A Monte Carlo simulation framework for testing cosmological models

Yuri Heymann

3 rue Chandieu, 1202 Geneva, Switzerland

(Dated: March 12, 2014)

Abstract

We tested alternative cosmologies using Monte Carlo simulations based on the sampling method of the zCosmos galactic survey. The survey encompasses a collection of observable galaxies with respective redshifts that have been obtained for a given spectroscopic area of the sky. Using a cosmological model, we can convert the redshifts into light-travel times and, by slicing the survey into small redshift buckets, compute a curve of the galactic density over time. Because foreground galaxies obstruct the images of more distant galaxies, we simulated the theoretical galactic density curve using an average galactic radius. By comparing the simulated galactic density curves versus the survey galactic density curve, we could assess the cosmologies. We applied the test to the expanding-universe cosmology of de Sitter and to a dichotomous cosmology introduced herein.

I. INTRODUCTION

We tested cosmological models using relatively small simulations that can be run on a home computer. Simulation is a promising and powerful tool for the field of cosmology. For example, the Millennium Simulation project at the Max Planck Institute for Astrophysics, the largest N-body simulation carried out so far, simulated the formation of large structures in the universe using a cluster of 512 processors. Our rationale was to slice a galactic survey into small redshift buckets. We then used cosmological models to compute the volume of each bucket and derived the galactic density curve versus the redshift, or light-travel time. We used the simulation to generate a uniform distribution of galaxies for each redshift bucket. We then computed the number of visible galaxies (i.e., those that were not covered by foreground galaxies) to derive a simulated galactic density curve. Our method requires only a cosmological model and a behavior for the galactic density and the average galactic radius versus the redshift.

We are interested in a special class of cosmological models: cosmologies with a Hubble constant that does not vary over time to conform to the linear relationship between the luminosity distance and the redshift observed for Type Ia supernovae [1]. This choice is motivated by the idea that the laws of nature follow simple principles. There are two distinct cosmologies that satisfy this condition: the de Sitter flat-universe cosmology and the dichotomous cosmology described herein.

The de Sitter cosmology is a solution to the Friedmann equation for an empty universe, without matter, dominated by a repulsive cosmological constant Λ corresponding to a positive vacuum energy density, which sets the expansion rate $H = \sqrt{\frac{1}{3}\Lambda}$. The dichotomous cosmology consists of a static material world and an expanding luminous world. It is not hard to think of a mechanism whereby light expands and matter is static. For example, consider that the light wavelength is stretched via a tired-light process when photons lose energy. The number of light wave cycles is constant, resulting in an expanding luminous world and static material world. In order to maintain a constant speed of light, we would still have to introduce a time-dilation effect.

For both the dichotomous and the de Sitter cosmologies, the same equation relates light-travel time to redshifts, making it easy to compare both models using our testing framework.

II. METHOD

A. The cosmological model

Consider an expanding luminous world, or an expanding universe, with a constant expansion rate H_0. Because of the expansion, the distance between two points is stretched. Let us introduce the Euclidean distance y, which is the equivalent distance measured if there were no expansion. The Euclidean distance is also the proper distance at the time light was emitted, which is the comoving distance times the scale factor at the time of emission. Hence, y must satisfy the following differential equation:

$$\frac{dy}{dt} = -c + H_0 y \, . \quad (1)$$

By setting time zero at a reference T_b in the past, we get $t = T_b - T$; therefore, $dt = -dT$. Hence, (1) becomes:

$$\frac{dy}{dT} = c - H_0 y \, , \quad (2)$$

with boundary condition $y(T = 0) = 0$. Solving (2) we get:

$$y = \frac{c}{H_0} \left(1 - \exp(-H_0 T)\right) \, . \quad (3)$$

Because $dt = \frac{da}{H\,a}$, where a is the scale factor, the proper light-travel time versus redshift is:

$$T = \int_{1/(1+z)}^{1} \frac{da}{H_0\, a} = \frac{1}{H_0} \ln(1+z)\,. \quad (4)$$

By substitution of (4) into (3), we get:

$$y = \frac{c}{H_0} \frac{z}{(1+z)}\,, \quad (5)$$

As $T_0 = \frac{y}{c}$, we finally get:

$$T_0 = \frac{1}{H_0} \frac{z}{(1+z)}\,, \quad (6)$$

where T_0 is the light-travel time in the temporal reference frame of the observer, H_0 the Hubble constant, and z the redshift. Eq. (6) is our cosmological model relating light-travel time to redshifts.

B. The sampling method

The zCosmos galactic survey [2] consists of a collection of visible galaxies with their respective redshifts obtained for a given spectroscopic area in the sky. Here, we used Data Release DR1, which contains galactic observations up to a redshift of 5.2. We sliced the collection of galaxies into small redshift buckets and counted the number of galaxies in each bucket. Using our cosmological model, we converted the redshifts into light-travel times. The volume of each bucket is equal to the volume of the slice for the whole sphere contained between the lower and upper radius boundaries of the bucket multiplied by the ratio of the spectroscopic survey area divided by the solid angle of the sphere.

For an observer at the center of a sphere, the volume of a slice of the sphere is:

$$V_i = \frac{4\pi}{3} \left(r_i^3 - r_{i-1}^3 \right)\,, \quad (7)$$

where r_{i-1} and r_i are the lower and upper radius boundaries of the bucket, respectively.

The spectroscopic area of the zCosmos galactic survey was determined to be 0.075 square degrees [3]. Hence, the ratio of the survey spectroscopic area divided by the solid angle of the sphere is as follows:

$$\eta_{surv} = \frac{0.075}{4\pi(180/\pi)^2} = 1.81806 \times 10^{-6}\,. \quad (8)$$

Thus, the volume of the i^{th} bucket of the survey is $\eta_{surv} V_i$. The galactic density of the bucket is the number of galaxies contained within the redshift boundaries of the bucket divided by the bucket volume. By computing the galactic density for each bucket, we get the survey galactic density curve versus the redshift or light-travel time.

C. The simulation method

To simulate the galactic density curve, in addition to a cosmological model, we need two other behaviors: the galactic density versus redshift and the relationship between the average galactic radius and redshifts.

For the sake of convenience, we used the same redshift slicing that we used to compute the survey galactic-density curve, say $z \in \{0, z_1, z_2, ..., z_n\}$, where $z_{i+1} = z_i + \delta z$. By iteration from redshifts z_1 to z_n, we generated N_i galaxies with a uniform distribution in an isotropic universe and then determined whether each galaxy is visible amongst the foreground galaxies. We determined the position of each galaxy using the astronomical spherical coordinates (r, θ, φ), where r is the radial distance, $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ is the declination, and $\varphi \in [0, 2\pi]$ is the right ascension. Each galaxy also has an associated radius.

First, we fixed the spectroscopic area of the simulation by taking boundaries for the declination and right ascension, say $\varphi \in [\varphi_{min}, \varphi_{max}]$ and $\theta \in [\theta_{min}, \theta_{max}]$. The spectroscopic area of the simulation is:

$$specArea = \left(\frac{180}{\pi}\right)^2 (\sin\theta_{max} - \sin\theta_{min}) \times (\varphi_{max} - \varphi_{min}) \,, \quad (9)$$

and the spectroscopic area of the simulation to solid angle of the sphere is:

$$\eta_{sim} = \frac{specArea}{4\pi(180/\pi)^2} \,. \quad (10)$$

To determine the number of galaxies to generate for a redshift bucket $[z_{i-1}, z_i]$, we computed the volume V_i of the spherical shell using (7) and then multiplied the galactic density by $\eta_{sim} V_i$, hence:

$$N_i = \rho_i \, \eta_{sim} V_i \,, \quad (11)$$

where N_i is the number of galaxies generated, ρ_i is the galactic density at redshift z_i, and η_{sim} and V_i are as defined previously.

To generate a galaxy, we drew two independent, uniform random variables, say X and Y, on the interval $[0, 1]$ and computed the declination and right ascension of the galaxy as follows:

$$\theta = \theta_{min} + X(\theta_{max} - \theta_{min}) \,,$$
$$\varphi = \varphi_{min} + Y(\varphi_{max} - \varphi_{min}) \,. \quad (12)$$

The newly generated galaxy was attributed the radial distance corresponding to the light-travel time at redshift z_i.

Next, we determined whether each generated galaxy was hidden by foreground galaxies. As an example, consider the calculations for galaxy B with galaxy A in the foreground. We compute the distance between the projection of galaxy A on the plan of galaxy B and galaxy B itself, which we call the "projected distance". If the projected distance is smaller than or equal to the critical distance, then galaxy B is determined to be not visible. The projected distance is calculated as:

$$projectedDist = \sqrt{squareDist} \,, \quad (13)$$

where the square distance is:

$$squareDist = (x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2 \,, \quad (14)$$

and (x, y, z) are the Cartesian coordinates of both galaxies projected in the plan of galaxy B, and subscripts A and B designate the coordinates of galaxies A and B, respectively.

The spherical coordinates are converted to Cartesian coordinates as follows:

$$x = r_B \cos\theta \sin\varphi \,,$$
$$y = r_B \cos\theta \cos\varphi \,,$$
$$z = r_B \sin\theta \,, \quad (15)$$

where r_B is the radial distance of galaxy B required to project galaxy A into the plan of galaxy B.

The critical distance is calculated as:

$$criticalDist = \frac{r_B}{r_A} R_A + R_B \,, \quad (16)$$

where R_A and R_B are the respective radii of galaxies A and B. The ratio of radial distances, $\frac{r_B}{r_A}$, applied to the radius of galaxy A represents the projection of galaxy A into the plan of galaxy B according to Thales' theorem.

For the special case when the foreground galaxy A is lies over galaxy B but covers it only partially (see Fig. 1), we consider galaxy B to be not visible. Because the zCosmos galactic survey was obtained using an automated device, an algorithm cannot identify a galaxy that is not isolated from other sources of light. Still, galaxy B could hide more distant galaxies.

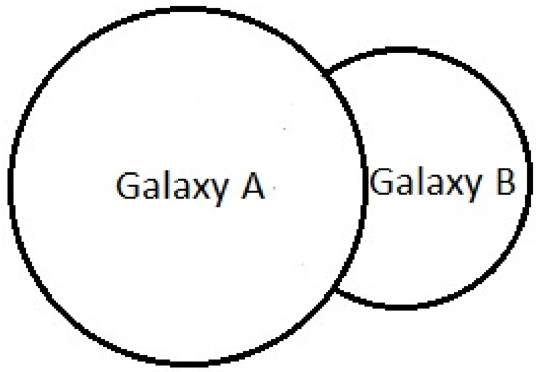

FIG. 1. A foreground galaxy partially covering a more distant galaxy

Finally, we count the visible galaxies in each redshift bucket and multiply the counts by the ratio of the survey area to the simulated spectroscopic area in order to have numbers that are comparable between the survey and the simulation.

To generate the declination and right ascension angles of a galaxy, we used the Mersenne Twister algorithm [4], which is a pseudo-random number generator based on the Mersenne prime $2^{19937}-1$. The algorithm has a very long period of $2^{19937}-1$ and passes numerous tests for statistical randomness.

D. Galactic density and radius function of redshifts

In the dichotomous cosmology, where the material world is static and the luminous world is expanding, the galactic density is constant over time, but the image of galaxies is dilated by a factor of $(1 + z)$, because the expanding luminous world acts like a magnifying glass. Because light is stretched, the apparent size of galaxies is also stretched by the same factor resulting in a lensing effect across the whole sky. In contrast, in the expanding universe theory, the galactic density increases by a factor $(1+z)^3$ as we look back in time.

The radius of a galaxy in an expanding universe can be tackled in two different ways. If we consider that the whole space expands, then the galactic radius expands over time and is divided by a factor $(1+z)$. Because the expanding universe has the same magnifying effect as the expanding luminous world, the galactic radius is also multiplied by a factor of $(1 + z)$. The net effect is that the galactic radius is constant over time, as in Expanding Cosmology A in Table 1. The other approach is to consider that galaxies do not expand in size, but because of the magnifying effect of the expansion, the image of the galaxies is dilated by a factor $(1 + z)$, as in Expanding Cosmology B in Table 1.

In Table 1, ρ_0 is the present galactic density, and R_0 is the present average galactic radius. Because of the cluster of galaxies around the Milky Way, the number of galaxies in the bucket with redshift 0.1 was generated to match the galactic density of the survey. For buckets with redshifts above 0.1, we used the functions in Table 1.

III. RESULTS AND DISCUSSION

A. Galactic density curves

For both the survey and simulated galactic density curves, we used redshift buckets of size $\delta z = 0.1$. We used 0.082 square degrees as the spectroscopic area for the dichotomous cosmology simulation. We used a smaller value of 0.025 square degrees for the expanding universe theory because of the large number of galaxies generated. For the Hubble constant employed in the cosmological model (6), we used a value of $H_0 = 67.3\, km\, s^{-1}\, Mpc^{-1}$, or $2.16 \times 10^{-18} sec^{-1}$ [1].

Figure 2 shows the simulated galactic density curve for the dichotomous cosmology versus the galactic density curve obtained from the survey. For this simulation, we used a constant galactic density of $\rho = 3 \times 10^6$ galactic counts per cubic Glyr (billion light years) and an average galactic radius of $R = 40,000\,(1 + z)$ light years. The factor $(1 + z)$

TABLE I. Galactic density and radius functions of redshifts for the dichotomous cosmology and expanding universe theory

	Galactic density	Galactic radius
Dichotomous Cosmology	ρ_0	$R_0(1+z)$
Expanding Cosmology A	$\rho_0(1+z)^3$	R_0
Expanding Cosmology B	$\rho_0(1+z)^3$	$R_0(1+z)$

accounts for the magnifying effect of the expanding luminous world in the dichotomous cosmology (see section II.D).

The present average galactic radius of 40,000 light years is within the range of dwarf galaxies and large galaxies. In [5], the galaxies were divided into two groups based on their respective mass: a group with $M_* \approx 10^{11} M_\odot$, corresponding to dwarf galaxies, and a group with $M_* > 10^{11.5} M_\odot$, corresponding to large galaxies. According to that study, the present average radius of dwarf galaxies is 20,200 light years, whereas that of large galaxies is 65,200 light years. Because dwarf galaxies are much more numerous than large galaxies, we would expect the overall average galactic radius to be smaller than 40,000 light years. The gravitational lensing effect that creates a halo around galaxies, and some blurring effect from the luminosity of galaxies, can be accounted for by the fact that foreground galaxies obstruct the images of distant galaxies in a larger area than that of the circle defined by the intrinsic radius of the foreground galaxies. Furthermore a minimum distance must be observed between galaxies for the selection algorithm of the telescope to be able to identify the galaxies as being distinct from one another.

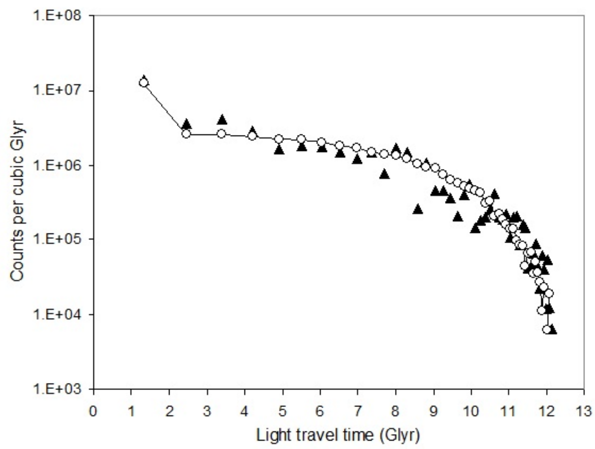

FIG. 2. Galactic density curve for the dichotomous cosmology. Glyr are billion light years. The solid triangles indicate densities based on the zCosmos survey. The open dots indicate densities obtained by Monte Carlo simulation for the Dichotomous Cosmology with a galactic radius of 40,000 light years.

Figure 3 shows the simulated galactic density curve for Expanding Cosmology A versus the galactic density curve obtained from the survey. The galactic density used for this simulation was $\rho = 3 \times 10^6\,(1+z)^3$ counts per cubic Glyr. Two curves were simulated with an average galactic radius of $48,000$ and $78,000$ light years, respectively. The grounds for using a constant galactic radius in Expanding Cosmology A are explained in Section II D. In this cosmology, we can vary ρ_0 and R_0, and there is no solution such that the simulated galactic density curve matches the survey galactic density curve.

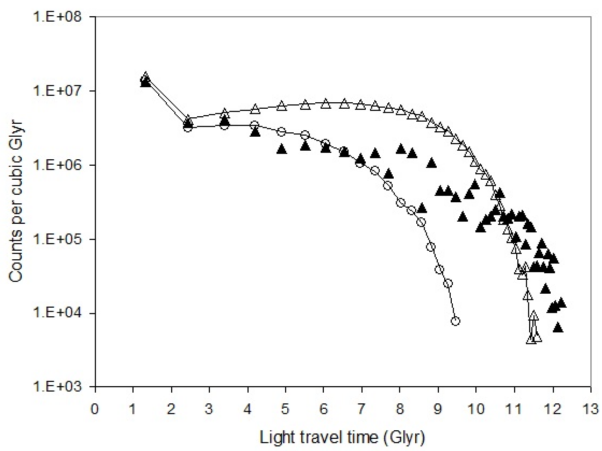

FIG. 3. Galactic density curve for Expanding Cosmology A, where Glyr are billion light years. The solid triangles indicate densities based on the zCosmos survey. The open dots indicate densities obtained by Monte Carlo simulation with a galactic radius of 78,000 light years. The open triangles are the simulated densities obtained with a galactic radius of 48,000 light years.

Figure 4 shows the simulated galactic density curve for Expanding Cosmology B versus the galactic density curve obtained from the survey. We again used a galactic density $\rho = 3\times10^6\,(1+z)^3$ counts per cubic Glyr. The two curves simulated for this cosmology have an average galactic radius of $R = 40,000\,(1+z)$ light years and $R = 13,000\,(1+z)$ light years, respectively. There is no solution for Expanding Cosmology B such that the simulated galactic density curve matches the survey galactic density curve.

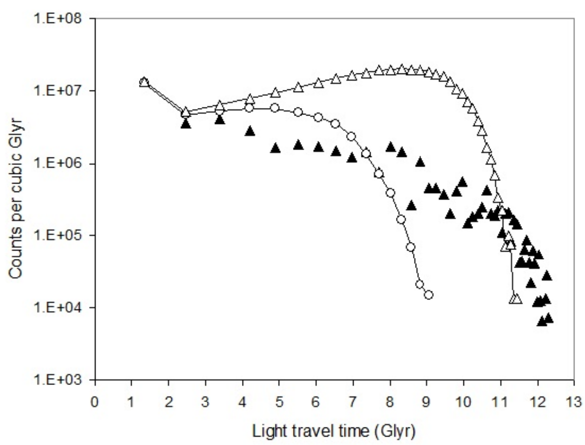

FIG. 4. Galactic density curve for Expanding Cosmology B. Glyr are billion light years. The solid triangles indicate densities based on the zCosmos survey. The open dots indicate densities obtained by Monte Carlo simulation with a galactic radius of 40,000 light years. The open triangles are the simulated densities obtained with a galactic radius of 13,000 light years.

B. Size-biased selection in galactic surveys

As the redshift increases, the number of foreground galaxies increases, leaving only small holes where more distant galaxies can be observed. This effect of increasing redshifts decreases the likelihood of selecting large galaxies. Hence, smaller galaxies are preferentially selected. This size-biased selection could have a significant impact on studies of the morphological evolution of galaxies. For example, in [5] the average radius of galaxies decreases as the redshift increases; dwarf galaxies decrease by $(1 + z)^{-0.7}$ and large galaxies decrease by $(1 + z)^{-1.8}$. As we could expect, the size-biased selection is more important for large galaxies than for small galaxies. The way to quantify the effect of the size-biased selection is by using Monte Carlo simulation to generate galactic radii with a size distribution obtained from galactic surveys at low redshifts.

IV. CONCLUSION

We developed a Monte Carlo simulation framework to test cosmologies. The framework is based on the sampling method of the zCosmos galactic survey. We used simulations to generate a theoretical galactic density curve for a given cosmology. The theoretical density curve was then compared with the galactic density curve obtained from the galactic survey. We applied the test to the flat-universe de Sitter cosmology and to a dichotomous cosmology.

The simulated galactic density curve of the dichotomous cosmology matched the survey galactic density curve remarkably well. For the classes of expanding universe that we considered, there was no solution such that the simulated galactic density curve matched the survey galactic density curve. On the basis of this test, we conclude that the dichotomous cosmology provides an accurate description of the physics underlying cosmological redshifts.

V. ACKNOWLEDGEMENTS

This work is based on zCOSMOS observations carried out using the Very Large Telescope at the ESO Paranal Observatory under Programme ID: LP175.A-0839. I would also like to thank Prof. Arto Annila for insightful comments and our inspiring discussion.

[1] Y. Heymann, Progress in Physics **3**, 5 (2013).

[2] S. Lilly, O. Le Fevre, A. Renzini, G. Zamorani, M. Scodeggio, T. Contini, C. Carollo, G. Hasinger, J.-P. Kneib, A. Lovino, V. Le Brun, V. Mainieri,

M. Mignoli, J. Silverman, T. L.A.M, M. Bolzonella, A. Bongiorno, D. Bottini, P. Capak, K. Caputi, A. Cimatti, O. Cucciati, E. Daddi, R. Feldmann, P. Franzetti, B. Garilli, L. Guzzo, O. Ilbert, P. Kampczyk, K. Kovak, F. Lamareille, A. Leauthaud, J.-F. Le Borgne, H. McCracken, C. Marinoni, R. Pello, E. Ricciardelli, C. Scarlata, D. Vergani, D. Sanders, E. Schinnerer, N. Scoville, Y. Taniguchi, S. Arnouts, H. Aussel, S. Bardelli, M. Brusa, A. Cappi, P. Ciliegi, A. Finoguenov, S. Foucaud, R. Franceshini, C. Halliday, C. Impey, C. Knobel, A. Koekemoer, J. Kurk, D. Maccagni, S. Maddox, B. Marano, G. Marconi, B. Meneux, B. Mobasher, C. Moreau, J. Peacock, C. Porciani, L. Pozzetti, R. Scaramella, D. Schiminovich, P. Shopbell, I. Smail, D. Thompson, L. Tresse, G. Vettolani, A. Zanichelli, and E. Zucca, The Astrophysical Journal Supplement Series **172**, 70 (2007).

[3] Y. Heymann, Progress in Physics **4**, 63 (2011).

[4] M. Matsumoto and T. Nishimura, ACM Transactions on Modeling and Computer Simulation - Special issue on uniform random number generation **8**, 3 (1998).

[5] M. Stringer, F. Shankar, G. Novak, M. Huertas-Company, F. Combes, and B. Moster, http://arxiv.org/abs/1310.3823 (2013).