise from gravity or, equivalently, the geometries of space-time. The latter energies are conventionally characterized as increasing the material energies of the star as momentums and pressures and as contributing to increasing energy densities in the Einstein field equations. The collapsing stellar materials transfer the energies due to gravity to the center of the star and concentrate them there. For the anti-gravity hypothesis, if those energies that arise from gravity are expressed as photons, unconfined gluons, mesons, Z bosons or W bosons, those energies express no net geo-energies; they do not contribute to gravitational collapse; and, significantly, they can result in particle/anti-particle production. Fundamentally and in contrast to conventional theory, energies that arise from gravity become both positive and negative geo-energies; in order for gravitational stellar collapse to stop, it is necessary only for there to be sufficient generation of the two types of geo-energies and for the positive and negative geo-energies to organize into configurations that stop the collapse. For conventional theory, there is no such possibility, and theorists have reluctantly accepted the formation of gravitational singularities, black holes, as nearly settled science.

For a matter star experiencing gravitational collapse, anti-matter production of sufficient quantity and energy density would provide the necessary material for an anti-matter core to form in a region of space-time of matching gravitational polarity; that formation would stop gravitational collapse without violating conservation of energy; and this is why the anti-gravity alternative to black holes is possible. The energy configuration of a matter shell and an anti-matter core is at a lower energy state than the conventional alternative of a black hole, and that makes the anti-gravity alternative a preferred end state.

I observe that if the anti-matter particles generated at the center of a collapsing matter star are sufficiently dense and energetic, their momentum will decrease, and they will flip the gravitational polarity of space-time; in comparison to its energy at the moment of particle/anti-particle creation, the anti-particle will have lost kinetic energy, as I describe above for the circumstances of a collider. Assuming sufficient mobility and lifetimes for particles and anti-particles at the stellar center, for those created anti-matter particles, with that loss of kinetic energy and in contrast to their sibling matter particles, the anti-matter particles will stay near the center of the star. At the center, the anti-particles that have flipped the polarity of space-time will gravitationally attract one another and repel particles or anti-particles that exist in space-time of opposite gravitational polarity. The existence of the antimatter at the center of the collapse decreases the energy density required there for other anti-particles to flip the polarity of space-time to a matching polarity. An antimatter core will form.

I hypothesize that stars, currently classified as candidate black holes, including galactic centers, are actually these composite configurations. The formation of such a stable composite body of matter and antimatter would appear to be a black hole: the existing limits on redshifts for celestial bodies are predicated on the densest material sinking away from the surface [25], and the anti-gravity of the core makes such material in the shell fall towards the exterior surface. In the absence of being able to exclude the possibility of the conjecture and with the incentive of validating an alternative to the formation of gravitational singularities, I advocate attempting to validate that such composite bodies of matter and anti-matter form during gravitational collapse before the details of that possible formation are more fully understood. The most compelling validation of the hypothesis of anti-gravity would be observations consistent with anti-gravity preventing the formation of black holes.

Consider a composite body of matter and antimatter orbiting a conventional non-composite body: the conventional body and the shell of the composite body are either both composed of matter or both composed of antimatter. For the velocity of the composite star, there is a conventional geodesic path; however, the composite star does not follow that geodesic path for an orbit of the conventional star. The core within its region of space-time with opposite polarity, independent of the shell, would follow a different geodesic that accelerates away from the conventional star. The shell and core alter the geometries in which they each exist such that their geodesic paths are different from a conventional orbit of the conventional star, and it is necessary to use Newtonian approximations of anti-gravity to approximate the dynamics. Using the Newtonian approximation and using m_0 for the mass of the conventional star, m_c for the mass of the core and m_s for the mass of the shell, all positive masses, the aggregate force on the composite star is $-Gm_0(m_s-m_c)/r^2$, and the inertial mass of the composite star is m_s+m_c. For the effective gravitational mass m_g equal to m_s-m_c and the effective inertial mass m_i equal to m_s+m_c, a circular orbit for the composite body requires the gravitational force to equal the negative of the centrifugal force:

$$-Gm_0m_g/r^2 = -m_iV(r)^2/r \text{ and} \qquad (21)$$
$$V(r) = (Gm_0/r)^{1/2}(m_g/m_i)^{1/2}.$$

For the circular approximation, the orbital velocity of a composite star is reduced by a factor of $(m_g/m_i)^{1/2}$. Using Bondi's characterizations of masses [8], the effective active and passive gravitational masses are equal, and both are less than the apparent inertial mass.

For a conventional star, the galactic orbital velocity of a star is approximated by $V(r)=(G\ M(r)/r)^{1/2}$, for $V(r)$ equal to the circular galactic orbital velocities at radius r, G equal to the gravitational constant and $M(r)$ equal to the effective active gravitational mass within the radius r [26]. For the proposed theory, the galactic circular orbits for a composite configuration of matter and antimatter equal $(G(m_g/m_i)M(r)/r)^{½}$, slower than the orbit of conventional visible stars. Conventional visible stars would overtake such presumably dark composite bodies, and the visible star(s) and the dark composite body could become gravitationally entangled. The aggregate average orbital velocity of such entangled stars would be:

$$V(r) = (G(m_v+m_g)(m_v+m_i)^{-1}M(r)/r)^{½} \tag{22}$$

for m_v equal to the mass of the entangled conventional visible stars. In 1962, Rubin *et al.* examined the kinematics of 888 early type stars in our Galaxy beyond the sun and reported that "the rotation curve is approximately flat" [26, 27]. Slower orbits should be readily apparent. If the observed galactic orbital velocities of entangled candidate black holes were consistently slower than those of un-entangled conventional stars at approximately the same radii, that observation would validate the possibility that candidate black holes were composite bodies of matter and antimatter, confirming the hypothesis of anti-gravity.

Alternatively, the gravitational behaviors of the entangled configurations of conventional stars and candidate black holes can be considered. For conventional binary stars consisting of a visible star and a dark star, the following approximation holds [5, 28, 29]:

$$V_v^3P/2\pi G = m_d\sin^3\theta/(m_v/m_d+1)^2 \tag{23}$$

The left-side value is calculated from observed measurements of the visible star. The V_v is the velocity component of the visible star inferred by the observed relativistic red shift as the star travels towards and away from the Earth. The P is the period of the orbits. The G is the gravitational constant. The m_d is the mass of the dark star. The θ is the angle of the orbital plane, which can be approximated by curve fitting. The m_v is the mass of the visible star. The m_v is inferred from the type of the visible star. The equation can be solved for m_d.

For a dark composite star with unequal effective active gravitational and apparent inertial masses, the second expression becomes:

$$V_v^3P/2\pi G = m_g\sin^3\theta/(m_v/m_i+1)^2. \tag{24}$$

The m_g is the effective gravitational mass of the dark star. The m_i is the apparent inertial mass of the dark star. In the limit of m_i much greater than m_v, only the m_g matters, and the visible star rotates around the dark star at velocities and radii consistent with only m_g. In such circumstances, the m_g may be less than the m_v, and high velocities and small periods for the visible star may be the consequence of small orbital radii. It is not possible to solve for both m_g and m_i with this single equation. In order to identify that m_g does not equal m_i, it is necessary to infer the size of the orbit of the dark star by some independent observation, for instance by some indirect observation of its velocity. If that is not possible, it will be necessary to consider more than 2 gravitationally coupled stars. For example, in a configuration of 3 stars for which one of the stars has significantly greater apparent inertial mass than effective active gravitational mass, that star will have a dampened gravitational acceleration for the combined gravitational attraction of the other two stars as the 2 periodically come together.

If the core within a composite body escapes its shell of opposite-type matter, the liberated core will accelerate out of the space-time region of opposite gravitational polarity. Observation of such bodies accelerating out of galaxies would validate the hypothesis of anti-gravity. If the hypervelocity stars that were observed to be exiting our galaxy by Brown *et al* [30, 31] were accelerating as they exit, they would likely be such escaped cores of composite stars. The observation of such acceleration would validate the hypothesis of anti-gravity.

V. Conclusion

John Wheeler was first to use the term "black hole" in publication, in 1968, and, in his autobiography, he recounts how physicists in 1939, at the dawn of the atomic age, readily verified the "postulate of fission" devised by Otto Frisch and Lise Meitner because atomic fission had an easily detected energy signature, and many physicists verified that energy signature within a day of receiving reports of the postulate [32]. Prior to those directed observations, that signature had never been both observed and reported. Signatures, or indicators, of a Universe with anti-gravity and without black holes are observed candidate black holes with unequal apparent inertial mass and effective gravitational mass, and there are currently no reports of whether that signature does or does not exist.

The presentation of the derivation of the radially symmetric vacuum anti-gravity metric solution establishes that geometric gravity is compatible with anti-gravity. The mathematical relationships of the metric are derived and not speculative; it is only the applicability of the metric to the physical realm that is subject to the confirmation of the anti-gravity hypothesis.

The presentation of the anti-gravity hypothesis establishes that the hypothesis is credible and possibly a better characterization of the physical realm than conventional General Relativity. All empirical validations of General Relativity across the 98 years since its proposal are also consistent with the hypothesis of anti-gravity. I distinguish validation from inference. For anti-gravity and in contrast to conventional theory, a different geometric structure for the Universe, one that includes boundaries between space-time regions of opposite gravitational polarities, and a timeline for its evolution will be inferred from existing and future observations.

My hope is that my presentation on the detection of anti-gravity will motivate astronomers to look for candidate black holes with a greater apparent inertial mass than effective gravitational mass or for accelerating escaped composite star cores and that their observations would validate the hypothesis of anti-gravity. If the hypothesis of anti-gravity is correct, the long delayed unification of gravity with the other physical dynamics will likely happen only in the context of gravity with anti-gravity.

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