LETTERS TO PROGRESS IN PHYSICS

On the Equation which Governs Cavity Radiation

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In this work, the equation which properly governs cavity radiation is presented. Given thermal equilibrium, the radiation contained within an arbitrary cavity depends upon the nature of its walls, in addition to its temperature and its frequency of observation. With this realization, the universality of cavity radiation collapses. The constants of Planck and Boltzmann can no longer be viewed as universal.

Science enhances the moral value of life, because it furthers a love of truth and reverence…

Max Planck, Where is Science Going? 1932 [1]

When Max Planck formulated his law [2, 3], he insisted that cavity radiation must always be black or normal, as first proposed by Gustav Robert Kirchhoff [4, 5]. The laws of thermal emission [2–7] were considered universal in nature. Based on Kirchhoff's law [4, 5], cavity radiation was said to be independent of the nature of the walls and determined solely by temperature and frequency. Provided that the cavity walls were opaque, the radiation which it contained was always of the same nature [2–5]. All cavities, even those made from arbitrary materials, were endowed with this property.

Cavity radiation gained an almost mystical quality and Planck subsequently insisted that his equation had overarching consequences throughout physics. The constants contained within his formulation, those of Planck and Boltzmann (h and k), became fundamental to all of physics, leading to the development of Planck length, Planck mass, Planck time, and Planck temperature [3, p. 175].

However, it can be demonstrated that cavity radiation is not universal, but depends on the nature of the cavity itself [8–15]. As such, the proper equation governing cavity radiation is hereby presented.

It is appropriate to begin this derivation by simply considering Kirchhoff's law [4, 5]

$$\frac{\epsilon_\nu}{\kappa_\nu} = f(T, \nu)\,, \tag{1}$$

where $f(T, \nu)$ is the function provided by Max Planck [2]. As Eq. 1 was hypothesized to be applicable to all cavities, we can adopt the limits of two extremes, namely the perfect absorber and the perfect reflector.

First, the condition under which Kirchhoff's law is often presented, the perfectly absorbing cavity, can be considered (emissivity $(\epsilon) = 1$, absorptivity $(\kappa) = 1$, reflectivity $(\rho) = 0$; at the frequency of interest, ν). In setting κ to 1, it is apparent that the mathematical form of the Eq. 1 remains valid.

Second, if a perfectly reflecting cavity is utilized ($\epsilon = 0$, $\kappa = 0$, $\rho = 1$), it is immediately observed that, in setting κ to 0, Kirchhoff's law becomes undefined. This simple mathematical test indicates that arbitrary cavities cannot be black, as Kirchhoff's law cannot be valid over all conditions. The problem arises because reflection has not been properly included in Kirchhoff's formulation. In this respect, it is possible to invoke Stewart's law of thermal emission [16] which states that, under conditions of thermal equilibrium, the emissivity and absorptivity are equal

$$\epsilon_\nu = \kappa_\nu\,. \tag{2}$$

It is recognized that, for any material, the sum of the emissivity and reflectivity is always equal to 1. This constitutes another formulation of Stewart's law [10, 16] which can also be expressed in terms of absorptivity

$$\epsilon_\nu + \rho_\nu = \kappa_\nu + \rho_\nu = 1\,. \tag{3}$$

As such, let us substitute these relations into Kirchhoff's law (Eq. 1). This mathematical operation is permitted in this exceptional case, since Stewart's law (Eqs. 2, 3) is valid under conditions of thermal equilibrium. The effects of reflection are inserted into Kirchhoff's law. In so doing, the possibility that Eq. 1 can become undefined, when absorptivity is equal to 0, is removed, as reflectivity will be equal to 1. Hence, the following is obtained:

$$\frac{\epsilon_\nu + \rho_\nu}{\kappa_\nu + \rho_\nu} = f(T, \nu)\,. \tag{4}$$

Since, $\kappa_\nu + \rho_\nu$ can be set to 1, with rearrangement, the law for arbitrary cavity radiation under conditions of thermal equilibrium, can be obtained. This law is now properly dependent on the nature of the cavity walls, because it includes the reflectivity observed in real materials

$$\epsilon_\nu = f(T, \nu) - \rho_\nu\,. \tag{5}$$

Dedication

This work is dedicated to our mothers on whose knees we learn the most important lesson: love.

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