Derivation of the Born rule from many-worlds interpretation and probability theory

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Abstract

We try to derive the Born rule from the many-worlds interpretation in this paper.

Although many researchers have tried to derive the Born rule (probability interpretation) from Many-Worlds Interpretation (MWI), it has not resulted in the success. For this reason, derivation of the Born rule had become an important issue of MWI. We try to derive the Born rule by introducing an elementary event of probability theory to the quantum theory as a new method.

We interpret the wave function as a manifold like a torus, and interpret the absolute value of the wave function as the surface area of the manifold. We put points on the surface of the manifold at a fixed interval. We interpret each point as a state that we cannot divide any more, an elementary state. We draw an arrow from any point to any point. We interpret each arrow as an event that we cannot divide any more, an elementary event.

Probability is proportional to the number of elementary events, and the number of elementary events is the square of the number of elementary state. The number of elementary states is proportional to the surface area of the manifold, and the surface area of the manifold is the absolute value of the wave function. Therefore, the probability is proportional to the absolute square of the wave function.

CONTENTS

1 Introduction

1.1 Subject

According to Born rule, the probability of observing a particle is proportional to the absolute square of the wave function. On the other hand, according to the many-worlds interpretation, we observe the particle of the various places in the various events. It is the subject of this paper to derive the Born rule by counting the number of the events.

1.2 The importance of the subject

Wave function collapse and Born rule are principle of the quantum mechanics. We can eliminate the wave function collapse from the quantum mechanics by Many-Worlds Interpretation (MWI), but we cannot eliminate the Born rule.

For this reason, many researchers have tried to derive the Born rule from MWI. However, it has not resulted in the success. Therefore, it has become an important subject to derive the Born rule.

1.3 Past derivation method

Hugh Everett III[2] claimed that he derived the Born rule from Many-Worlds Interpretation (MWI) in 1957. After that, many researchers claimed that they derived the Born rule from the method that is different from the method of Everett. James Hartle[3] claimed in 1968, Bryce DeWitt[4] claimed in 1970 and Neil Graham[5] claimed in 1973 that they derived the Born rule.

However, Adrian Kent pointed out that their method of deriving Born rule was insufficient[6] in 1990. Though David Deutsch[7] in 1999, Sumio Wada[8] in 2007 tried to derive the Born rule, many researchers do not agree the method of deriving the Born rule in 2012.

1.4 New derivation method of this paper

In the probability theory, we explain the probability by the concept of the elementary event. Therefore, we might be able to explain the probability of the quantum theory by the same concept. We try to derive the probability of the quantum theory by introducing a concept of the elementary event to the quantum theory as the new method of this paper.

2 Traditional method of deriving and the problem

2.1 Born rule

Max Born[9] proposed Born rule in 1926. It is also called "probability interpretation." Born rule is a principle of quantum mechanics. We express the state of the particle by the wave function $\psi(x)$ in quantum mechanics. We see an example of a wave function in the following figure.

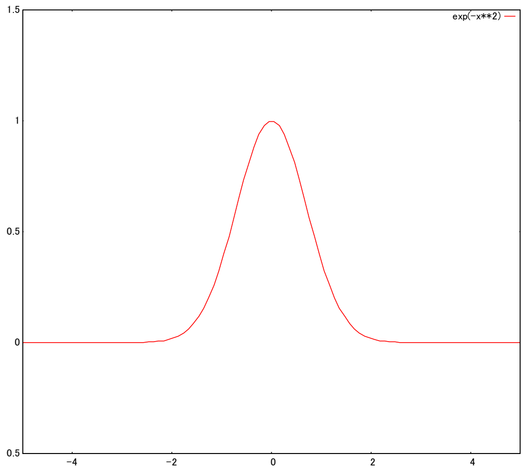

Figure 2.1: An example of a wave function

We observe the particle with a probability that is proportional to the absolute square of the wave function. We write the probability "$P(x)$" of observing the particle at the position "x" as follows.

$$P(x) = |\psi(x)|^2 \tag{2.1}$$

According to the Copenhagen interpretation that is a general interpretation of quantum mechanics, we cannot mention the state of the particle before observation because the wave function does not exist physically. However, the wave function might exist physically. One of the interpretations based on the existence of a wave function is a many-worlds interpretation.

2.2 Everett's many-worlds interpretation

Everett proposed Many-Worlds Interpretation (MWI) in order to deal with the universal wave function. He tried to derive the Born rule from the measure theory.

We express a ket vector "$|\psi>$" in the Hilbert space that represents the state of the system by certain basis vectors "$|\psi_k>$" as follows.

$$|\psi> = \sum_{k=1}^{n} a_k |\psi_k> \tag{2.2}$$

Here, we have normalized "$|\psi>$" and "$|\psi_k>$." The coefficients "a" and "a_k" are complex number. In order to derive the probability Everett introduced a new concept, measure. He expressed the measure by a positive function "m (a)." He requested the following equation for the measure.

$$m(a) = \sum_{k=1}^{n} m(a_k) \tag{2.3}$$

He adduced the probability conservation law to justify the request. We write the function "m (a)" satisfying the above equation by using a positive constant "c" as follows.

$$m(a) = c|a|^2 \tag{2.4}$$

Andrew Gleason[10] generally proved the above equation in 1957. His proof is called "Gleason's theorem." Everett considered the infinite time measurement, and concluded that the measure behaves like the probability. However, MWI of Everett has "basis problem" and "probability problem." I will explain them in the following sections.

2.2.1 Basis problem of many-worlds interpretation

If we define the measure by using a particular basis, we need to show how to select a particular basis. However, Everett did not show how to select a particular basis in his paper.

2.2.2 Probability problem of many-worlds interpretation

Everett tried to derive the Born rule from the measure theory. Then, Everett did not give the physical meaning to the measure. However, to request the conservation law of the probability for the equation of measure is equivalent to define the measure as the probability. Therefore, it is circular reasoning to show that measure acts like a probability for infinite time measurement.

If the number of each world is proportional to the measure, it is necessary to clarify the mechanism by which each number is proportional to the measure of the world. If the number of each world is not proportional to the measure, it is necessary to explain how the probability of occurrence of each world is proportional to the measure.

3 Review of existing ideas

3.1 Universal Wave function of Wheeler and DeWitt

John Wheeler and Bryce DeWitt[11] proposed the Universal wave function in 1967. We have the wave function by the Hamiltonian operator "H" and the ket vector "$|\psi>$" as follows.

$$H|\psi> = 0 \tag{3.1}$$

This ket vector "$|\psi>$" is not a normal function but a functional.

A functional is mathematically almost equivalent to a function of many variables. Since the discussion based on the functional is difficult, we use a function of many variables for discussion in this paper. The following sections describe the many-particle wave function, which is functions of many variables.

3.2 Barbour's many-particle wave function of the universe

Julian Barbour[12] expressed the universe by using the many-particle wave function in his book "The End of Time" in 1999.

We suppose that the number of the particles in the universe is n, and the k-th particle's position is $r_k = (x_k, y_k, z_k)$. Then we express the many-particle wave function ψ as follows.

$$\psi = \psi(r_1, r_2, r_3, \cdots, r_n) \qquad (3.2)$$

The many dimensional space expressing the positions of all the particles is called "configuration space."

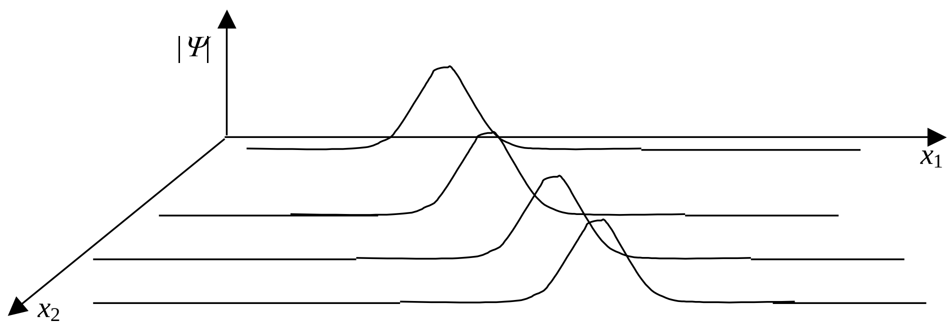

Figure 3.1: Many-particle wave function

The configuration space expresses all the possible worlds that exist physically in the past, the present and the future, because a point in the configuration expresses the positions of all the particles. In other words, many-particle wave function expresses all the possible worlds in many-worlds interpretation.

If the combination of the positions of the all particles of a certain world is decided, the state of the clock of the world will be decided. If the state of the clock of the world is decided, the time of the clock of the world is decided. Therefore, many-particle wave function does not need time as the argument of the function.

The probability P that we observe each world in the configuration space is shown below.

$$P = |\psi(r_1, r_2, r_3, \cdots, r_n)|^2 \qquad (3.3)$$

In order to consider the reason why we express the probability by this equation, we will review the probability theory in the following section.

3.3 Laplace's Probability Theory

Pierre-Simon Laplace[13] summarized the classical probability theory in 1814. He defined probability as follows.

> If equally possible case exists, the probability of the expected event is the ratio of the number of the suitable cases for the expected event to the number of all cases.

This "equally possible case" is an elementary event in probability theory. All the elementary events have a same probability of occurrence.

We suppose that the number of all elementary events is "N_Ω," and the number of elementary events of a certain event "N." Then we express the probability of occurrence of the event "P" as follows.

$$P = \frac{N}{N_\Omega} \propto N \qquad (3.4)$$

$$N \ll N_\Omega \qquad (3.5)$$

For example, we suppose that the five balls are in the bag. Three of five balls are red and two balls are blue. We suppose that the probability of the event that we take out the red ball is P. Then, the probability is 3/5.

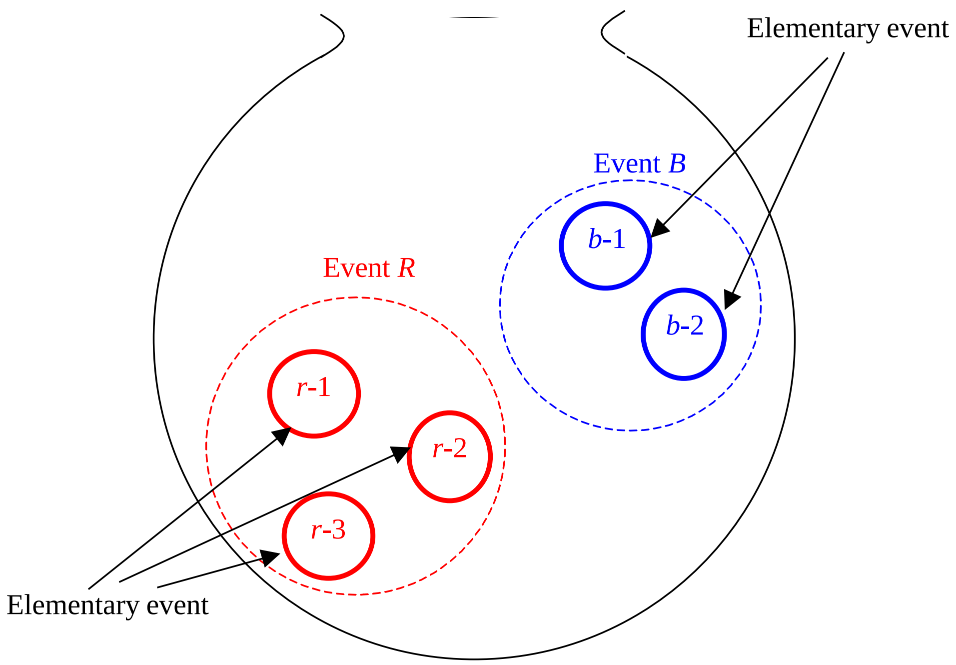

Figure 3.2: Event is a set of elementary events

We explain the reason by the concept of an elementary event. According to the probability theory, we interpret the event that we take out each ball as an elementary event. We interpret an event as a set of elementary events.

In order to derive the Born rule, we need to find "**elementary event**" of quantum theory. An elementary event of probability theory generally we cannot divide any more, so it is expected that an elementary event of quantum theory also cannot be divided anymore.

3.4 Penrose's spin networks

Roger Penrose[14] proposed spin networks in 1971. According to the spin networks, we express the space as a graph with a line that connects a point and the other point. This graph is called "spin network." Since the space-time is discrete, the space-time has a minimum length and minimum time.

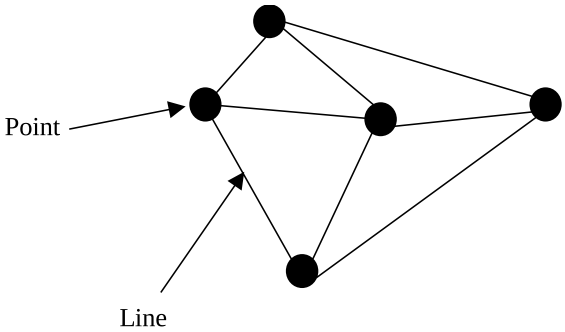

Figure 3.3: Penrose's spin network

In this paper, though we do not use a spin network, we assume that space-time is discrete as well as by this theory and the space is a graph that connects the points. In this paper, we assume that the minimum length is Planck length "ℓ_P" and the minimum time is Planck time "t_P."

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.6 \times 10^{-35}[m] \tag{3.6}$$

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \approx 5.4 \times 10^{-44}[s] \tag{3.7}$$

We call the minimum domain that is constructed by the Planck length "elementary domain."

If the space-time is discrete, we need to review the theory that has been constructed based on the continuous space-time. Therefore, in the next section, we review what happens in the path integral in the case of discrete space-time.

3.5 Feynman's path integral

Richard Feynman[15] proposed path integral in 1948. It provides a new quantization method. In the path integral, we need to take the sum of all the possible paths of the particle.

We express the probability amplitude $K(b, a)$ from the position "a" to the position "b" as follows.

$$K(b,a) = \int_a^b Dx(t)\, exp\left(\frac{i}{\hbar} S[b,a]\right) \tag{3.8}$$

The probability amplitude "$K(b, a)$" is called "propagator." The symbol $Dx(t)$ represents the sum of the probability amplitudes for all paths. The "exp $\left(\frac{i}{\hbar} S[b,a]\right)$" is called the phase factor.

We express the wave functions by the propagator "K (b, a)" as follows.

$$\psi(b, t_b) = K(b, a)\psi(a, t_a) \tag{3.9}$$

In the path integral, an event that a particle moves from a position "a" to the other position "b" is made to correspond to the propagator "K (b, a)". We get the wave function of time "t_b" by multiplying the propagator "K (b, a)" to a wave function of time "t_a."

As shown in the following figure, there is not only a normal path of "α" but also the other path of "β" to travel long distance in short period of time. Such path might have a speed that is greater than the speed of light. Since the path is contrary to the special relativity, the path is not allowed. In this paper, we call such a movement of the path "long-distance transition."

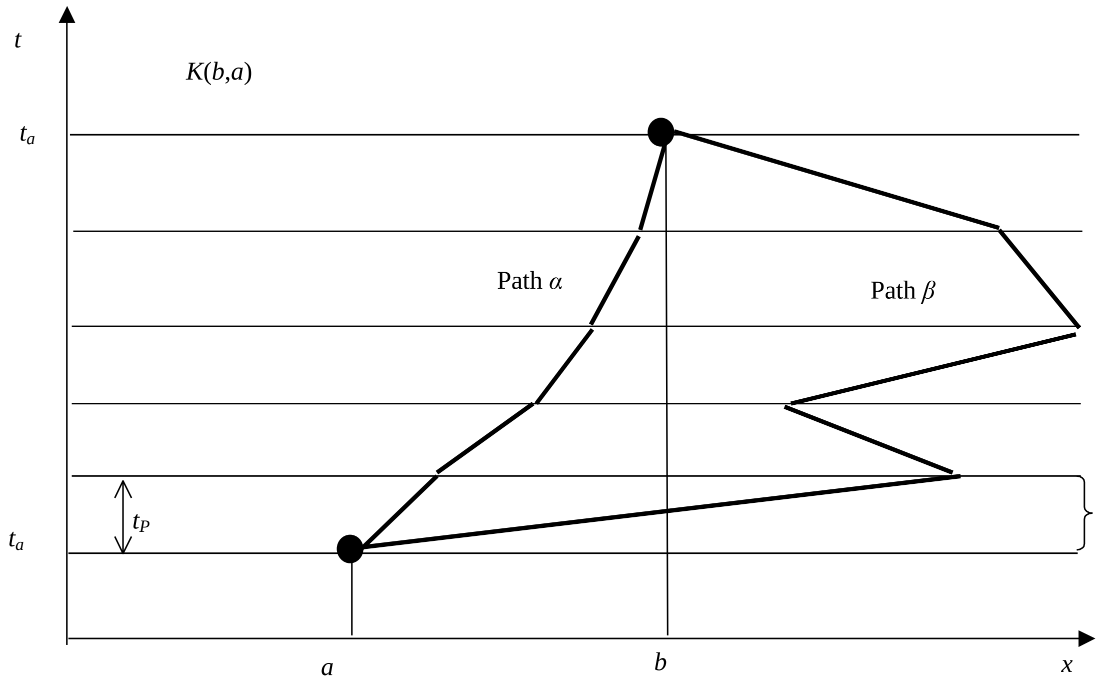

Figure 3.4: Feynman's path integral

Generally, the textbook of a path integral explains as follows.

The sum of a minutely different path near a path of "α" becomes large. On the other hand, the sum of a minutely different path near a path of "β" becomes small. For this reason, long-distance transition is suppressed and the path of "β" does not remain.

Then, what happens after the minimum time "TP"? We express the propagator from a position of "a" to a position of "b" after minimum time "t_P" as follows.

$$K(b, a) = \exp\left(\frac{i}{\hbar} S[b, a]\right) \tag{3.10}$$

In this paper, we assume the discrete time. Since we cannot divide minimum time any more, when the departure point and the point of arrival are decided, it cannot take a minutely different path near a path of "β." For this reason, we cannot suppress long-distance transition and the path of "β" remains.

Therefore, if we apply path integral to the discrete space-time and the position of a particle is determined like a delta function of the Dirac, long-distance transition occurs after the minimum time "t_P."

$$\psi(x', t_b) = K(x', x)\delta(x - a) \tag{3.11}$$

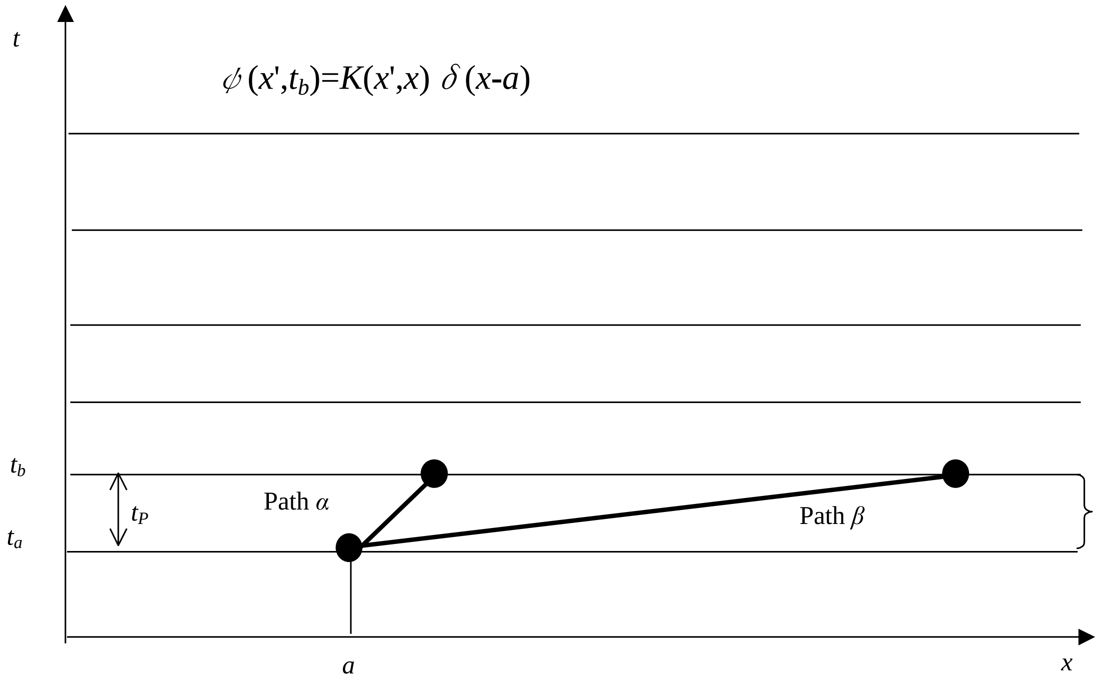

Figure 3.5: Long-distance transition in the path integral

However, we do not observe the long-distance transition. We deduce the reason is that **the position of the particle is distributed with a normal distribution** like the following figure.

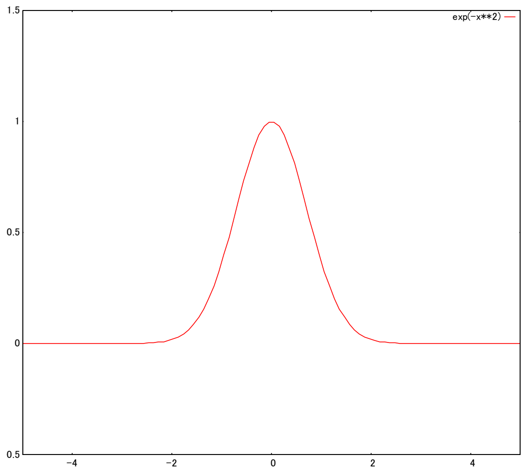

Figure 3.6: A wave function of a localized state

Therefore, position x is distributed with deviation "Δx," momentum "p" is also distributed with deviation "Δp." According to the Uncertainty Principle, the product of "Δx" and "Δp" is close to Planck constant "$\hbar/2$."

$$\Delta x \, \Delta p \approx \frac{\hbar}{2} \tag{3.12}$$

We call the state of the wave function with a normal distribution "**localized state**."

In the short distance, the sum of the phase factor of localized state becomes large. On the other hand, in long distance, the sum of the phase factor of localized state becomes small. We call this phenomenon "**suppression of long-distance transition due to localized states**."

If the state is localized state, the long-distance transition does not occur after the minimum time "TP." Therefore, the localized state is localized near place after the time t_P. For this reason, we deduce that network structure of the path integral is realized, as shown in the following figure.

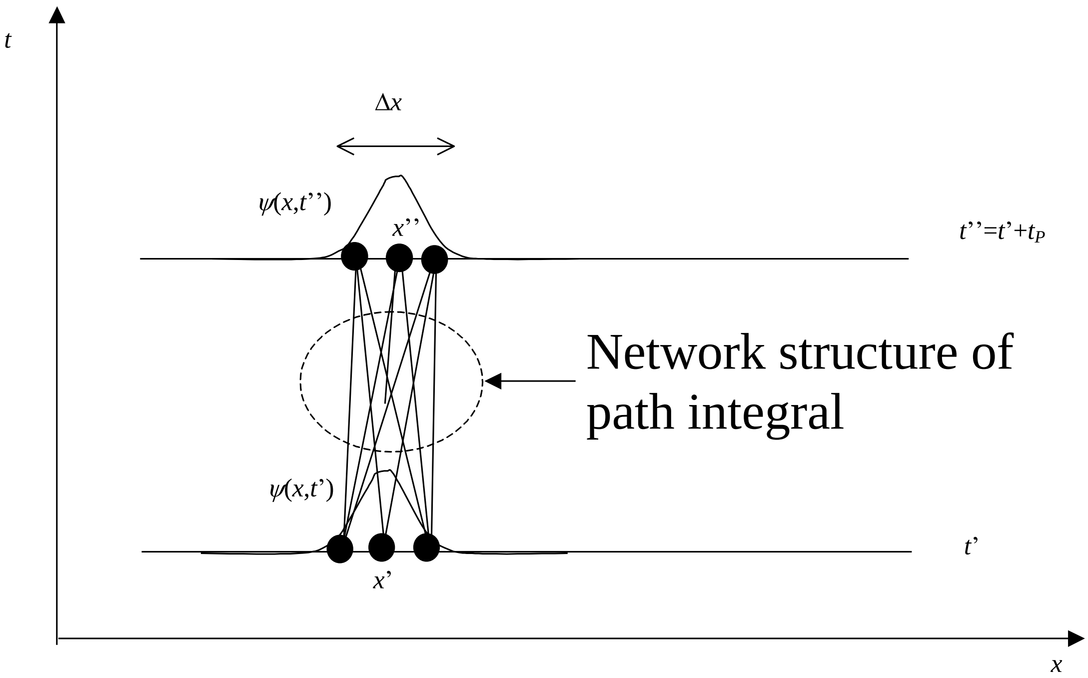

Figure 3.7: Network structure of path integral

In this paper, we call the network structure of path "network structure of path integral."

We suppose that there is an event "*A*-*B*" that is a transition from a state "*A*" to a state "*B*." If the state "*A*" has three positions and the state "*B*" has three positions, the event "*A*-*B*" has 3 × 3 = 9 paths.

In "network structure of the integral path", the number of paths is the square of the number of positions. On the other hand, according to the Born rule, the probability becomes the absolute square of the wave function. In this paper, we discuss the similarities of these "square."

3.6 Dirac's quantum field theory

Paul Dirac[16] proposed the quantum field theory to explain the emission and absorption of electromagnetic waves in 1927. We express the fundamental commutation relation[17] of the quantum field theory in the case of one-dimensional space as follows.

$$[\psi(x), \pi(y)] = i\hbar\delta(x - y) \quad (3.13)$$

Then "ψ" is the field and "π" is the conjugate operator of the field "ψ." The variable "x" and "y" are position. The function "δ" is Dirac's delta function.

This commutation relation is similar to the following commutation relation between position "x" and momentum "p."

$$[x, p] = i\hbar \quad (3.14)$$

This indicates that field "ψ" is a physical quantity that has a property similar to the position "x." In this paper, we call the physical quantity "positional physical quantity."

We got a field "$\psi(x)$" by the first quantization for the position "x." On the other hand, the field $\psi(x)$ is "positional physical quantity" like the position "x." Therefore, we get a new field $\Psi[\psi(x)]$ by the second quantization for the field "ψ." We call the field "$\Psi[\psi(x)]$" "second wave function." We express the second wave function "$\Psi[\psi(x)]$" in the following figure.

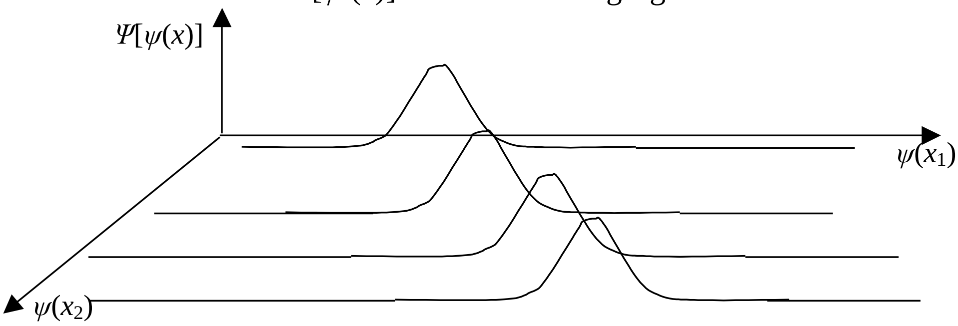

Figure 3.8: The second wave function

If we express the position by using the number n of the points $x_1, x_2, x_3, ..., x_n$ we express the second wave function as a wave of the n-dimensional space. In the view of 3-dimensional normal space, it becomes a functional.

Normally, we need to express the universal wave function by a functional rather than a function of many variables. On the other hand, the functional is mathematically almost equivalent to the function of many variables. Therefore, in this paper, we discuss by using the many-particle wave function in order to simplify the discussion.

3.7 Kaluza-Klein theory

Theodor Kaluza[18] proposed in 1921 and Oskar Klein[19] proposed in 1926 the introduction of extra space that is ultra-fine round, in order to unify the electromagnetic field and gravity. This theory is called "Kaluza-Klein theory."

Though normal space is three-dimensional space, if we consider general relativity in the four-dimensional space, we explain the electromagnetic field.

We express a new space ψ^4 by using a normal space G^3 and an extra space S^1 like a one-dimensional circle as follows.

$$\psi^4 = G^3 \times S^1 \tag{3.15}$$

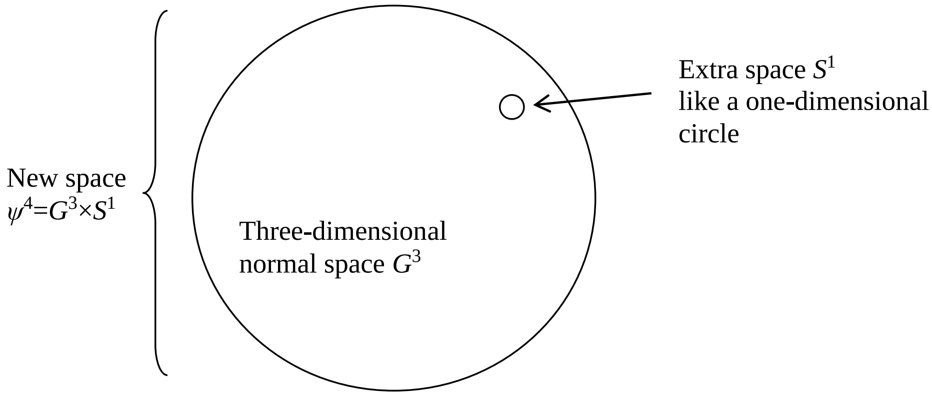

Figure 3.9: Kaluza-Klein theory

We use a term of "space" as a same meaning of a manifold in this paper.

We express the solid angle of the manifold as the subscript or the argument of the manifold in this paper. For example, we express the manifold G^3 specified with the solid angle γ by G_γ or "G (γ)." We also express the manifold S^1 specified with the angle φ by S_φ or "S (φ)."

In this paper, we interpret the symbol "G (γ)" of the manifold as a vector in a high-dimensional space (for example space ψ^4).

In order to interpret the wave function as a manifold we assume that there are extra spaces like Kaluza-Klein theory. In order to interpret the wave function as a manifold we need to define the superposition of the manifold like a superposition of the wave function. Therefore, we consider the superposition of the manifold in the next section.

3.8 Cartan's differential form

Elie Cartan[20] defined differential form in 1899 in order to describe manifold by the method that is independent to the coordinates. In order to define the superposition of the manifold, we use this differential form.

We sum the complex numbers of wave functions every position for the superposition of a wave function. Therefore, we deduce that we sum the surface areas of manifolds at every solid angle for the superposition of manifolds.

We express the surface area S of the manifold ψ by a solid angle ω and surface element "ds (ω)" as follows.

$$S = \int \; ds(\omega) \tag{3.16}$$

However, the surface area S that is integrated with "ds" may become zero because the surface element "ds" of differential form may become negative. To avoid this case, we define the surface area S by the sum of the absolute value of the surface element as follows.

$$S = \int |ds(\omega)| \tag{3.17}$$

We express the surface element $ds\ (\omega)$ by using total differential $h\ (\omega)$ and elementary solid angle $d\omega$ as follows.

$$ds(\omega) = h(\omega)\, d\omega \tag{3.18}$$

Then, we express the surface area of the manifold ψ as follows.

$$S = \int |h(\omega)\, d\omega| \tag{3.19}$$

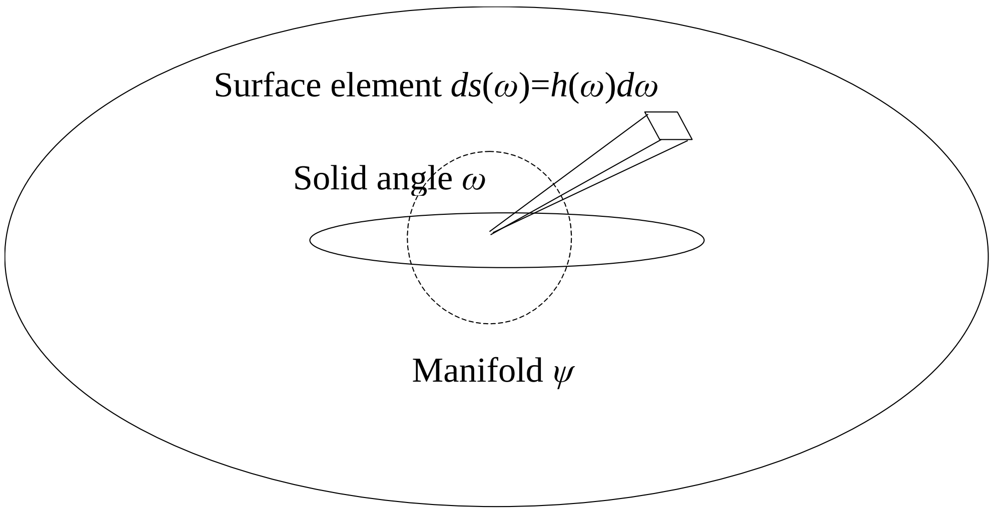

Figure 3.10: Manifold

Now, we express the surface element ds_2 of a manifold ψ_2 as follows.

$$ds_2 = ds_2(\omega) \tag{3.20}$$

Then, we express the surface element ds_3 of the other manifold ψ_3 as follows.

$$ds_3 = ds_3(\omega) \tag{3.21}$$

In order to get a manifold ψ_1, we calculate the superposition of manifold ψ_2 and ψ_3.

$$\psi_1 = \psi_2 + \psi_3 \tag{3.22}$$

We express the surface element ds_1 of the manifold ψ_1 as follows.

$$ds_1(\omega) = ds_2(\omega) + ds_3(\omega) \tag{3.23}$$

We define the superposition of the manifold as the above equation.

4 A new method of deriving

4.1 Elementary event of many-worlds interpretation

In the case of the Copenhagen interpretation, we cannot introduce an elementary event to the quantum theory, because we always observe one event at one observation.

Therefore, we introduce the elementary events to quantum mechanics by embracing the Many-Worlds Interpretation (MWI) in this paper. In MWI all the events those occur in one observation occur. However, one observer cannot observe all the events at the same time, because the observer itself is involved in each event.

If we interpret an event of quantum theory as a set of elementary events, we can derive the probability that each event occurs from the number of the elementary events. If the event R or event B occur in some observations, a world branched to the world that event R occurs and the other world that event B occurs in the MWI.

For example, if the number of elementary events of the event R is three, and B is two, the probability of occurrence of the event R is 3/5. We call a world that an elementary event occurs "elementary world." We interpret a world as a set of elementary worlds.

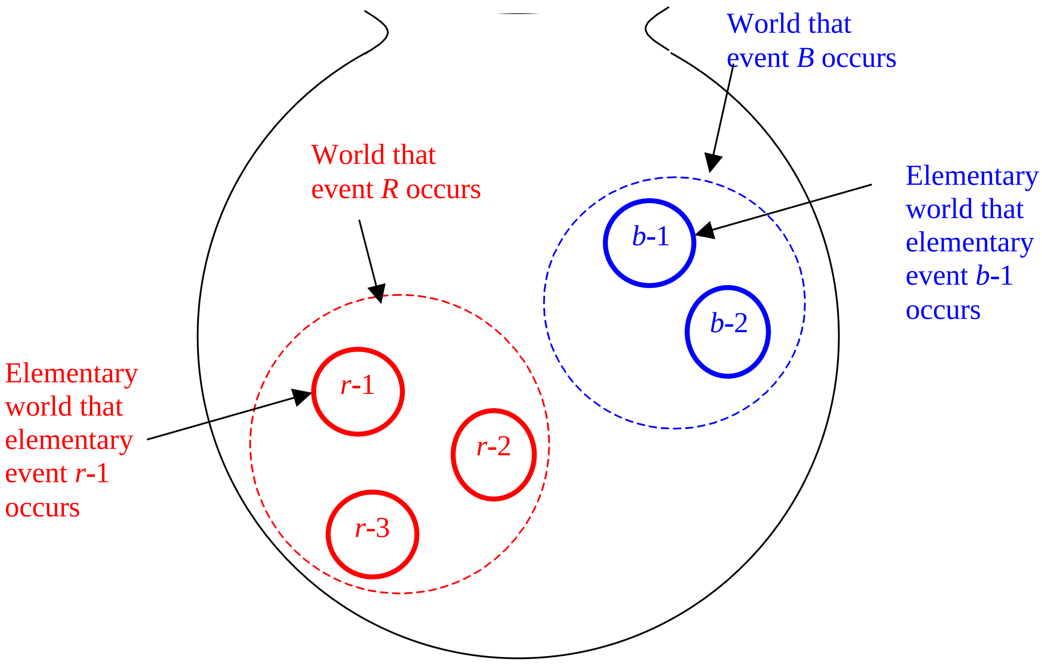

Figure 4.1: World is a set of elementary world in many-worlds interpretation

The concrete implementation method of the elementary events is described in the following sections.

4.2 Elementary state of many-worlds interpretation

In the wave function of a many-particle system in configuration space (many-particle wave function), we call the state that positions of all particles are decided "position eigenstate."

However, in the actual experiment, each particle spreads in the narrow range. Therefore, actual state spreads in the narrow range in configuration space. We call the state "localized state."

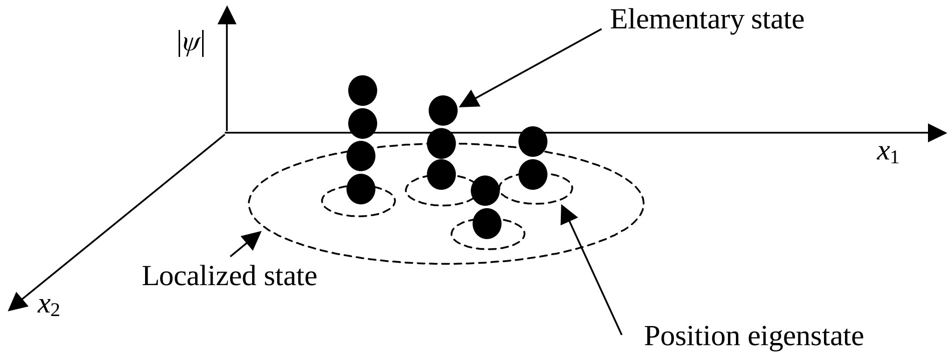

Figure 4.2: Elementary state in configuration space

In addition, we interpret a wave function of "position eigenstate" as a manifold. We interpret an absolute value of the wave function as the surface area of the manifold. We put a point on the surface of the manifold at a fixed interval. We interpret the point as "elementary state." The number of the elementary events is proportional to an absolute value of the wave function, because the number of the elementary events is proportional to the surface area of the manifold.

In the discussion of this paper, there is no difference between the discussion using the many-particle wave function and the discussion using the wave function of one particle. Therefore, in the discussion of this paper, we do not use the many-particle wave function but the wave function of one particle.

4.3 Geometric interpretation of the wave function

In order to consider the geometric interpretation of the wave function, we introduce the extra space like the Kaluza-Klein theory.

Because a complex wave function has two degrees of freedom, two-dimensional manifolds are candidates. There are manifolds with boundary and manifolds without boundary. Because the normal three-dimensional space does not have a boundary, we choose the candidates in the manifolds without boundary. Two-dimensional manifolds without boundary are as follows.

Two-dimensional sphere: S^2
Two-dimensional torus: $S^1 \times S^1 = T^2$
Two-dimensional real projective plane: RP^2
Klein bottle: K^2

A complex number has an absolute value and a phase. Because the phase is periodical quantity, we consider that we use S^1 in order to express the phase. Then we pick up a two-dimensional torus manifold T^2 including the S^1. We express the torus as follows.

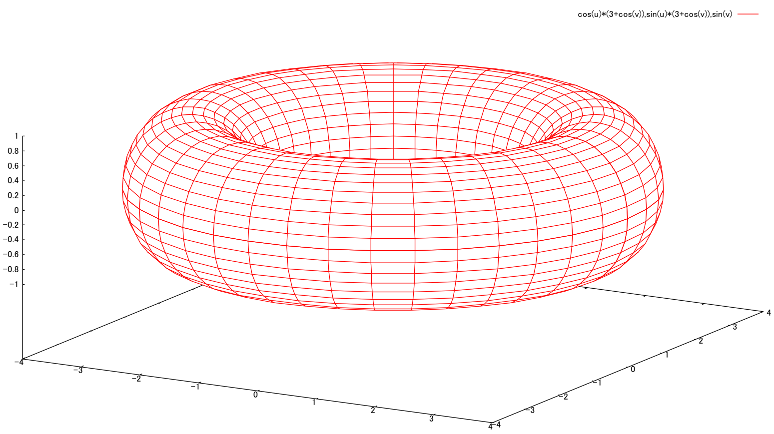

Figure 4.3: Torus

$$X = \cos(\varphi)\left(R_\varphi + R_\alpha \cos(\alpha)\right) \tag{4.1}$$

$$Y = \sin(\varphi)\left(R_\varphi + R_\alpha \cos(\alpha)\right) \tag{4.2}$$

$$Z = R_\alpha \sin(\alpha) \tag{4.3}$$

Wherein coordinates X, Y, Z are used for showing the figure and has nothing to do with the coordinates x, y, z of the normal space.

We call the circle of major radius of this torus "phase circle." We call the circle of minor radius of this torus "amplitude circle."

We suppose that the angle is φ, the position is q, and the radius is R_φ of the phase circle.
We suppose that the angle is α, the position is a, and the radius is R_α of the amplitude circle.

We express the space T^2 as the direct product space of the phase circle and the amplitude circle as follows.

$$T_{\varphi\alpha} = S_\varphi \times S_\alpha \tag{4.4}$$

We call the space "torus space."
We express the position $b = (q, a)$ on the torus space T^2 by the solid angle $\beta = (\varphi, \alpha)$ as follows.

$$b = T(\beta) \tag{4.5}$$

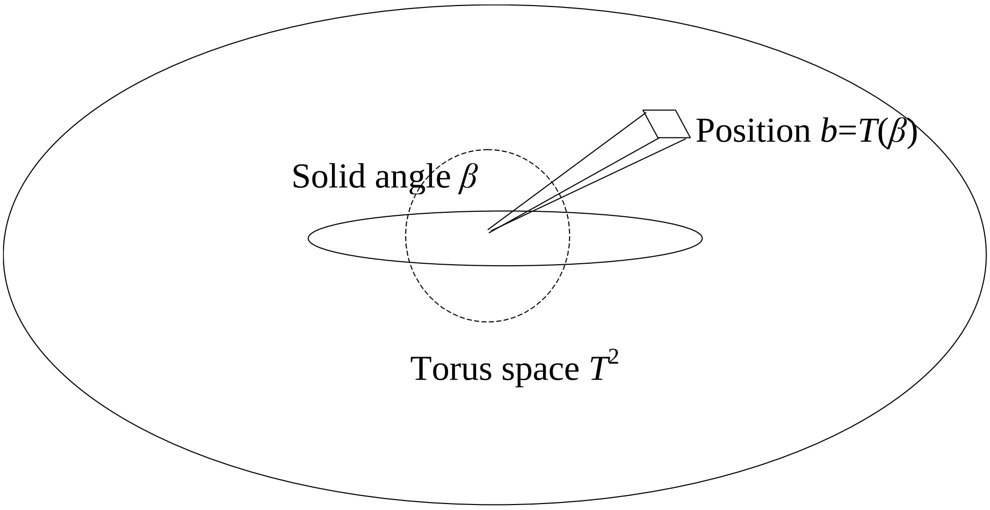

Figure 4.4: Torus space

We express the position $r = (x, y, z)$ of the normal space G^3 by the solid angle $\gamma = (\xi, \eta, \zeta)$ as follows.

$$r = G(\gamma) \tag{4.6}$$

We express the wave space ψ^5 as the direct product space of a torus space T^2 and a normal space G^3 as follows.

$$\psi_{\beta\gamma} = T_\beta \times G_\gamma \tag{4.7}$$

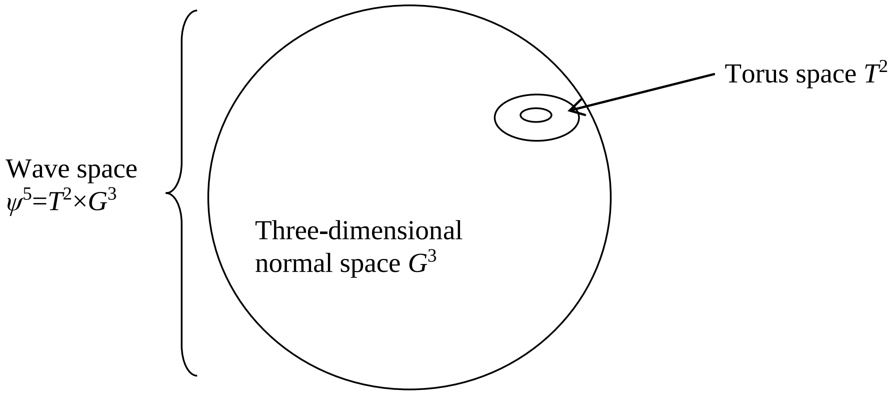

Figure 4.5: Wave space

We express the position $w = (b, r)$ of the wave space ψ^5 by the solid angle $\omega = (\beta, \gamma)$ as follows.

$$w = \psi(\omega) \tag{4.8}$$

We interpret the angle of the phase circle as the phase of the wave function.

The sign of the wave function changes when the phase φ is shifted by π radians. It is shown as follows.

$$\psi(\varphi) = -\psi(\varphi + \pi) \tag{4.9}$$

Therefore, summation of ψ (φ) and ψ ($\varphi + \pi$) become zero.

In the case of the torus $T_{\varphi\alpha}$, the sign of the surface element *ds* does not change though the phase φ is shifted by π radians. It is shown as follows.

$$ds(\varphi, \alpha) = ds(\varphi + \pi, \alpha) \tag{4.10}$$

Therefore, summation of *ds* (φ, α) and *ds* ($\varphi + \pi$, α) does not become "0." For this reason, we cannot interpret the torus as the wave function.

Then, we increase or decrease the radius R_α of the amplitude circle with the proportional to the cosine function of φ. We change the figures and the equation as follows.

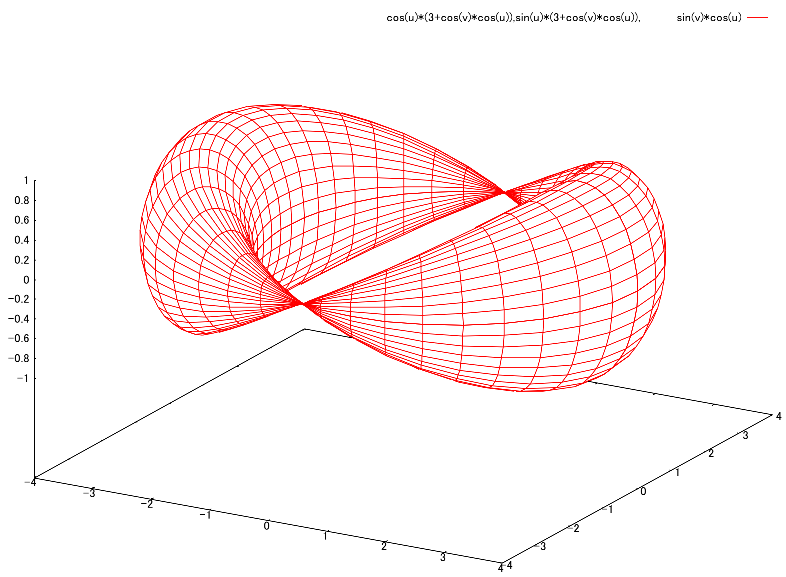

Figure 4.6: Wave function of the particle of spin 1

$$X = \cos(\varphi)\big(R_\varphi + R_\alpha \cos(\alpha) \times \cos(\varphi)\big) \tag{4.11}$$

$$Y = \sin(\varphi)\big(R_\varphi + R_\alpha \cos(\alpha) \times \cos(\varphi)\big) \tag{4.12}$$

$$Z = R_\alpha \sin(\alpha) \times \cos(\varphi) \tag{4.13}$$

In the case of the torus $T_{\varphi\alpha}$, the sign of the surface element *ds* changes when the phase φ is shifted by π radians. It is shown as follows.

$$ds(\varphi, \alpha) = -ds(\varphi + \pi, \alpha) \tag{4.14}$$

Therefore, summation of ds (φ, α) and ds ($\varphi + \pi$, α) becomes zero. For this reason, we can interpret this torus as the wave function.

We might be able to express the wave function of a particle of spin 1 by using the torus.

4.4 Introduction of an elementary state to the quantum theory

We express the wave function ψ (x, t) by Dirac delta function as follows.

$$\psi(x, t) = \int a(y, t)\delta(x - y)dy \tag{4.15}$$

We interpret the state a (y, t) as the state that the position y of the particle is fixed, "position eigenstate." Then we compose the elementary state that cannot be separated any more by dividing the "position eigenstate."

We divided the surface by using "elementary domain" and we suppose that each domain has one elementary state.

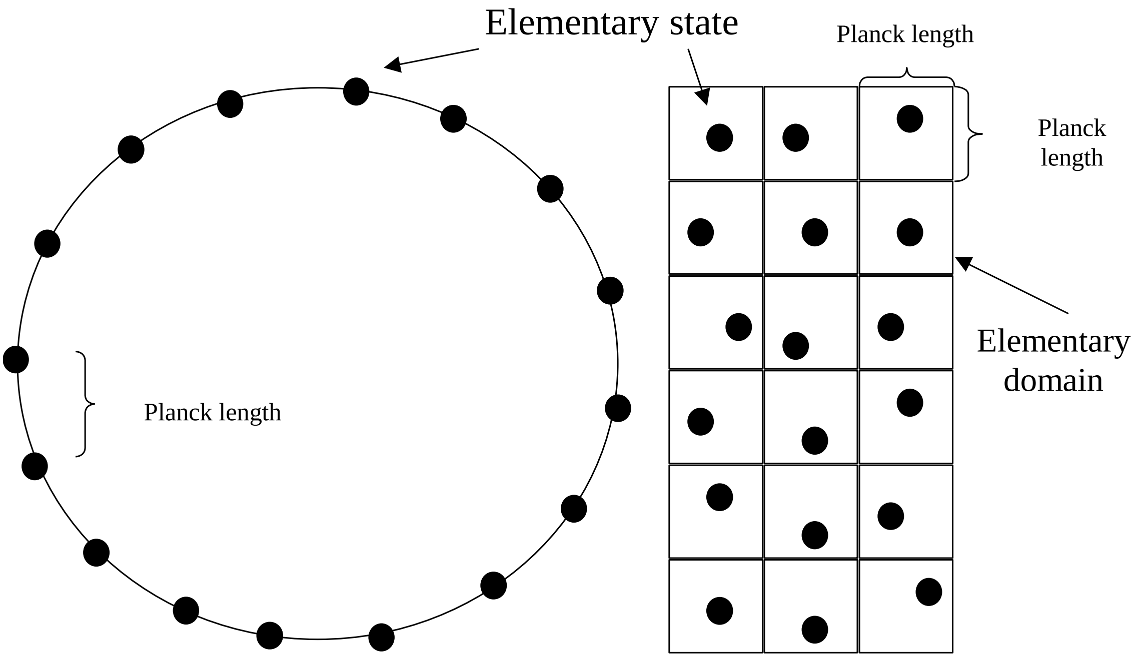

Figure 4.7: Elementary state of many-worlds interpretation

Since the surface area S of the manifold is the absolute of the wave function ψ (x, t), we can describe the number M (x, t) of the elementary state by using Planck length ℓ_P as follows.

$$M(x, t) = \frac{S}{\ell_P^2} = \frac{|\psi|}{\ell_P^2} \tag{4.16}$$

4.5 Application of path integral to the field

In quantum field theory, we quantize the field itself. One method of quantization is the path integral. Therefore, we apply the path integral to the field itself.

We were able to apply the path integral to the position x that is the "positional physical quantity." Therefore, we deduce that we can apply the path integral to the field ψ that is "positional physical quantity." Then, we apply the path integral to the new function $\Psi\,[\psi\,(x, t)]$.

There was a network structure of the path integral for the position x that is "positional physical quantity." Therefore, we apply a network structure of the path integral for the field ψ that is "positional physical quantity" like the following figure.

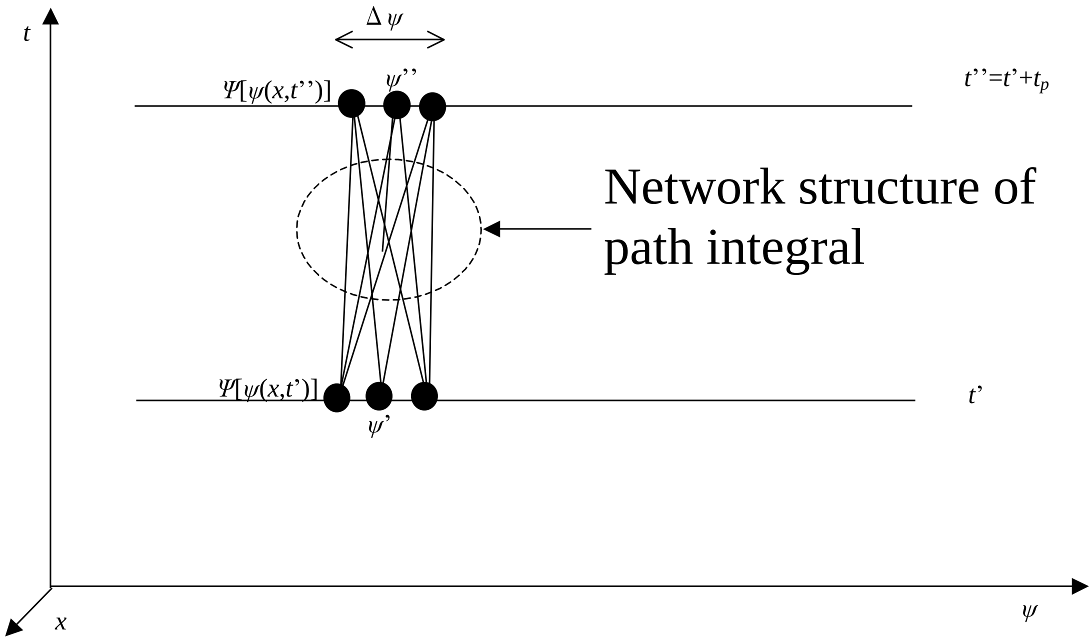

Figure 4.8: Application of network structure of path integral to field itself

We call the space that is specified by position ψ "second wave space" Ψ.

In the above figure, we apply "network structure of the path integral" to the region that is smaller than $\Delta\psi$.

4.6 Introduction of elementary event to the quantum theory

We introduce a new concept, elementary event to the quantum theory in this paper.

We express an event as a transition from one state to the other state in quantum theory. Therefore, we express an elementary event as a transition from one elementary state to the other elementary state.

We interpret an elementary state as a point. We interpret an elementary event as an arrow from a point to the other point. Since we can draw a line from any point to any point, we deduce that an elementary event from any elementary state to any elementary state exists.

If the arrow from the point A to the point B exists, the arrow from the point B to the point A also exists conversely. If the number of points is M, the number of arrows becomes M^2. In other words, if the number of elementary states is M, the number of elementary events becomes M^2.

Though there is no clear evidence of the existence of an elementary event, we deduce it by the following reasons.

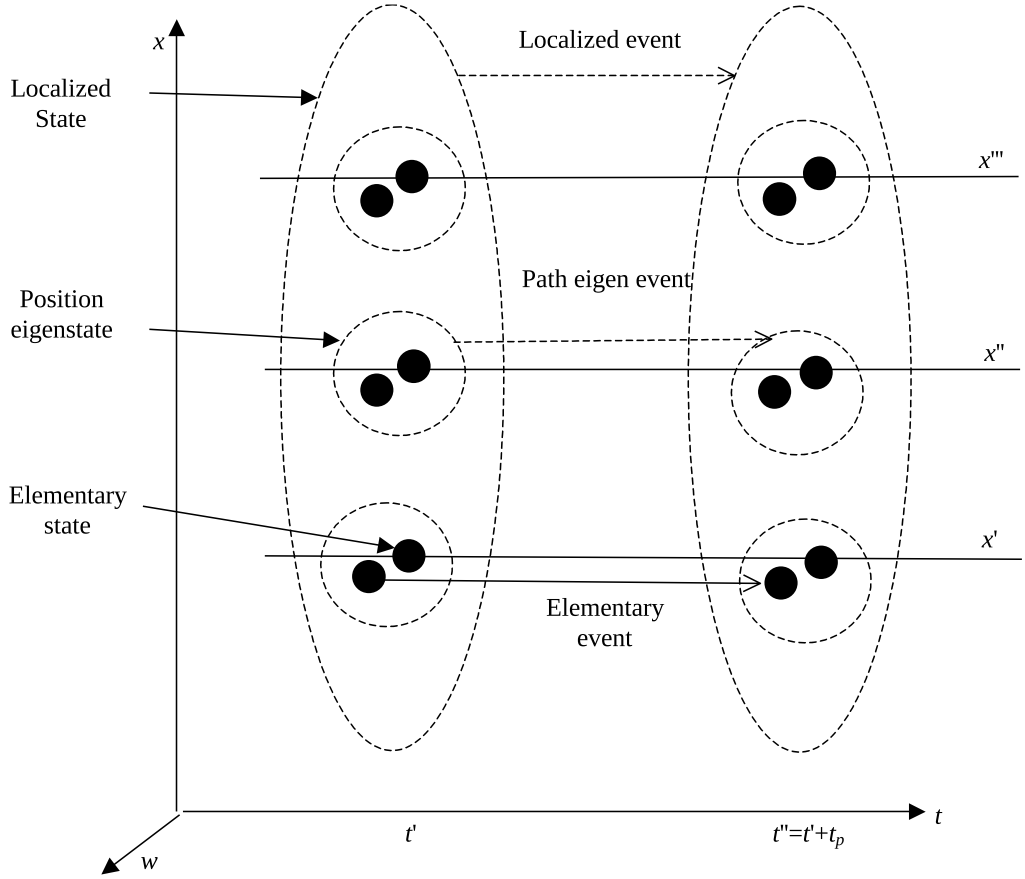

Figure 4.9: Elementary event and elementary state of many-worlds interpretation

We assume that an elementary event of quantum theory has the same properties as elementary events of probability theory. In other words, the probability of occurrence of an event is proportional to the number of elementary events those are included in the event.

In addition, we define the event that is transition from any position eigenstate to any position eigenstate "path eigen event." The path eigen event is a set of elementary events.

Actual state is localized by the uncertainty principle. We call the state "localized state. We call the event from any localized state to any localized state "localized event."

If we apply the path integral to the discrete space-time, the long-distance transition from position eigenstate occurs. However, "long-distance transition" is suppressed due to the localized states. It means that the number of elementary events of localized event that is Long-distance transition is very rare.

The existence of an elementary state and an elementary event suggests that an existence probability and a probability of occurrence are different concepts. If the number of elementary states of a certain state is "*m*," the state's existence probability is proportional to "*m*." If the number of elementary events of a certain event is "*n*," the event's existence probability is proportional to "*n*."

4.7 Derivation of the Born rule

This section describes how to derive this probability.

We express the observation probability P (x, t) of the particle by the wave function ψ (x, t) as follows.

$$P(x,t) = |\psi(x,t)|^2 \tag{4.17}$$

On the other hand, we express the probability P based on the Laplace's definition of probability as follows.

$$P = \frac{N}{N_\Omega} \tag{4.18}$$

Here N_Ω is the number of all elementary events and N is the number f the elementary events those are expected. If N_Ω is sufficiently larger than N, P is proportional to N.

$$P \propto N \tag{4.19}$$

Actual state is localized state. We apply the "network structure of path integral" to the localized state. Since "long-distance transition" does not occur for localized state, the length of transition is small after minimum time t_P.

The number N (x', t') of elementary events of the localized state "ψ (x', t') Δx" is proportional to the surface area of the manifold. We apply the "network structure of path integral" to the position on the surface area of the manifold.

Since the manifold after the minimum time almost same as the original manifold, we approximate it by the same manifold. We express the number M (x, t) of elementary states on the surface area of the manifold as follows.

$$M(x,t) = \frac{S}{\ell_P^2}\frac{\Delta x}{\ell_P} = \frac{|\psi|\Delta x}{\ell_P^3} \tag{4.20}$$

The number N of elementary events is the number of the transition from all elementary states at time t' to all elementary states at time t''. Therefore, the number N of the elementary events is the square of the number M of the elementary states.

$$N = M^2 \tag{4.21}$$

We express those elementary events in the following figure.

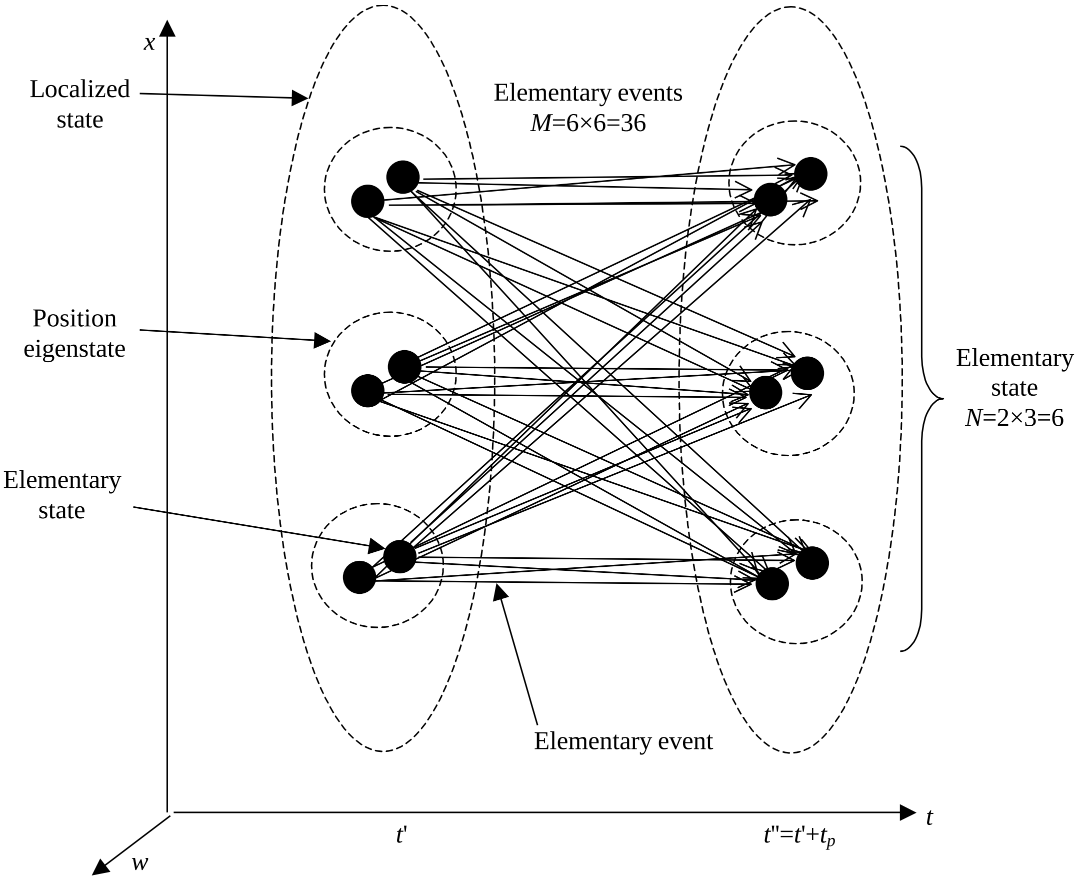

Figure 4.10: The number of elementary events is the square of the number of elementary states

According to the uncertainty principle, deviation Δp of momentum is almost constant if Δx is almost constant. Therefore, the number of elementary events is proportional to the absolute square of the wave function.

$$P \propto N = M^2 = \left(\frac{S}{\ell_P^2}\frac{\Delta x}{\ell_P}\right)^2 = \frac{(\Delta x)^2|\psi|^2}{\ell_P^6} \propto |\psi|^2 \qquad (4.22)$$

The probability of occurrence of an event is proportional to the number of the elementary events that is involved in the event. The number of the elementary events is proportional to the absolute square of the wave function. Therefore, the probability of occurrence of an event is proportional to the square of the absolute value of the wave function.

5 Conclusion

We explained the method to derive the Born rule from many-worlds interpretation and probability theory.

Probability is proportional to the number of the elementary events. The number of the elementary events is the square of the number of elementary state because we apply the "network structure of path integral" to the elementary state. The number of the elementary states is proportional to the absolute value of the wave function. Therefore, the probability is proportional to the absolute value of the wave function.

6 Supplement

6.1 Supplement of the many-particle wave function

We call an elementary state, a position eigenstate and a localized state for the universe "elementary world", "position eigen world" and "localized world" respectively.

In addition, we call an elementary event, a path eigen event and a localized event for the universe "elementary history", "path eigen history" and "localized history" respectively.

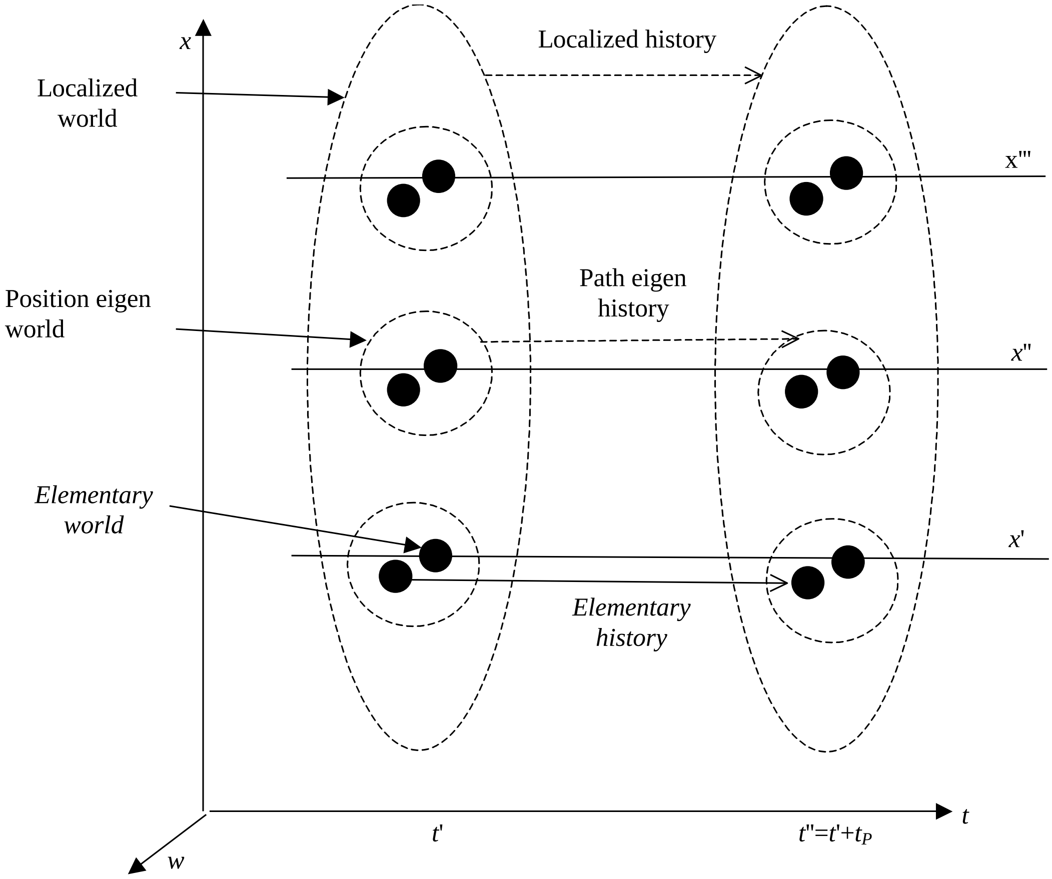

Figure 6.1: Elementary history and elementary world of many-worlds interpretation

We interpret one point of the configuration space of the many-particle wave function as the state that the positions of all particles are determined. The state is "position eigen world."

In the view of classical mechanics, the point is our world. In the view of the quantum mechanics, localized world is our world.

I guess that the absolute value of the many-particle wave function of the universe is most nearly zero in the almost area. The domain that the absolute value is large is localized like a network structure.

6.2 Supplement of the method of deriving the Born rule

The simplest way to derive the Born rule from Many-Worlds Interpretation (MWI) is that we connect the number of the world to the probability.

If the probability of occurrence of event *A* is higher than the probability of occurrence of event *B*, we deduce that the number of the world that event *A* occurred is greater than the number of the world that event *B* occurred.

For example, we suppose that we make the 100 planets those are exactly same as Earth. If the event *A* occurred on 80 planets and the event *B* occurred on 20 planets, then we interpret that the probability of the occurrence of the event *A* is 80%.

However, it is not clear how to count the world. Therefore, we count the number of elementary worlds of the localized world that event *A* occurred.

We express the number *M* of elementary worlds of the localized world by the wave function ψ (A) that event *A* occurred as follows.

$$M = \frac{|\psi(A)|}{\ell_P^2} \times \frac{(\Delta x)^{3n}}{\ell_P^{3n}} = \frac{|\psi(A)|(\Delta x)^{3n}}{\ell_P^{2+3n}} \tag{6.1}$$

Δx is the position deviation, and n is the number of all particles. The number of elementary world is proportional to the absolute value of the wave function. On the other hand, the probability is proportional to the absolute square of the wave function. Therefore, we cannot explain the probability by using the number of the elementary worlds.

To solve this problem, we explain the probability by using the number of the history. We express the number *N* of the elementary history of the localized history that event *A* occurred as follows.

$$N = M^2 \tag{6.2}$$

The probability is proportional to the number of the elementary history. The number of the history is the square of the number of the elementary world. On the other hand, the number of the elementary worlds is proportional to the absolute values of wave functions. Therefore, the probability is proportional to the absolute square of wave functions.

$$P \propto N = M^2 = \left(\frac{|\psi(A)|(\Delta x)^{3n}}{\ell_P^{2+3n}}\right)^2 = \frac{(\Delta x)^{6n}}{\ell_P^{4+6n}} |\psi(A)|^2 \propto |\psi(A)|^2 \tag{6.3}$$

6.3 Supplement of basis problem in many-worlds interpretation

In many-worlds interpretation, there is a problem that a particular basis of the wave function does not exist.

For example, we consider the Stern-Gerlach experiment of the spin of electrons. In this experiment, we measure the spin by using a magnetic field gradient. Since the basis of the spin is determined by the direction of the gradient magnetic field, there is no particular basis for the spin.

In this paper, we chose position as the particular basis. We could also choose the momentum as the particular basis, but we did not do so, because we express the basis of the momentum by using a set of the elementary state that the position is basis.

For spin, there is no way to select a particular basis. In this paper, we are considering the manifold of a particle of spin 1/2. We might be able to express the spin by using the manifold.

6.4 Interpretation of time in many-worlds interpretation

The position of all particles is different for each point in the configuration space of many-particle wave function. Therefore, we define the time for each point in the configuration space. Since a point corresponds to a position eigen world, we interpret the time as a parameter to classify the position eigen worlds.

A position eigen world transits the minimum length continuously in the configuration space. I guess that we feel the transition as a time.

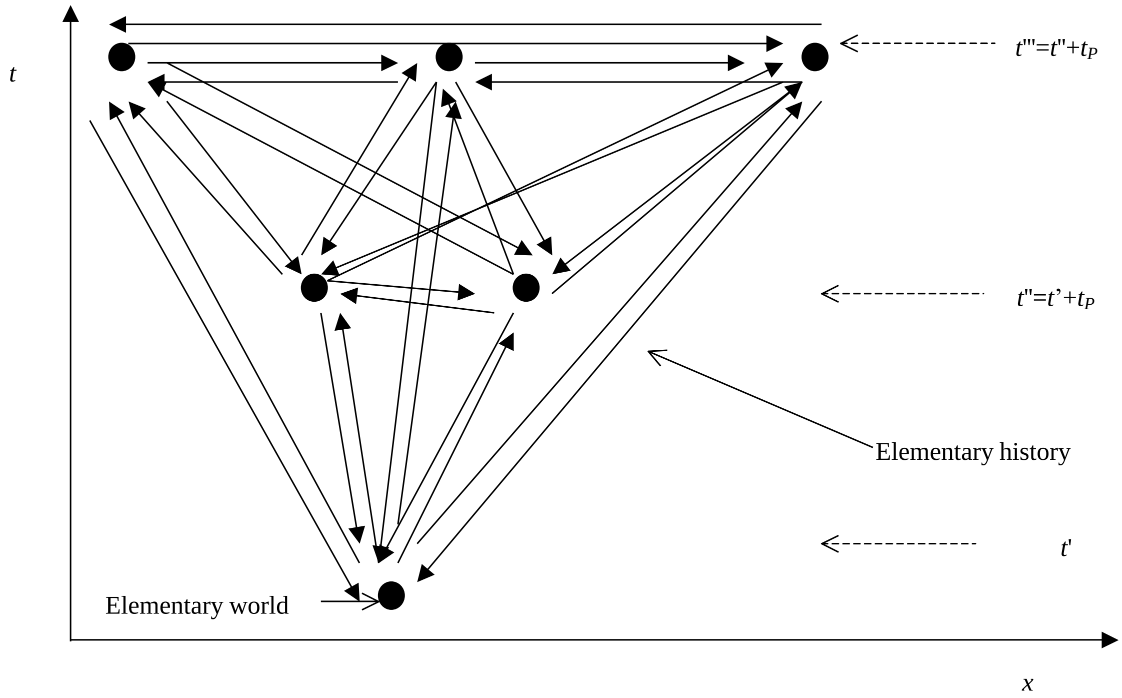

Figure 6.2: Many-worlds interpretation and arrow of time

If a transition of a direction exists, the transition of the opposite direction also exists. However, since there are many "elementary worlds" of future more overwhelmingly than the number of elementary worlds of past, we feel that our elementary world always transits to elementary world of the future. In this way, many-worlds interpretation explains the arrow of time by.

6.5 Supplement of Long-distance transition

In this paper, we have been thinking about one particle is localized in one place. Here we consider the wave function of one particle that was localized in one place at a certain time. We suppose that the wave function was separated and localized in two places. We call the state "many localized states." In this case, what would happen?

Elementary event exists between any two elementary states. The world does not become disorder because long-distance transition is suppressed due to the "localized state". We determine the number of elementary events between two localized states only by the number of elementary states of the two localized states.

Therefore, if there are "many localized states", the transition between the states those are localized in two places will occur.

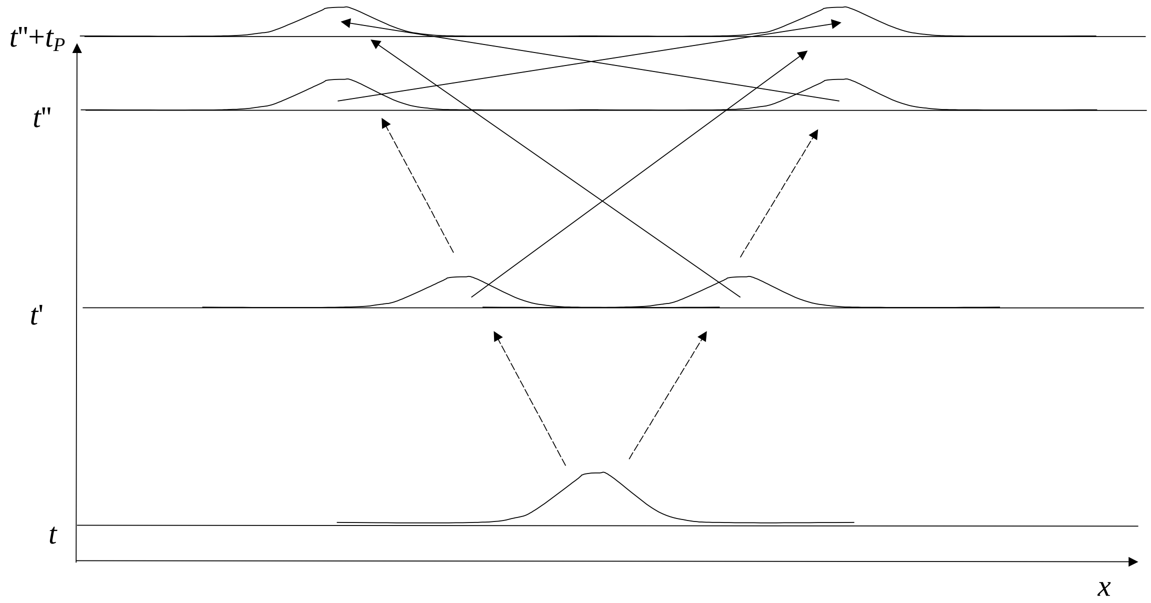

Figure 6.3: Long-distance transition between localized states

I call the phenomenon "localized long-distance transition" or "localized shift."

Then, will localized shift between localized states those have different time occur?

In this case, since the elementary event exists between any two elementary states, the localized shift occurs, too.

I do not deduce that the localized teleport send information, because we cannot send any information by using EPR correlation.

7 Future Issues

Future issues are shown as follows.

(1) Formulation by the manifold
(2) Formulation for the quantum field theory
(3) Formulation by the finite group theory
(4) Formulation by the principle
(5) Formulation for the spin
(6) Formulation for the relativistic mechanics
(7) Formulation for the gravity theory

We consider some of these issues in the following chapters.

8 Consideration of the future issues

8.1 Consideration of the formulation by the manifold

Since the wave space $\psi_{\varphi\alpha\gamma}$ which is the direct product space of torus space $T_{\varphi\alpha} = S_\varphi \times S_\alpha$ and normal space G_γ is a function of φ, α and γ, we express ψ $(\varphi, \alpha, \gamma)$ like a function.

$$T_{\varphi\alpha} = S_\varphi \times S_\alpha \tag{8.1}$$

$$\psi_{\varphi\alpha\gamma} = T_{\varphi\alpha} \times G_\gamma \tag{8.2}$$

$$\psi_{\varphi\alpha\gamma} = \psi(\varphi, \alpha, \gamma) \tag{8.3}$$

We express the surface S (γ) in the position γ of the wave space ψ (φ, α, γ) as follows.

$$S(\gamma) = \int \; |h(\omega)\, d\omega| \tag{8.4}$$

$$h(\omega, \gamma) = \psi(\varphi, \alpha, \gamma) \tag{8.5}$$

$$\omega = (\varphi, \alpha) \tag{8.6}$$

When the radius of the angle α becomes the maximum value for the angle φ, we interpret the angle φ as the angle $\varphi_{\max}$ (γ). Then we express the wave function of ψ (γ) as follows.

$$\psi(\gamma) = S(\gamma) \exp\big(i\, \varphi_{max}(\gamma)\big) \tag{8.7}$$

The above result means that we can interpret the wave space as the wave function.

8.2 Consideration of the formulation for the quantum field theory

Position eigenstate includes information on the absolute value of the wave function and the phase information. On the other hand, the elementary state does not include the information about the absolute value of the wave function and the phase information. Therefore, for the elementary state, we cannot use the logic of "deterrence of long-distance transition localized states." In order to solve this problem, consider the quantum field theory. The theory is also called "second quantization."

In the quantum field theory, we set the commutation relation to the height of the wave function itself. This means that the height of the wave function spread. Therefore, the new wave function, "second wave function" $\Psi\,[\psi\,(r)]$ exists. It has an argument of the height of the wave function.

We interpret "an elementary state" of this paper as the position eigenstate of the second wave function. Therefore, we call the new position eigenstate "second position eigenstate." If we define "second position eigenstate", we can also define "second localized state." Thus, we can use "suppression of long-distance transition due to localized states", again.

However, if we require the "third position eigenstate" and "fourth position eigenstate", the logic becomes infinite regress. I guess we can solve the problem by using the formulation of a finite group.

If the second wave function $\Psi\,[\psi\,(r)]$ exists, the second wave space Ψ exists. We express the second wave space Ψ by the second torus T_β and a wave space ψ_ω. This is a functional.

$$\Psi = T_\beta[\psi_\omega] \tag{8.8}$$

When the number of the points is d in the wave space ψ_ω, we can interpret the above function as follows.

- The second wave function Ψ is the vector function in the two-dimensional space T_β.
- The argument of the vector function is the vector ψ_ω in the d-dimensional space.

As the result, we interpret the second wave function Ψ as the vector in the (d + 2) dimensional space. We interpret the functional as the vector function that has a vector as the argument.

8.3 Consideration of the formulation by the finite group

We consider how to formulate mathematically elementary state. Because the elementary state is defined in the discrete wave space, we consider the formulation of a finite group.

In this paper, we express the group specified the index m as W_m. When the order of the group is M_w, we interpret the group W_m as the M_w-dimensional vector M_w.

We suppose that there is a finite group W_m that has M_w elements and acts to d-dimensional discrete wave space V_m. We call this finite group "wave group."

We express m-th element of the wave group W_m as d-dimensional matrix w_m. Then, w_1 is the unit matrix. We express m-th point of the wave space V_m as v_m. If we act all matrix w_m to one point v_1 of the wave space V_m, we get all points of the wave space V_m.

$$w_m \in W_m \tag{8.9}$$

$$v_m \in V_m \tag{8.10}$$

$$v_\mathrm{m} = w_m v_1 \tag{8.11}$$

We interpret the point as an elementary state because the point v_m is a point of the wave space V_m. We also interpret the matrix w_m as an elementary state because a point v_m corresponds to a matrix w_m uniquely.

Because the elementary event is a transition from any elementary state to any elementary state, we can interpret a pair of a point v_m and v_n as an elementary event. We can also interpret a pair of matrix w_m and w_n as an elementary event. Therefore, we can express all elementary events by the direct product of W_m and W_n, a finite group H_{mn} as follows.

$$W_{mn} = W_m \times W_n \tag{8.12}$$

We call the finite group W_m "world group." We call the finite group H_{mn} "history group." The number of the elementary events is $M_w{}^2$.

8.4 Consideration of the construction by the principle

We construct "event theory" by "event principle."

8.4.1 Event principle

We propose the following "event principle."

- A state is a set of elementary states.
- An event is a set of elementary events.
- An elementary event is a pair of elementary states.
- A certain state's existing probability is proportional to the number of elementary states the state includes.
- A certain event's occurring probability is proportional to the number of elementary events the event includes.

8.5 Consideration of the formulation for spin

In this section, we consider the manifold of the wave function of the particle of the spin 1/2.

A particle rotates in the normal space G_γ. We suppose that the rotation angle of the spin θ. We express the rotation angle θ by the circle S_θ. We call the circle "normal circle." The direct product of the phase circle S_φ and the amplitude circle S_α and the normal circle S_θ is shown as follows.

$$\psi_{\varphi\alpha\theta} = S_\varphi \times S_\alpha \times S_\theta \tag{8.13}$$

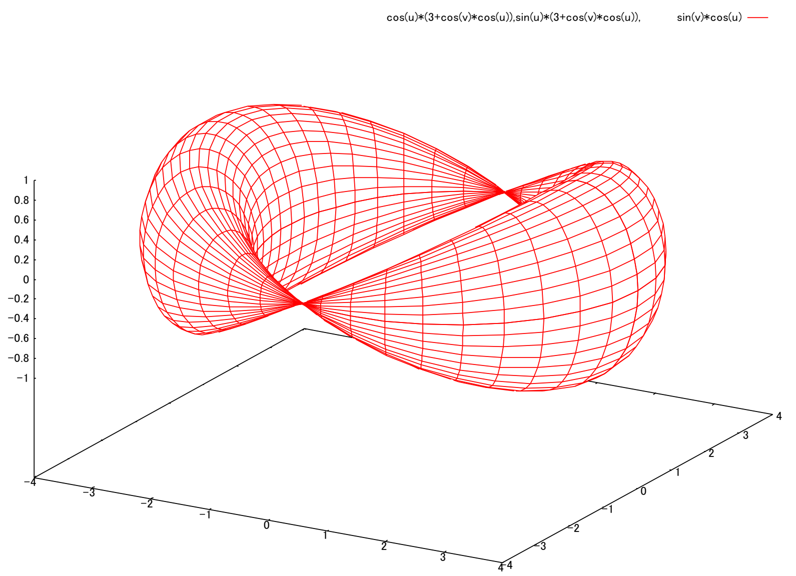

Figure 8.1: Direct product of circle and torus

$$X = \cos(\varphi)\big(R_\varphi + R_\alpha \cos(\alpha) \times \cos(\varphi + \theta)\big) \tag{8.14}$$

$$Y = \sin(\varphi)\big(R_\varphi + R_\alpha \cos(\alpha) \times \cos(\varphi + \theta)\big) \tag{8.15}$$

$$Z = R_\alpha \sin(\alpha) \times \cos(\varphi + \theta) \tag{8.16}$$

The sign of the wave function changes when the particle of the spin 1/2 rotates the angle $\theta = 2\pi$ in the normal space. The change of the sign of the wave function means the manifold is turned inside out. Though the particle rotates the angle $\theta = 2\pi$, the manifold is not turned inside out. Therefore, this manifold does not express the wave function of the particle of spin 1/2.

In order to express the wave function of the particle of spin 1/2, we divide the angle φ and θ by 2.

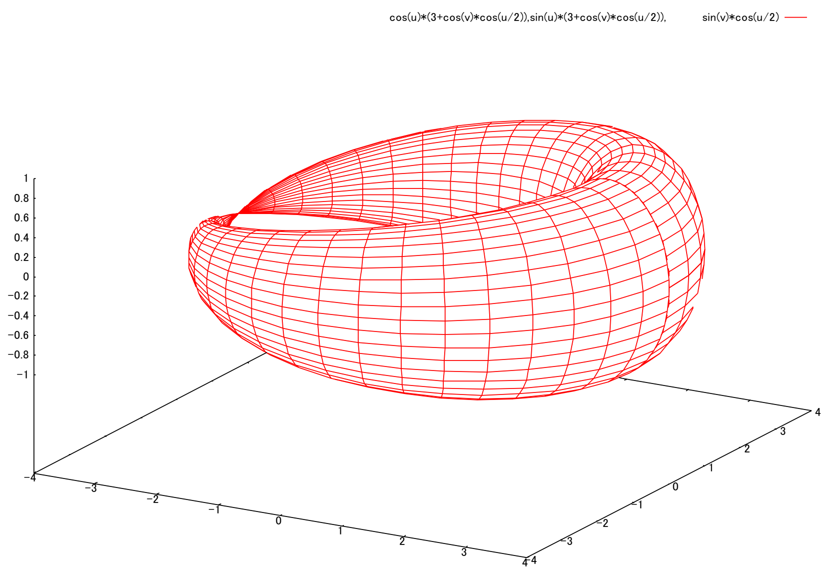

Figure 8.2: Wave function of the particle spin 1/2

$$X = \cos(\varphi)\left(R_\varphi + R_\alpha \cos(\alpha) \times \cos\left(\frac{\varphi}{2} + \frac{\theta}{2}\right)\right) \tag{8.17}$$

$$Y = \sin(\varphi)\left(R_\varphi + R_\alpha\, cos(\alpha) \times \cos\left(\frac{\varphi}{2} + \frac{\theta}{2}\right)\right) \tag{8.18}$$

$$Z = R_\alpha \sin(\alpha) \times \cos\left(\frac{\varphi}{2} + \frac{\theta}{2}\right) \tag{8.19}$$

When the node of the manifold rotates the angle $\theta = 2\pi$, the manifold is turned inside out. Therefore, this manifold might express the wave function of the particle of spin 1/2.

9 Appendix

9.1 Definition of Terms

We define terms in the following table.

Table 9.1: Normal space, etc.

TERM	DEFINITION
Normal space	Three-dimensional normal space
Phase circle	Extra space like circle that describes the phase of the wave function
Amplitude circle	Extra space like circle that describes the amplitude of the wave function
Torus space	Direct product space of the phase circle and the amplitude circle
Wave space	Direct product space of the torus space and the normal space

Table 9.2: Elementary domain, etc.

TERM	DEFINITION
Elementary domain	The minimum domain of the wave space
Elementary position	Position of the wave space
Elementary path	An arrow from any elementary position to any elementary position
Normal domain	The minimum domain of the normal space
Normal position	Position of the normal space
Normal path	An arrow from any normal position to any normal position

Table 9.3: Elementary state, etc.

TERM	DEFINITION
Elementary state	Point of the wave space
Position eigenstate	State that the position is fixed
Localized state	State that the distribution is a normal distribution
Elementary event	A transition from any elementary state to any elementary state
Path eigen event	A transition from any position eigenstate to any position eigenstate
Localized event	A transition from any localized state to any localized state
Elementary world	Elementary state of the universe
Position eigen world	Position eigenstate of the universe
Localized world	Localized states of the universe
Elementary history	Elementary event of the universe
Path eigen history	Path eigen event of the universe
Localized history	Localized events of the universe

Table 9.4: Wave space, etc.

TERM	DEFINITION
Wave space	Direct product space of the torus space and the normal space
World space	Wave space
History space	Direct product of the wave space
Wave group	The finite group which acts on the wave space
World group	Wave group
History group	Direct product of the world groups

Table 9.5: Localized displacement, etc.

TERM	DEFINITION
Localized displacement	Localized transition of short-distance
Localized transition	Localized transition
Localized shift	Localized transition of long-distance
Localized teleportation	Localized transition of ultra-long-distance

9.2 Arrangement of Terms

We arrange terms in the following table.

Table 9.6: Wave space, etc.

CATEGORY	SPACE	GROUP
Normal	Normal space	-
Phase	Phase circle	-
Amplitude	Amplitude circle	-
Torus space	Torus space	Torus group
Wave space	Wave space	Wave group

Table 9.7: Elementary domain, etc.

CATEGORY	ELEMENTARY	NORMAL
Domain	Elementary domain	Normal domain
Position	Elementary position	Normal position
Path	Elementary path	Normal path

Table 9.8: Wave space, etc.

CATEGORY	ELEMENTARY	EIGEN	LOCALIZED
State	Elementary state	Position eigenstate	Localized state
Event	Elementary event	Path eigen event	Localized event
World	Elementary world	Position eigen world	Localized world
History	Elementary history	Path eigen history	Localized history

Table 9.9: Wave space, etc.

CATEGORY	FUNCTION	SPACE	FINITE GROUP
Wave	Wave function	Wave space	Wave group
World	World function	World space	World group
History	History function	History space	History group

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