

# Relations Between Significant Masses in the Micro and Macro World Based on the Boscovich's Theory

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**Abstract:** Starting from "A Theory of Natural Philosophy" by Roger Boscovich (1711–1787), we are showing that some significant masses in the micro and macro world,  $m_i$ , (the Planck mass, fundamental mass, background mass, hypothetical mass quantum  $m_q$ , as well as the masses of vacuum, infinite universe and our universe) can be calculated by a simple equation  $m_i = m_q N^{1/i}$ , where  $N$  is a large non-dimensional number ( $6.3870772 \cdot 10^{+121}$ ) and  $i = -1, 0, 1, 2, 3, 4, \infty$ . Knowing these significant masses is useful for the calculation of other real masses. We are also showing the examples of calculating the masses of proton, electron and up quark.

## Introduction

The goal of this work is to determine, by using a simple formula, the relations between masses of the non-cohesion limits of the zero order and determine their values, then to show that knowing them is useful for determining the values of real masses. We will determine the relations between significant masses:

- Planck mass,  $m_{pl} = 2.176\,51 \cdot 10^{-8}$  kg [4];
- Fundamental mass,  $m_f = 1.088622 \cdot 10^{-28}$  kg [5], [6];
- Background mass,  $m_b$ , for the first time mentioned here
- Hypothetical mass quantum,  $m_q = 2.7233883 \cdot 10^{-69}$  kg [8]
- Vacuum mass,  $m_v = 0$
- Mass of the infinite universe,  $m_{ui} \rightarrow \infty$
- Mass of our universe,  $M_u = 1.7394491196 \cdot 10^{+53}$  kg [10].

The Planck mass  $m_{pl}$  is well-known for the numerous formulas it appears in and for its very value. Its essence is far less known, although there have been countless attempts to explain it. Here we will not get into that, although the formulas which will be presented here will further explain the place and role of the Planck mass.

The fundamental mass  $m_f$  was mentioned by Weinberg [6] as the mass which would be useful for explaining the relation between the gravitational and Coulomb force. Weinberg is comparing the value of this mass with the real mass of pion, disregarding or not knowing about Roger Boscovich's force curve.

Mass  $m_b$  we named the "background mass", because by using it one can easily determine the temperature of background radiation.

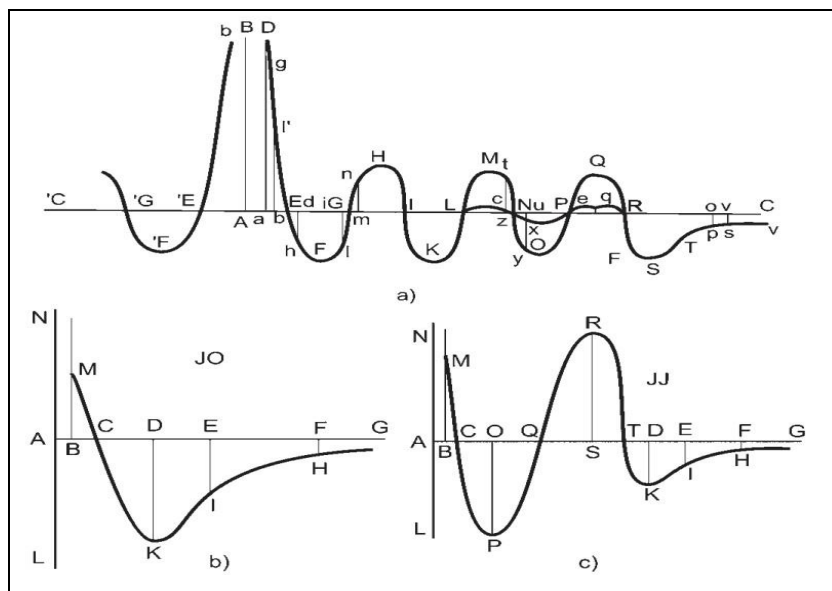
Hypothetical mass quantum  $m_q$  is often mentioned under various names and different assumptions and formulas lead to its approximate value of  $m_q \approx 10^{-69}$  kg, while the article [8] calculates its exact value, mentioned above.

In this group of masses we also included the mass of the infinite universe, the mass of our universe and the mass of vacuum.

To determine the relations among the mentioned masses, here it is assumed that they are situated at the unstable positions on Roger Boscovich's force curve [2], and we will calculate them similarly as the relations among characteristic volumes of the matter and the relations among the mean density of planets and the density of the Sun.

### Boscovich's theory of natural philosophy [7]

Boscovich's comprehension of nature is partly based on the ideas of Leibniz and Newton, and partly deviates from them. From Leibniz he accepts the assumption that the basic elements of matter are tiny dots (monads), which do not have size (non-extended) and are indivisible. However, Boscovich does not accept Leibniz's assumption that the points can touch each other. Instead, he believes that the points are distanced from each other over some space, which can infinitely increase or decrease, but not completely disappear. From Newton, he accepts the existence of mutual forces between these points. Unlike Newton, who believes that at very short distances there is strong attractive force between the particles, Boscovich believes that there is a strong repulsive force, which becomes greater as the distance becomes less. This is similar to the views of Empedocles that there are forces of love and forces of strife, where Boscovich believes that force can be attractive or repulsive, and which alternates depending on the distance between points (Figure 1).



**Figure 1.** General (a) and particular (b, c) shapes of curves that present the attractive and repulsive forces (bottom and upper ordinates, respectively) vs. distance (abscissa) between the elementary points and particles of [2].

Boscovich accepted Newton's notion that assembling points together in collections forms more complicated particles of the first order, connecting these first order particles together then forms particles of the second order, then the third order and so on, and by further assembling atoms, molecules and macromolecules are formed. All worlds of smaller dimensions are like a single point in relation to the larger world. He believes that his curve shown in Figure 1 is valid for each pair of particles at any level of the hierarchy of matter. The number of arches, their size and shape can be different. Boscovich curve displayed: curve with one attractive and repulsive arch (Figure 1b), two attractive and repulsive two arches (Fig. 1c), as well as with a number of arches (Fig. 1a).

Boscovich especially points out that there are distances (E, G, I, L, N, P, and R in Figure 1a) at which the repulsive and attractive forces are equal. The particles are in balance if they are at such distances. However, there are two types of cases. In some cases (E, I, N and R) by increasing the distance the attractive force increases, and by reducing the distance the repulsive force increases. In this case, the particles are in a stable equilibrium, for if you accidentally increase the distance between the particles, then it creates an attractive force, and brings them back to the previous distance. If, however, the distance is reduced, then the resulting repulsive force brings them back to the previous distance. These distances (from A to E, I, N and R) are called **the limits of cohesion**.

In other cases, if the distances correspond to the positions G, L and P, the particles are in an unstable equilibrium because you either have (a) a small increase leading to the appearance of the repulsive force and to an even greater separation of the particles, or (b) a small decrease in distance leading to the appearance of the attractive force and to an even greater coming together of the particles. These positions Boscovich called **the limits of non-cohesion**.

Boscovich's philosophical and natural-scientific concepts, however, were known to many of his contemporaries and subsequent scientists, since in the 18<sup>th</sup> and 19<sup>th</sup> century his theory [2] and other works were in the curricula in many universities and educational institutions in cities that are now within Austria, Italy, France, Germany, England, Poland, Hungary, Croatia... Also, his theory was in contents of many books and encyclopaedias in the 18<sup>th</sup> and 19<sup>th</sup> century. Hence, many well-known scientists declared themselves as supporters and followers of at least some of his comprehension: Ampere, Cauchy, Fehner, Priestley, Gay Lisak, Faraday, J. J. Thomson, W. Thomson (Lord Kelvin), Mendelejev, Helmholtz, Hertz, Maxwell, Lorentz, Davy, Bohr, Heisenberg, Lederman and many others [7].

There is plenty of evidence in modern science that confirms the validity of Boscovich's curve [7]. In 1993, Nobel Prize laureate Leon Lederman wrote that Boscovich was "ahead of his time": "His Theory was incomplete and limited, but the idea of particles having a radius equal to zero and looking like a point, and yet produces around itself a 'force field' **is the key to the whole of modern physics**".

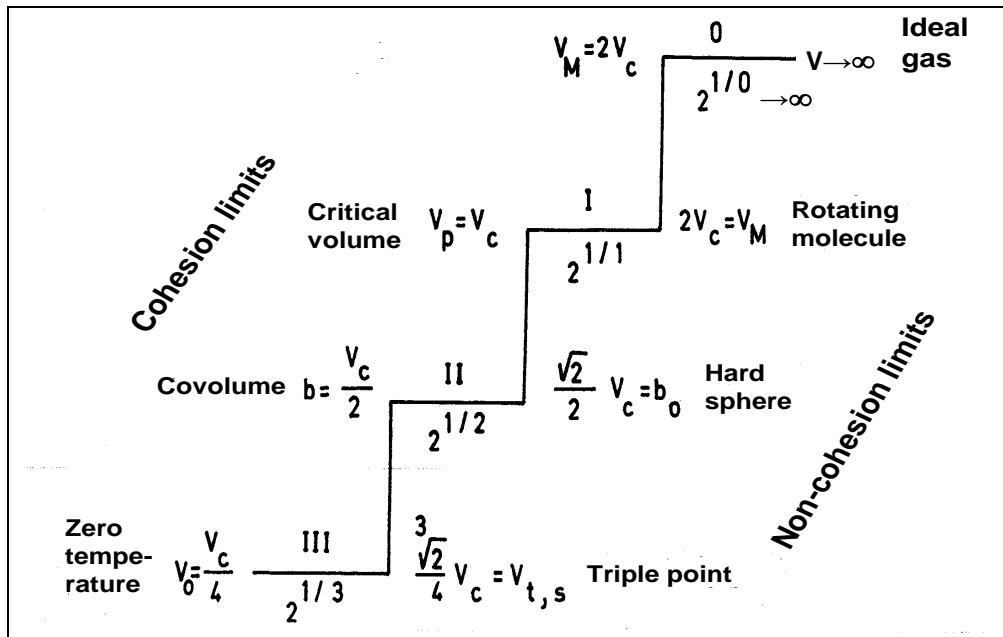
### **Some physical meanings of cohesion and non-cohesion limits [7]**

The existence of stable and unstable positions at cohesion and non-cohesion limits for each pair of particles at any level of the hierarchy of matter has great importance in science and its history.

For example, at the beginning of the 20<sup>th</sup> century Lord Kelvin and J. J. Thomson used that concept to propose a "planetary" structure of atom, i.e. with a positively charged nucleus at the centre and negatively charged electrons rotating at allowed orbits having the radius equal to cohesion limits. The forbidden orbits are situated at non-cohesion limits. Moreover, Boscovich indicates that it stems from his theory that as a particle approaches another particle, and when it passes from one to the other limits of cohesion, it will lose or gain exactly a certain amount of energy. That "quantum energy", as it is now called, between the two limits of cohesion is equal to the difference between areas delimited by repulsive and attractive arches. Hence, the Boscovich theory is the very first **quantum theory**. The planetary model of atom was based on Boscovich's orbits and concept of energy quantum and was confirmed by E. Rutherford and N. Bohr at the beginning of the 20<sup>th</sup> century.

### Relations between characteristic volumes of matter [7]

Furthermore, it was shown that molecules situated at these cohesion and non-cohesion limits contribute to some very characteristic states of matter. A very simple staircase model is used to calculate the volumes of matter in these states (Figure 2).



**Figure 2.** The relations between the specific volumes of matter in characteristic states and the critical volume  $V_c$  [7]

Mathematical model of the staircase diagram is:

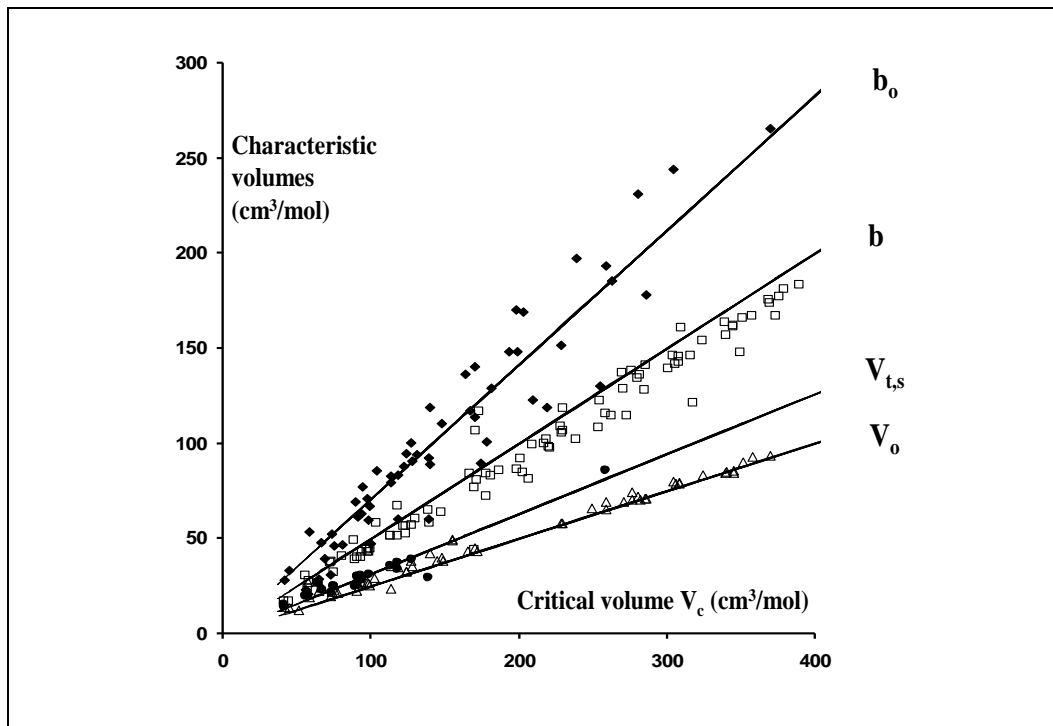
$$\text{For cohesion limits: } V_x/V_c = 2^{(1-i)} \quad (1a)$$

where  $V_x$  is  $V_M$ ,  $V_c$ ,  $b$  and  $V_0$  for  $i=0$ ,  $i=1$ ,  $i=2$  and  $i=3$ , respectively

$$\text{For non-cohesion limits: } V_y/V_c = 2^{1/i} * V_x/V_c = 2^{1/i} * 2^{(1-i)} = 2^{(1-i+1/i)} \quad (1b)$$

where  $V_y$  is  $V_M$ ,  $b_0$  and  $V_{t,s}$  for  $i=1$ ,  $i=2$  and  $i=3$ , respectively

The relations between the specific volumes of matter in characteristic states (i.e. cohesion and non-cohesion limits) and the critical volume  $V_c$  (Figure 2 and eq. 1) has been confirmed for 143 substances (Figure 3).

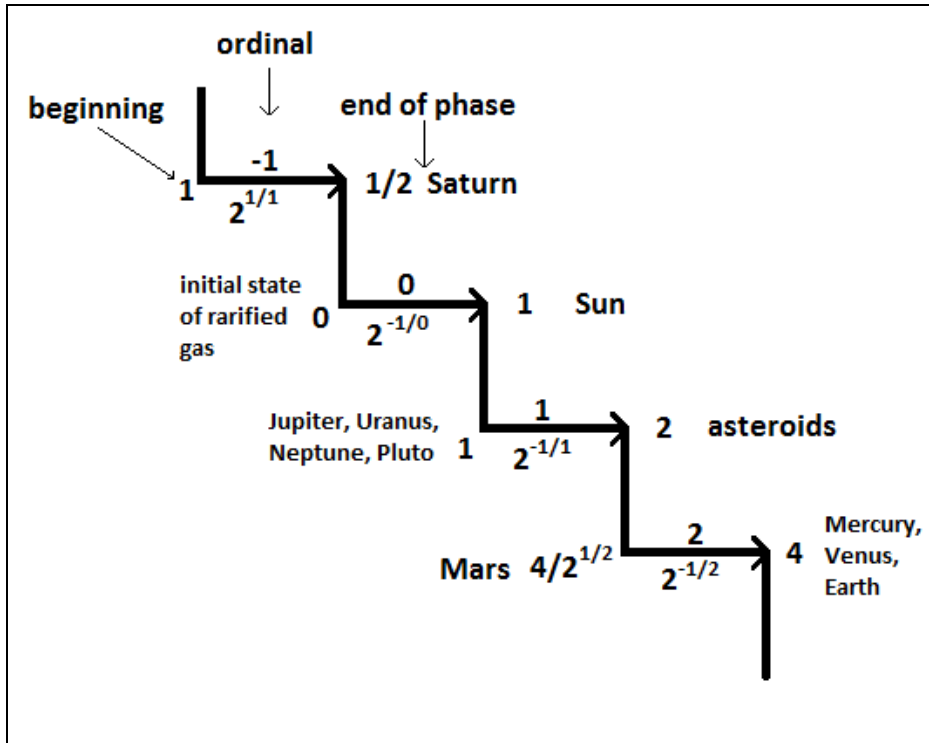


**Figure 3.** The relations between the critical volume  $V_c$  and other characteristic volumes: hard sphere volume  $b_0$ , covolume  $b$ , the volume of the solid phase in the triple point  $V_{t,s}$ , volume of matter at absolute zero  $V_0$ . Lines represent the theoretical expected values based on our model (Figure 2), and points are experimentally determined values taken from the literature for the 143 substances: metals, inert gases, elements, saturated and unsaturated hydrocarbons, aromatic hydrocarbons, organic or inorganic compounds of oxygen, nitrogen, sulphur and halogens [7].

### Relations between mean densities of planets and the Sun

The identical staircase model was derived to represent the ratio of mean density of planets to mean density of the Sun (Figure 4). A similar mathematical model as Equation (1) is for density of planets as for specific volumes of matter. It should be taken only that the specific volume ( $V$ ) is equal to the reciprocal value of density ( $d$ ), i.e.  $V=1/d$ . Results of calculation are presented in Table 1.

Boscovich's theory evolved from the interaction of "non-extended material points" leading to the quantum model of the atom, to describing how these atoms make up the planets in the Solar system, as well as how molecules interact forming characteristic states of matter. These physical meaning of cohesion and non-cohesion limits, as well as the similarities of above mathematical models give to Boscovich's theory such universality, which is attributive only to those general laws of nature, on which foundations rests the magnificent edifice of modern science.



**Figure 4.** The ratio of mean density of the planets with that of the Sun ( $d_{\text{sun}}=1.41 \text{ g/cm}^3$ ) [7]

Table 1. Mean density of planets in the Solar system [7]

Planet	Mean density ( $\text{g/cm}^3$ )	
	Empirical data	Calculated using staircase model (Figure 3)
Mercury	5.4	5.64
Venus	5.2	5.64
Earth	5.5	5.64
Mars	3.9	4.00
Jupiter	1.3	1.41
Saturn	0.7	0.71
Uranus	1.6	1.41
Neptune	1.7	1.41

### Significant masses in the micro and macro world

On Figure 1, the neighboring positions (E, G, I...) are always one stable and one unstable. The hypothesis is that in the unstable positions (limits of non-cohesion) we can imagine the existence of the unstable mass. Here the first goal is to calculate the values of the mentioned masses in relation to the smallest of them and then to determine the real masses at stable positions from these values.

The most important and the most famous of the masses is the Planck mass, so we could start determining the masses from that one. However, **hypothetical mass quantum  $m_q$**  is smaller than the Planck mass; therefore, we decided to start from that smallest mass  $m_q$ , so that the other ones would be its product. There is a question; however, what is physical meaning and what is the value of  $m_q$ ? Let's consider the comprehensions of Boscovich.

According to Boscovich, "*matter consists of points that are perfectly simple, indivisible, of no extent and separated from one another. Each of these points has a property of inertia, and in addition a mutual active force depending on the distance...*" as it is presented in Figure 1. The areas delimited by attractive and repulsive arches and abscissa represent the minimum quants of energy by interaction of two elementary points.

A property of inertia of Boscovich's point, however, should represents its hypothetical mass, and could represents a hypothetical mass quantum,  $m_q$ . It is described in the literature that  $m_q$  is order of magnitude  $\approx 10^{-69}$  kg. We showed in [8] a more exact value,  $m_q = 2.7233883 \cdot 10^{-69}$  kg.

According to Boscovich, "*the number of existing points of matter is finite*" [2, Supplement I, Section 16]. He claims that the number of points in the universe is very large, but – no matter how large – it is finite, since "*the number of points present in any given mass is finite*" [2, Section 394].

Suppose that  $N$  represents the number of hypothetical Boscovich's points in our universe, then the mass of our universe  $M_u$  is given by (2):

$$M_u = N \cdot m_q \quad (2)$$

Knowing  $M_u = 1.7394491196 \cdot 10^{+53}$  kg [10], the number of hypothetical points  $N$  in our universe can be calculated:

$$N = M_u / m_q = 6.3870772 \cdot 10^{+121} \quad (3)$$

Earlier we showed [8, 9] that this large non-dimensional number is in relations with other important physical quantities (Equation 4), so it makes sense that it is also used in this article for calculating masses.

$$\begin{aligned} N &= (M_u / m_{pl})^2 = R_u M_u c / \hbar = M_u / m_q = (m_f / m_q)^3 = (m_{pl} / m_f)^6 = \\ &= (R_u / l_{pl})^2 = (T_u^2 c^5 G^{-1} h^{-1})^3 = 6.3870772 \cdot 10^{+121} \end{aligned} \quad (4)$$

where  $M_u$ ,  $R_u$ ,  $T_u$  are the mass, radius and cycle of our universe, respectively. The Codata values of the reduced Planck constant, Planck constant and speed of light in vacuum are [1]:  $\hbar = 1.054\ 571\ 726\ (47) \cdot 10^{-34}$  Js;  $h = 6.626\ 069\ 57\ (29) \cdot 10^{-34}$  Js and  $c = 299\ 792\ 458\ \text{ms}^{-1}$ .

Let's assume that other masses can be expressed via the hypothetical mass quantum  $m_q$  and number  $N$ . It should be noticed that relations of characteristic volumes of matter at cohesion and non-cohesion limits is presented by coefficient  $2^{1/i}$ , where  $i = -1, 0, 1, 2, 3, \dots$  (Figure 2 and Equation 1). The same coefficient and exponent are for ratio of mean density of the planets

with that of the Sun (Figure 4). Hence, it is reasonable to use the same exponent  $1/i$  for  $N$  which multiplies the hypothetical mass  $m_q$  in the simplest relation (1) with other masses (5).

$$m_i = m_q N^{1/i} \quad (5)$$

Here  $i = -1, 0, 1, 2, 3, 4$  and  $\infty$

Results of calculation are presented in Table 2.

Table 2. The expressions and values of significant masses calculated by Equation (5)

<b>i</b>	<b><math>m_i</math></b>	<b>Value (kg)</b>	<b>Mass of</b>
-1	$m_v = m_q N^{(-1)}$	4.26390383E-191 $\approx$ 0	Vacuum
0	$m_{ui} = m_q N^{\infty}$	$\infty$ (see note)	Infinite universe
1	$M_u = m_q N^1$	1.7394491196 $\cdot 10^{+53}$	Our universe
2	$m_{pl} = m_q N^{1/2}$	2.1765099 $\cdot 10^{-8}$	Planck mass
3	$m_f = m_q N^{1/3}$	1.088622 $\cdot 10^{-28}$	Fundamental mass
4	$m_b = m_q N^{1/4}$	7.6990139 $\cdot 10^{-39}$	Background mass
$\infty$	$m_q = m_q N^0$	2.7233883 $\cdot 10^{-69}$	Hypothetical mass quantum

Note: Boscovich differentiates between mathematical infinity (unlimited) and the actual (real) infinity in the real world. The mathematical infinity is something that has no end, as mathematically we can always imagine something larger, at least by a unit. However, the actual (real) infinity in the real world is something that is represented by a large, but still definite number. That number is not and cannot be, not even by a unit, larger than some definite, but high, finite value. It is as large as it is. Still, even for that large finite number we use the symbol of mathematical infinity ( $\infty$ ) and calculate with it as with a mathematical infinite number.

Regarding formula (5), for  $i = \infty$ , of course, we have the very value of the hypothetical mass quantum. This mass we can find in many articles, almost always related to the cosmological constant. Since in this approach there is no need to use that term and in order not to bring confusion to the reader, we are not referencing the mentioned literature. For hypothetical mass quantum it is interesting that its generalized radius is the Planck length [5].

For  $i = 2$  we have the Planck mass [4]. We will not go into detail about this mass, since it has already been extensively covered in literature. Its CODATA 2010<sup>th</sup> value [1] is:  $m_{pl} = 2.17651(13) \cdot 10^{-8}$  kg.

Fundamental mass  $m_f$ , whose  $i = 3$ , is presented in detail in [5]. We can talk about the fundamental particle as an intersection between the matter, represented by the proton, and the electromagnetic part of the universe.

For  $i = 4$ , let's call it the background mass  $m_b$ . It's relation to the background temperature is given in [9].  $T_{BG} = T_{pl} m_b / (d^{1/2} \pi^{1/4} m_{pl}) = 2.72571687$  K, where  $d = q/3 = \log_2(N)/3 = 134.8761518$ .

For  $i = 1$ , we have the mass of our universe  $M_u$ , which can be understood as mass existing and acting in one quantum of time equals to Cycle of 13.7 billion years in space of the volume equals to  $R_u^3$ .



For  $i=0$ , we have the mass of the infinite universe  $m_{ui}$ , which comprises our universe and the universe beyond it.

For  $i=-1$ , we have the absence of every mass, i.e. almost an ideal vacuum.

In a somewhat more complicated form, the above-mentioned significant masses can be expressed through fundamental physical constants ( $c$  - speed of light,  $G$  - universal gravitational constant,  $\hbar$  - reduced Planck constant) [9, 10].

## Calculation of the values of real masses

Knowing the values of these significant masses is useful since the values of real masses can be more easily calculated by them. For example, we can calculate the masses of proton  $m_p$ , electron  $m_e$  and up quark  $m_{up}$ , knowing the values of fundamental mass  $m_f$  and the values only two additional non-dimensional constants:  $\alpha'$  and  $\beta$ . The first one is the inverse fine-structure constant which has a value  $\alpha'=137.035\ 999\ 074$  [1]. The second one  $\beta$  is the ratio between classical electron radius  $r_e=2.817\ 940\ 3267\ (27) \cdot 10^{-15}$  m [1] and the proton-Compton wavelength  $\lambda_p=1.321\ 409\ 856\ 23\ (94) \cdot 10^{-15}$  m [1], i.e.  $\beta=r_e/\lambda_p=2.132525585$ . The mathematical expressions for calculation as well as the results are presented by Equations (6, 7 and 8).

$$m_p = 2\pi m_f 2^{(2/3)*(1/(1+\frac{1}{2\pi\beta+1}))+1)} = 1.672621777 * 10^{-27} \text{ kg} \quad (6)$$

$$m_e = (\alpha' \beta)^{-1} m_f 2^{(2/3)*(1/(1+\frac{1}{2\pi\beta+1}))+1)} = 9.109382908 * 10^{-31} \text{ kg} \quad (7)$$

$$m_{up} = m_f \alpha'^{-2/3} = 4.0956280301 * 10^{-30} \text{ kg} \quad (8)$$

The values calculated by Equations (6 and 7) are equal to CODATA [1] values for proton and electron masses:

$$m_p=1.672\ 621\ 777\ (74) \cdot 10^{-27} \text{ kg}$$

$$m_e=9.109\ 382\ 91\ (40) \cdot 10^{-31} \text{ kg}$$

## Conclusion

Starting from "A Theory of Natural Philosophy" of Roger Boscovich, we have shown that certain significant masses in the micro and macro world,  $m_i$ , (the Planck mass, fundamental mass, background mass, hypothetical mass quantum  $m_q$ , as well as the masses of vacuum, infinite universe and our universe) can be calculated via a simple equation  $m_i=m_q N^{1/i}$ , where  $N$  is a large non-dimensional number ( $6.3870772 \cdot 10^{+121}$ ), while  $i=-1, 0, 1, 2, 3, 4, \infty$ .

Knowing these significant masses is useful for calculating other real masses. We showed examples of calculating the mass of proton, electron and up quark.

All the formulas shown are simple and derived from a logical starting point that a whole and parts are immanently dependent on each other. They are also showing that the quantum character of phenomena is connected to the relations between the whole and parts.

**Novi Sad, March 2014**

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