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An holomorphic study of the Smarandache concept in loops

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Abstract If two loops are isomorphic, then it is shown that their holomorphs are also isomorphic. Conversely, it is shown that if their holomorphs are isomorphic, then the loops are isotopic. It is shown that a loop is a Smarandache loop if and only if its holomorph is a Smarandache loop. This statement is also shown to be true for some weak Smarandache loops (inverse property, weak inverse property) but false for others (conjugacy closed, Bol, central, extra, Burn, A-, homogeneous) except if their holomorphs are nuclear or central. A necessary and sufficient condition for the Nuclear-holomorph of a Smarandache Bol loop to be a Smarandache Bruck loop is shown. Whence, it is found also to be a Smarandache Kikkawa loop if in addition the loop is a Smarandache A-loop with a centrum holomorph. Under this same necessary and sufficient condition, the Central-holomorph of a Smarandache A-loop is shown to be a Smarandache K-loop.

Keywords Holomorph of loops; Smarandache loops.

§1. Introduction

The study of Smarandache loops was initiated by W.B. Vasantha Kandasamy in 2002. In her book [19], she defined a Smarandache loop (S-loop) as a loop with at least a subloop which forms a subgroup under the binary operation of the loop. For more on loops and their properties, readers should check [16], [3], [5], [8], [9] and [19]. In her book, she introduced over 75 Smarandache concepts on loops. In her first paper [20], she introduced Smarandache : left(right) alternative loops, Bol loops, Moufang loops, and Bruck loops. But in this paper, Smarandache : inverse property loops (IPL), weak inverse property loops (WIPL), G-loops, conjugacy closed loops (CC-loop), central loops, extra loops, A-loops, K-loops, Bruck loops, Kikkawa loops, Burn loops and homogeneous loops will be introduced and studied relative to the holomorphs of loops. Interestingly, Adeniran [1] and Robinson [17], Oyebo [15], Chiboka and Solarin [6], Bruck [2], Bruck and Paige [4], Robinson [18], Huthnance [11] and Adeniran [1] have respectively studied the holomorphs of Bol loops, central loops, conjugacy closed loops, inverse property loops, A-loops, extra loops, weak inverse property loops and Bruck loops.

In this study, if two loops are isomorphic then it is shown that their holomorphs are also isomorphic. Conversely, it is shown that if their holomorphs are isomorphic, then the loops are isotopic.

It will be shown that a loop is a Smarandache loop if and only if its holomorph is a Smarandache loop. This statement is also shown to be true for some weak Smarandache loops

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(inverse property, weak inverse property) but false for others (conjugacy closed, Bol, central, extra, Burn, A-, homogeneous) except if their holomorphs are nuclear or central. A necessary and sufficient condition for the Nuclear-holomorph of a Smarandache Bol loop to be a Smarandache Bruck loop is shown. Whence, it is found also to be a Smarandache Kikkawa loop if in addition the loop is a Smarandache A-loop with a centrum holomorph. Under this same necessary and sufficient condition, the Central-holomorph of a Smarandache A-loop is shown to be a Smarandache K-loop.

§2. Definitions and Notations

Let $(L,\cdot)$ be a loop. Let $Aum(L,\cdot)$ be the automorphism group of $(L,\cdot)$, and the set $H = (L,\cdot)\times Aum(L,\cdot)$. If we define '$\circ$' on H such that $(\alpha,x)\circ(\beta,y) = (\alpha\beta, x\beta\cdot y)\ \forall\ (\alpha,x),(\beta,y)\in H$, then $H(L,\cdot) = (H,\circ)$ is a loop as shown in Bruck [2] and is called the Holomorph of $(L,\cdot)$.

The nucleus of $(L,\cdot)$ is denoted by $N(L,\cdot) = N(L)$, its centrum by $C(L,\cdot) = C(L)$ and center by $Z(L,\cdot) = N(L,\cdot)\cap C(L,\cdot) = Z(L)$. For the meaning of these three sets, readers should check earlier citations on loop theory.

If in L, $x^{-1}\cdot x\alpha \in N(L)$ or $x\alpha\cdot x^{-1}\in N(L)\ \forall\ x\in L$ and $\alpha\in Aum(L,\cdot)$, $(H,\circ)$ is called a Nuclear-holomorph of L, if $x^{-1}\cdot x\alpha\in C(L)$ or $x\alpha\cdot x^{-1}\in C(L)\ \forall\ x\in L$ and $\alpha\in Aum(L,\cdot)$, $(H,\circ)$ is called a Centrum-holomorph of L hence a Central-holomorph if $x^{-1}\cdot x\alpha\in Z(L)$ or $x\alpha\cdot x^{-1}\in Z(L)\ \forall\ x\in L$ and $\alpha\in Aum(L,\cdot)$.

For the definitions of automorphic inverse property loop (AIPL), anti-automorphic inverse property loop (AAIPL), weak inverse property loop (WIPL), inverse property loop (IPL), Bol loop, Moufang loop, central loop, extra loop, A-loop, conjugacy closed loop (CC-loop) and G-loop, readers can check earlier references on loop theory.

Here, a K-loop is an A-loop with the AIP, a Bruck loop is a Bol loop with the AIP, a Burn loop is Bol loop with the conjugacy closed property, an homogeneous loop is an A-loop with the IP and a Kikkawa loop is an A-loop with the IP and AIP.

Definition 2.1. *A loop is called a Smarandache inverse property loop (SIPL) if it has at least a non-trivial subloop with the IP.*

A loop is called a Smarandache weak inverse property loop (SWIPL) if it has at least a non-trivial subloop with the WIP.

A loop is called a Smarandache G-loop (SG-loop) if it has at least a non-trivial subloop that is a G-loop.

A loop is called a Smarandache CC-loop (SCCL) if it has at least a non-trivial subloop that is a CC-loop.

A loop is called a Smarandache Bol-loop (SBL) if it has at least a non-trivial subloop that is a Bol-loop.

A loop is called a Smarandache central-loop (SCL) if it has at least a non-trivial subloop that is a central-loop.

A loop is called a Smarandache extra-loop (SEL) if it has at least a non-trivial subloop that is a extra-loop.

A loop is called a Smarandache A-loop (SAL) if it has at least a non-trivial subloop that is a A-loop.

A loop is called a Smarandache K-loop (SKL) if it has at least a non-trivial subloop that is a K-loop.

A loop is called a Smarandache Moufang-loop (SML) if it has at least a non-trivial subloop that is a Moufang-loop.

A loop is called a Smarandache Bruck-loop (SBRL) if it has at least a non-trivial subloop that is a Bruck-loop.

A loop is called a Smarandache Kikkawa-loop (SKWL) if it has at least a non-trivial subloop that is a Kikkawa-loop.

A loop is called a Smarandache Burn-loop (SBNL) if it has at least a non-trivial subloop that is a Burn-loop.

A loop is called a Smarandache homogeneous-loop (SHL) if it has at least a non-trivial subloop that is a homogeneous-loop.

§3. Main Results

Holomorph of Smarandache Loops

Theorem 3.1. Let $(L,\cdot)$ be a Smarandanche loop with subgroup $(S,\cdot)$. The holomorph H_S of S is a group.

Theorem 3.2. A loop is a Smarandache loop if and only if its holomorph is a Smarandache loop.

Proof. Let L be a Smarandache loop with subgroup S. By Theorem 3.1, $(H_S,\circ)$ is a group where $H_S = Aum(S,\cdot)\times(S,\cdot)$. Clearly, $H_S \not\subset H(L,\cdot)$. So, let us replace $Aum(S,\cdot)$ in H_S by $A(S,\cdot)=\{\alpha\in Aum(L,\cdot)\ :\ s\alpha\in S\ \forall\ s\in S\}$, the group of Smarandache loop automorphisms on S as defined in [19]. $A(S,\cdot)\le Aum(L,\cdot)$ hence, $H_S = A(S,\cdot)\times(S,\cdot)$ remains a group. In fact, $(H_S,\circ)\subset(H,\circ)$ and $(H_S,\circ)\le(H,\circ)$. Thence, the holomorph of a Smarandache loop is a Smarandache loop.

To prove the converse, recall that $H(L) = Aum(L)\times L$. If $H(L)$ is a Smarandache loop then $\exists\ S_H\subset H(L)\ \ni\ S_H\le H(L)$. $S_H\subset H(L)\Rightarrow\ \exists\ Bum(L)\subset Aum(L)$ and $B\subset L\ \ni\ S_H = Bum(L)\times B$. Let us choose $Bum(L)=\{\alpha\in Aum(L)\ :\ b\alpha\in B\ \forall\ b\in B\}$, this is the Smarandache loop automorphisms on B. So, $(S_H,\circ)=(Bum(L)\times B,\circ)$ is expected to be a group.

Thus, $(\alpha,x)\circ[(\beta,y)\circ(\gamma,z)]=[(\alpha,x)\circ(\beta,y)]\circ(\gamma,z)\ \forall\ x,y,z\in B,\ \alpha,\beta,\gamma\in Bum(L)\Leftrightarrow x\beta\gamma\cdot(y\gamma\cdot z)=(x\beta\gamma\cdot y\gamma)\cdot z\Leftrightarrow x'\cdot(y'\cdot z)=(x'\cdot y')\cdot z\ \forall\ x',y',z\in B$. So, $(B,\cdot)$ must be a group. Hence, L is a Smarandache loop.

Remark 3.1. It must be noted that if $Aum(L,\cdot)=A(S,\cdot)$, then S is a characteristic subloop.

Theorem 3.3. Let L and L' be loops. $L\cong L'$ implies $H(L)\cong H(L')$.

Proof. If $L\cong L'$ then $\exists$ a bijection $\alpha\ :\ L\to L'\ \ni\ (\alpha,\alpha,\alpha)\ :\ L\to\ L'$ is an isotopism. According to [16], if two loops are isotopic, then their groups of autotopism are isomorphic. The automorphism group is one of such since it is a form of autotopism. Thus ; $Aum(L) \cong Aum(L') \Rightarrow H(L) = Aum(L) \times L \cong Aum(L') \times L' = H(L')$.

Theorem 3.4. Let $(L, \oplus)$ and $(L', \otimes)$ be loops. $H(L) \cong H(L') \Leftrightarrow x\delta \otimes y\gamma = (x\beta \oplus y)\delta \ \forall \ x, y \in L, \ \beta \in Aum(L)$ and some $\delta, \gamma \in Sym(L')$. Hence, $\gamma\mathcal{L}_{e\delta} = \delta$, $\delta\mathcal{R}_{e\gamma} = \beta\delta$ where e is the identity element in L and $\mathcal{L}_x$, $\mathcal{R}_x$ are respectively the left and right translations mappings of $x \in L'$.

Proof. Let $H(L, \oplus) = (H, \circ)$ and $H(L', \otimes) = (H, \odot)$. $H(L) \cong H(L') \Leftrightarrow \ \exists \ \phi \ : \ H(L) \to H(L') \ \ni \ [(\alpha, x) \circ (\beta, y)]\phi = (\alpha, x)\phi \odot (\beta, y)\phi$. Define $(\alpha, x)\phi = (\psi^{-1}\alpha\psi, x\psi^{-1}\alpha\psi) \ \forall \ (\alpha, x) \in (H, \circ)$ and where $\psi \ : \ L \to L'$ is a bijection.
$H(L) \cong H(L') \Leftrightarrow (\alpha\beta, x\beta \oplus y)\phi = (\psi^{-1}\alpha\psi, x\psi^{-1}\alpha\psi) \odot (\psi^{-1}\beta\psi, y\psi^{-1}\beta\psi) \Leftrightarrow (\psi^{-1}\alpha\beta\psi, (x\beta \oplus y)\psi^{-1}\alpha\beta\psi) = (\psi^{-1}\alpha\beta\psi, x\psi^{-1}\alpha\beta\psi \otimes y\psi^{-1}\beta\psi) \Leftrightarrow (x\beta \oplus y)\psi^{-1}\alpha\beta\psi = x\psi^{-1}\alpha\beta\psi \otimes y\psi^{-1}\beta\psi \Leftrightarrow x\delta \otimes y\gamma = (x\beta \oplus y)\delta$ where $\delta = \psi^{-1}\alpha\beta\psi$, $\gamma = \psi^{-1}\beta\psi$.

Furthermore, $\gamma\mathcal{L}_{x\delta} = L_{x\beta}\delta$ and $\delta\mathcal{R}_{y\gamma} = \beta R_y\delta \ \forall \ x, y \in L$. Thus, with $x = y = e$, $\gamma\mathcal{L}_{e\delta} = \delta$ and $\delta\mathcal{R}_{e\gamma} = \beta\delta$.

Corollary 3.1. Let L and L' be loops. $H(L) \cong H(L')$ implies L and L' are isotopic under a triple of the form (δ, I, δ).

Proof. In Theorem 3.4, let $\beta = I$, then $\gamma = I$. The conclusion follows immediately.

Remark 3.2. By Theorem 3.3 and Corollary 3.1, any two distinct isomorphic loops are non-trivialy isotopic.

Corollary 3.2. Let L be a Smarandache loop. If L is isomorphic to L', then $\{H(L), H(L')\}$ and $\{L, L'\}$ are both systems of isomorphic Smarandache loops.

Proof. This follows from Theorem 3.2, Theorem 3.3, Corollary 3.1 and the obvious fact that the Smarandache loop property in loops is isomorphic invariant.

Remark 3.3. The fact in Corollary 3.2 that $H(L)$ and $H(L')$ are isomorphic Smarandache loops could be a clue to solve one of the problems posed in [20]. The problem required us to prove or disprove that every Smarandache loop has a Smarandache loop isomorph.

Smarandache Inverse Properties

Theorem 3.5. Let L be a loop with holomorph $H(L)$. L is an IP-SIPL if and only if $H(L)$ is an IP-SIPL.

Proof. In an IPL, every subloop is an IPL. So if L is an IPL, then it is an IP-SIPL. From [2], it can be stated that L is an IPL if and only if $H(L)$ is an IPL. Hence, $H(L)$ is an IP-SIPL. Conversely assuming that $H(L)$ is an IP-SIPL and using the same argument L is an IP-SIPL.

Theorem 3.6. Let L be a loop with holomorph $H(L)$. L is a WIP-SWIPL if and only if $H(L)$ is a WIP-SWIPL.

Proof. In a WIPL, every subloop is a WIPL. So if L is a WIPL, then it is a WIP-SWIPL. From [11], it can be stated that L is a WIPL if and only if $H(L)$ is a WIPL. Hence, $H(L)$ is a WIP-SWIPL. Conversely assuming that $H(L)$ is a WIP-SWIPL and using the same argument L is a WIP-SWIPL.

Smarandache G-Loops

Theorem 3.7. Every G-loop is a SG-loop.

Proof. As shown in [Lemma 2.2, [7]], every subloop in a G-loop is a G-loop. Hence, the claim follows.

Corollary 3.3. CC-loops are SG-loops.

Proof. In [10], CC-loops were shown to be G-loops. Hence, the result follows by Theorem 3.7.

Theorem 3.8. Let G be a CC-loop with normal subloop H. G/H is a SG-loop.

Proof. According to [Theorem 2.1, [7]], G/H is a G-loop. Hence, by Theorem 3.7, the result follows.

Smarandache Conjugacy closed Loops

Theorem 3.9. Every SCCL is a SG-loop.

Proof. If a loop L is a SCCL, then there exist a subloop H of L that is a CC-loop. CC-loops are G-loops, hence, H is a G-loop which implies L is a SG-loop.

Theorem 3.10 Every CC-loop is a SCCL.

Proof. By the definition of CC-loop in [13], [12] and [14], every subloop of a CC-loop is a CC-loop. Hence, the conclusion follows.

Remark 3.4. The fact in Corollary 3.3 that CC-loops are SG-loops can be seen from Theorem 3.9 and Theorem 3.10.

Theorem 3.11. Let L be a loop with Nuclear-holomorph $H(L)$. L is an IP-CC-SIP-SCCL if and only if $H(L)$ is an IP-CC-SIP-SCCL.

Proof. If L is an IP-CCL, then by Theorem 3.5, $H(L)$ is an IP-SIPL and hence by [Theorem 2.1, [6]] and Theorem 3.10, $H(L)$ is an IP-CC-SIP-SCCL. The converse is true by assuming that $H(L)$ is an IP-CC-SIP-SCCL and using the same reasoning.

Smarandache : Bol loops, central loops, extra loops and Burn loops

Theorem 3.12. Let L be a loop with Nuclear-holomorph $H(L)$. L is a Bol-SBL if and only if $H(L)$ is a Bol-SBL.

Proof. If L is a Bol-loop, then by [17] and [1], $H(L)$ is a Bol-loop. According to [Theorem 6, [20]], every Bol-loop is a SBL. Hence, $H(L)$ is a Bol-SBL. The Converse is true by using the same argument.

Theorem 3.13. Let L be a loop with Nuclear-holomorph $H(L)$. L is a central-SCL if and only if $H(L)$ is a central-SCL.

Proof. If L is a central-loop, then by [15], $H(L)$ is a central-loop. Every central-loop is a SCL. Hence, $H(L)$ is a central-SCL. The Converse is true by using the same argument.

Theorem 3.14. Let L be a loop with Nuclear-holomorph $H(L)$. L is a extra-SEL if and only if $H(L)$ is an extra-SEL.

Proof. If L is a extra-loop, then by [18], $H(L)$ is a extra-loop. Every extra-loop is a SEL. Hence, $H(L)$ is a extra-SEL. The Converse is true by using the same argument.

Corollary 3.4. Let L be a loop with Nuclear-holomorph $H(L)$. L is a IP-Burn-SIP-SBNL if and only if $H(L)$ is an IP-Burn-SIP-SBNL.

Proof. This follows by combining Theorem 3.11 and Theorem 3.12.

Smarandache : A-loops, homogeneous loops

Theorem 3.15. Every A-loop is a SAL.

Proof. According to [Theorem 2.2, [4]], every subloop of an A-loop is an A-loop. Hence, the conclusion follows.

Theorem 3.16. Let L be a loop with Central-holomorph $H(L)$. L is an A-SAL if and only if $H(L)$ is an A-SAL.

Proof. If L is an A-loop, then by [Theorem 5.3, [4]], $H(L)$ is a A-loop. By Theorem 3.15, every A-loop is a SAL. Hence, $H(L)$ is an A-SAL. The Converse is true by using the same argument.

Corollary 3.5. Let L be a loop with Central-holomorph $H(L)$. L is an homogeneous-SHL if and only if $H(L)$ is an homogeneous-SHL.

Proof. This can be seen by combining Theorem 3.5 and Theorem 3.16.

Smarandache : K-loops, Bruck-loops and Kikkawa-loops

Theorem 3.17. Let $(L,\cdot)$ be a loop with holomorph $H(L)$. $H(L)$ is an AIPL if and only if $x\beta^{-1}J\cdot yJ=(x\cdot y\alpha^{-1})J\ \forall\ x,y\in L$ and $\alpha\beta=\beta\alpha\ \forall\ \alpha,\beta\in Aum(L,\cdot)$. Hence, $xJ\cdot yJ=(z\cdot w)J$, $xJ\cdot yJ=(x\cdot w)J$, $xJ\cdot yJ=(y\cdot w)J$, $xJ\cdot yJ=(z\cdot x)J$, $xJ\cdot yJ=(z\cdot y)J$, $xJ\cdot yJ=(x\cdot y)J$, $xJ\cdot yJ=(y\cdot x)J\ \forall\ x,y,z,w\in S$.

Proof. $H(L)$ is an AIPL $\Leftrightarrow\ \forall\ (\alpha,x),(\beta,y)\in H(L)$, $[(\alpha,x)\circ(\beta,y)]^{-1}=(\alpha,x)^{-1}\circ(\beta,y)^{-1}\Leftrightarrow(\alpha\beta,x\beta\cdot y)^{-1}=(\alpha^{-1},(x\alpha^{-1})^{-1})\circ(\beta^{-1},(y\beta^{-1})^{-1})\Leftrightarrow((\alpha\beta)^{-1},[(x\beta\cdot y)(\alpha\beta)^{-1}]^{-1})=(\alpha^{-1}\beta^{-1},(x\alpha^{-1})^{-1}\beta^{-1}\cdot(y\beta^{-1})^{-1})\Leftrightarrow\alpha\beta=\beta\alpha\,\forall\,\alpha,\beta\in Aum(L,\cdot)$ and $(x(\beta\alpha)^{-1})^{-1}\cdot(y\beta^{-1})^{-1}=[x\alpha^{-1}\cdot y(\alpha\beta)^{-1}]^{-1}\Leftrightarrow Aum(L,\cdot)$ is abelian and $(x(\beta\alpha)^{-1})J\cdot y\beta^{-1}J=[x\alpha^{-1}\cdot y(\alpha\beta)^{-1}]J\Leftrightarrow Aum(L,\cdot)$ is abelian and $(x\alpha^{-1}\beta^{-1})J\cdot y\beta^{-1}J=[x\alpha^{-1}\cdot y\beta^{-1}\alpha^{-1}]J\Leftrightarrow Aum(L,\cdot)$ is abelian and $(x(\beta\alpha)^{-1})J\cdot y\beta^{-1}J=[x\alpha^{-1}\cdot y(\alpha\beta)^{-1}]J\Leftrightarrow Aum(L,\cdot)$ is abelian and $x'\beta^{-1}J\cdot y'J=(x'\cdot y'\alpha^{-1})J$ where $x'=x\alpha^{-1},\ y'=y\beta$.

What follows can be deduced from the last proof.

Theorem 3.18. Let $(L,\cdot)$ be a Bol-SBL with Nuclear-holomorph $H(L)$. $H(L)$ is a Bruck-SBRL if and only if $x\beta^{-1}J\cdot yJ=(x\cdot y\alpha^{-1})J\ \forall\ x,y\in L$ and $\alpha\beta=\beta\alpha\ \forall\ \alpha,\beta\in Aum(L,\cdot)$. Hence,

1. L is a Moufang-SML and a Bruck-SBRL.
2. $H(L)$ is a Moufang-SML.
3. if L is also an A-SAL with Centrum-holomorph $H(L)$ then L is a Kikkawa-SKWL and so is $H(L)$.

Proof. By Theorem 3.12, $H(L)$ is a Bol-SBL. So by Theorem 3.17, $H(L)$ is a Bruck-SBRL $\Leftrightarrow Aum(L,\cdot)$ is abelian and $x\beta^{-1}J\cdot yJ=(x\cdot y\alpha^{-1})J\ \forall\ x,y\in L$.

1. From Theorem 3.17, L is a Bruck-SBRL. From Theorem 3.17, L is an AAIPL, hence L is a Moufang loop since it is a Bol-loop thus L is a Moufang-SML.

2. L is an AAIPL implies $H(L)$ is an AAIPL hence a Moufang loop. Thus, $H(L)$ is a Moufang-SML.

3. If L is also a A-SAL with Centrum-holomorph, then by Theorem 3.5, L and $H(L)$ are both Kikkawa-Smarandache Kikkawa-loops.

Theorem 3.19. Let $(L,\cdot)$ be a SAL with an A-subloop S and Central-holomorph $H(L)$. $H(L)$ is a SKL if and only if $x\beta^{-1}J\cdot yJ = (x\cdot y\alpha^{-1})J\ \forall\ x,y\in S$ and $\alpha\beta=\beta\alpha\ \forall\ \alpha,\beta\in A(S,\cdot)$. Hence, L is a SKL.

Proof. By Theorem 3.16, $H(L)$ is a SAL with A-subloop $H_S = A(S,\cdot)\times(S,\cdot)$. So $H(L)$ is a SKL if and only if H_S is a K-loop $\Leftrightarrow$ $A(S,\cdot)$ is abelian and $x\beta^{-1}J\cdot yJ = (x\cdot y\alpha^{-1})J\ \forall\ x,y\in S\ ,\alpha,\beta\in A(S,\cdot)$ by Theorem 3.17. Following Theorem 3.17, S is an AIPL hence a K-loop which makes L to be a SKL.

References

[1] J. O. Adeniran, On holomorphic theory of a class of left Bol loops, Scientific Annals of Al.I.Cuza. Univ. (to appear).

[2] R. H. Bruck, Contributions to the theory of loops, Trans. Amer. Soc. **55**(1994), 245–354.

[3] R. H. Bruck, A survey of binary systems, Springer-Verlag, Berlin-Göttingen-Heidelberg, 1966, pp. 185.

[4] R. H. Bruck and L. J. Paige, Loops whose inner mappings are automorphisms, The annuals of Mathematics, **63**(1956), No. 2, 308-323.

[5] O. Chein, H. O. Pflugfelder and J. D. H. Smith, Quasigroups and Loops : Theory and Applications, Heldermann Verlag, 1990, pp. 568.

[6] V. O. Chiboka and A. R. T. Solarin, Holomorphs of conjugacy closed loops, Scientific Annals of Al.I.Cuza. Univ. **37**(1991), No.3, 277-284.

[7] V. O. Chiboka and A. R. T. Solarin, Autotopism characterization of G-loops, Scientific Annals of Al.I.Cuza. Univ. **39**(1993), No. 1, 19-26.

[8] J. Dene and A. D. Keedwell, Latin squares and their applications, the English University press Lts, 1974, pp. 549.

[9] E. G. Goodaire, E. Jespers and C. P. Milies, Alternative Loop Rings, NHMS(184), Elsevier, 1996, pp. 387.

[10] E. G. Goodaire and D. A. Robinson, A class of loops which are isomorphic to all their loop isotopes, Can. J. Math. **34**(1982), 662-672.

[11] E. D. Huthnance Jr., A theory of generalised Moufang loops, Ph.D. thesis, Georgia Institute of Technology, 1968.

[12] M. K. Kinyon, K. Kunen and J. D. Phillips, Diassociativity in conjugacy closed loops, Comm. Alg. **32**(2004), 767-786.

[13] M. K. Kinyon, K. Kunen, Power-Associative, conjugacy closed loops, J. Alg. (to appear).

[14] K. Kunen (2000), The structure of conjugacy closed loops, Trans. Amer. Math. Soc. **352**(2000), 2889-2911.

[15] Y. T. Oyebo and J. O. Adeniran, An holomorphic study of Central loops, Pre-print.

[16] H. O. Pflugfelder, Quasigroups and Loops : Introduction, Sigma series in Pure Math. 7, Heldermann Verlag, Berlin, 1990, pp. 147.

[17] D. A. Robinson, Bol loops, Ph. D thesis, University of Wisconsin, Madison, Wisconsin, 1964.

[18] D. A. Robinson, Holomorphic theory of extra loops, Publ. Math. Debrecen **18**(1971), 59-64.

[19] W. B. Vasantha Kandasamy, Smarandache Loops, Department of Mathematics, Indian Institute of Technology, Madras, India, 2002, pp. 128.

[20] W. B. Vasantha Kandasamy, Smarandache Loops, Smarandache notions journal, **13**(2002), 252-258.