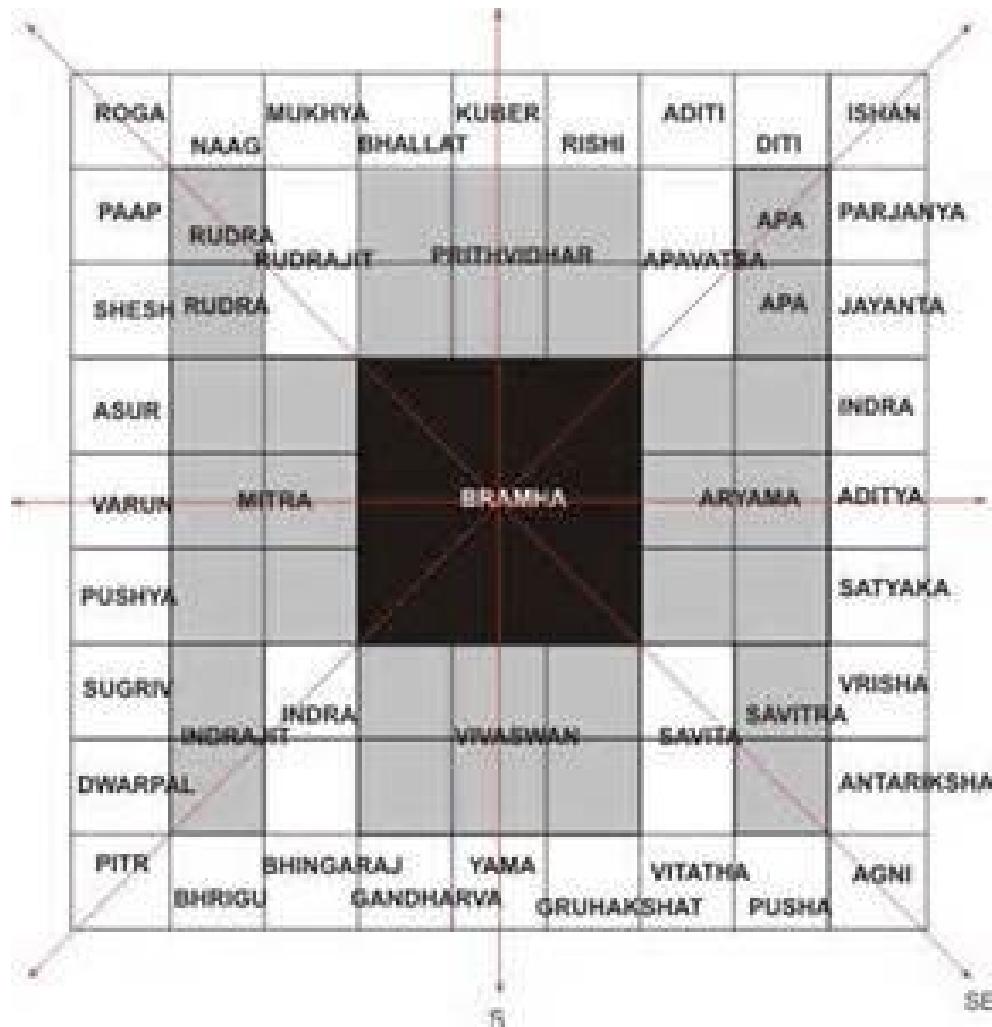


# 32 Cosmic Energies and their Clifford Algebras

By John Frederic Sweeney



Abstract

In a recent paper, the author explained the function and purpose of Brahma in Vedic Nuclear Physics, in relation to the 9 x 9 Vastu Purusha Mandala, commonly used in Vastu Shasta, known as the Indian form of Feng Shui, but which predates that form by many millennia. This paper overlays a grid of Clifford Algebras and Matrix Algebras over this ancient 9 x 9 Vastu Purusha in an attempt to derive correspondence between the 32 deities of the Vastu Purusha and contemporary mathematical physics. Next, a hypothesis: having established the veracity of the 9 x 9 Vastu Purusha and its 32 deities, is it possible to draw isomorphic relationships between the matrix algebras, Clifford Algebras and ancient Vedic deities? This paper attempts to do so. In addition, the paper solves elementary questions about Five Elements and rotation of the Svas Tika.

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# Matrix / Clifford Algebra Chart

By F. Reece Harvey

$M_{24}(R)$	$M_{16}(C)$	$M_{16}(H)$	$M_{16}(H) \oplus M_{16}(H)$	$M_{22}(H)$	$M_{16}(C)$	$M_{16}(R)$	$M_{16}(R) \oplus M_{16}(R)$	$M_{16}(R)$
$M_8(C)$	$M_8(H)$	$M_8(H) \oplus M_8(H)$	$M_{16}(H)$	$M_{22}(C)$	$M_{16}(R)$	$M_{16}(R) \oplus M_{16}(R)$	$M_{16}(R)$	$M_{16}(C)$
$M_4(H)$	$M_4(H) \oplus M_4(H)$	$M_4(H)$	$M_{16}(C)$	$M_{16}(R)$	$M_{16}(R) \oplus M_{16}(R)$	$M_{16}(R)$	$M_{16}(C)$	$M_{16}(H)$
$M_2(H) \oplus M_2(H)$	$M_4(H)$	$M_4(C)$	$M_{16}(R)$	$M_{16}(R) \oplus M_{16}(R)$	$M_{16}(R)$	$M_{16}(C)$	$M_{16}(H)$	$M_{32}(H) \oplus M_{32}(H)$
$M_2(H)$	$M_4(C)$	$M_8(R)$	$M_8(R) \oplus M_8(R)$	$M_{16}(R)$	$M_{16}(C)$	$M_{16}(H)$	$M_{16}(H) \oplus M_{16}(H)$	$M_{32}(H)$
$M_2(C)$	$M_4(R)$	$M_8(H) \oplus M_8(H)$	$M_8(R)$	$M_8(C)$	$M_8(H)$	$M_8(H) \oplus M_8(H)$	$M_{16}(H)$	$M_{32}(C)$
$M_2(R)$	$M_8(R) \oplus M_8(R)$	$M_8(R)$	$M_4(C)$	$M_4(H)$	$M_4(H) \oplus M_4(H)$	$M_8(H)$	$M_{16}(C)$	$M_{32}(R)$
$R \oplus R$	$M_8(R)$	$M_8(C)$	$M_8(H)$	$M_8(H) \oplus M_8(H)$	$M_8(H)$	$M_8(C)$	$M_{16}(R)$	$M_{32}(R) \oplus M_{32}(R)$
$R$	$C$	$H$	$H \oplus H$	$M_8(H)$	$M_4(C)$	$M_8(R)$	$M_8(R) \oplus M_8(R)$	$M_{16}(R)$

Green = Real Numbers

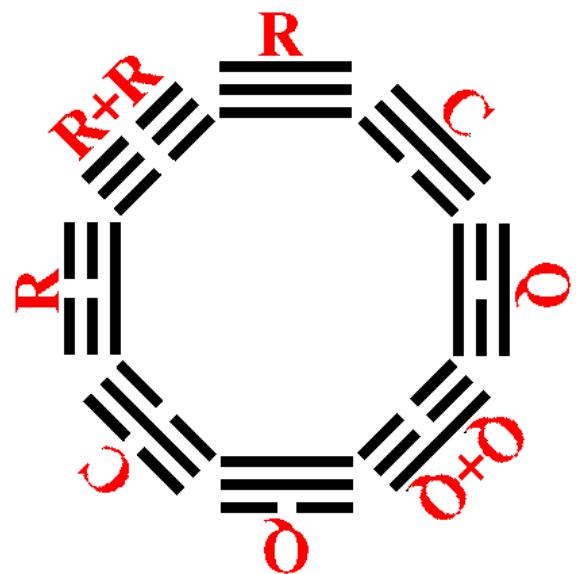
White = Complex Numbers

Yellow = Clifford Algebras

## North

$M_{14}(R)$ $CL(0,8)$	$M_4(R)$ $\oplus M_4(R)$	$M_4(R)$	$M_4(C)$	$M_2(H)$	$H \oplus H$ $CL(0,3)$	$H$	$C$	$R$
$M_{14}(R) \oplus M_{14}(R)$	$M_{16}(R)$ $CL(1,9)$	$M_8(C)$	$M_4(H)$	$M_2(H)$ $\oplus M_2(H)$ $CL(1,4)$	$M_2(H)$ $CL(1,3)$	$M_2(C)$	$M_2(R)$	$R \oplus R$
$M_{32}(R)$	$M_{24}(C)$	$M_8(H)$ $CL(2,8)$	$M_4(H)$	$M_4(H)$	$M_4(C)$	$M_4(R)$	$M_2(R)$ $\oplus M_2(R)$	$M_2(R)$
$M_{32}(C)$	$M_{32}(H)$	$M_6(H) \oplus M_6(H)$	$M_6(H)$	$M_6(C)$	$M_6(R)$ $CL(3,5)$	$M_4(C)$	$M_2(R)$ $\oplus M_2(R)$	$M_2(C)$
$M_{32}(H)$	$M_{24}(H) \oplus M_{24}(H)$	$M_{16}(H)$	$M_{32}(R)$	$M_{16}(R)$	$M_8(R) \oplus M_8(R)$	$M_4(R)$ $CL(4,4)$	$M_4(C)$	$M_2(H)$
$M_{32}(H) \oplus M_{32}(H)$	$M_{24}(H)$	$M_{64}(C)$	$M_{32}(R) \oplus M_{32}(R)$	$M_{32}(R) \oplus M_{32}(R)$	$M_{16}(R)$	$M_8(C)$	$M_4(H)$	$M_2(H) \oplus M_2(H)$
$M_{256}(H)$	$M_{128}(R) \oplus M_{128}(R)$	$M_{128}(R)$	$M_{64}(C)$	$M_{64}(C)$	$M_{16}(C)$	$M_8(H)$	$M_4(H) \oplus M_4(H)$	$M_4(H)$
$M_{256}(R)$	$M_{128}(R) \oplus M_{128}(R)$	$M_{128}(R)$	$M_{64}(C)$	$M_{32}(H)$	$M_{16}(H)$	$M_8(H) \oplus M_8(H)$	$M_8(H)$	$M_8(C)$
$M_{256}(R) \oplus M_{256}(R)$	$M_{128}(R) \oplus M_{128}(R)$	$M_{128}(R)$	$M_{64}(C)$	$M_{32}(H)$	$M_{16}(H) \oplus M_{16}(H)$	$M_{16}(H)$	$M_{16}(R)$	$M_{16}(C)$

## **Clifford Clock and 9 x 9 Square Matrix Algebras**



# Introduction

Indian writers expose a conundrum when stating where the 9 x 9 Vedic Square originates: either from the central Brahmanasthan spot or the peripheral North East direction, which pertains to the god Isana and from whence prana is supposed to enter into a home in Vastu Shasta. Contemporary writers simply assert one over the other, with none attempting to resolve the impasse or give logical reasons for their assumptions.

This author suggests that when Dark Matter emerges from the central square into the realm of visible matter, it does so by pulling tension from the periphery, which is governed by the gods of the Five Elements. In this way, the emergence of visible matter includes a Five Element characteristic, starting with the North East square of Isana, which corresponds to the element of Earth. In an earlier paper published on Vixra the author describes this nuclear process in Vedic Physics.

With some juxtaposition, we may shift the symbols of the chart of matrix and Clifford Algebras published by Reece Harvey and copied by Frank "Tony" Smith onto his monumental website such that the position of "R" for Real Numbers corresponds to the North East position of Isana. The rest of the numbers fill in accordingly until the final or 81<sup>st</sup> square gets filled with  $M_{256}(R)$  to represent Nimiti. (please see the charts after the table of contents).

In an earlier paper published on the Vixra server, this author drew an analogy to the 3 x 3 Magic Square of Qi Men Dun Jia (QMDJ), the ancient Egyptian divination method preserved in the stable and conservative Chinese culture for millennia. Just as the central square is considered the origin of things and events in QMDJ, yet symbols "ride" Kun 2 Earth palace, so in the Vedic Square, matter emerges from the center but takes on peripheral Five Element characteristics as it does, rotating through all Five Elements in turn, starting with Isana.

This solution seems to happily resolve the question, and gains support from millennia of Chinese metaphysics, perhaps most of which were culturally borrowed from India in the remote past, and then preserved in the culture of China, in the same way cultures are used in biology to preserve bacteria, amoeba and other cells in Petri dishes. The extremely culturally conservative China remains unlikely to have invented much of anything, but instead has probably preserved the valuable discoveries of Early Global Civilization.

Moreover, the rotation of the Five Elements fits in with the Svas Tik Magic Square, commonly known in the west as the swastika and highly esteemed in Nazi ideology as a key symbol of the Aryan race. Small wonder that the Aryans supposedly valued the symbol, that the Nazis adopted the symbol to represent their political movement, and that the Jews have since

outlawed its use - thus depriving westerners of a key symbol of occult knowledge. No fools, Asian Buddhists continue to use the symbol as they always have in recognition of its extreme importance, despite Jewish protests, and with the cruel irony that some 60 million Chinese died during WWII, while far fewer than the fictitious six million Jews died.

On a theoretical level, this much seems to hold true. The system of Brahma and the deities of the Vedic Square come down to us from remote antiquity, some 14,000 years or more in the past and remain more or less consistent, despite regional variations in spellings and names of deities. We might further draw an analogy to the Neters or numerical gods of remotely Ancient Egypt as described by Schwaller de Lubicz and his wife. The Vedic Square sets out the pantheon of Hindu gods and the Ancient Egyptians had their own similar pantheon.

At the same time, the order of the Matrix and Clifford Algebras in the Reece Harvey chart appears quite fixed and natural. That is to say, that matrix and Clifford Algebras seem to form natural patterns which continue on infinitely, as demonstrated by Frank "Tony" Smith. One may re - design the chart to match the flow of prana in the Vedic Square or Vastu Purusha, as the author has done above.

*Purusha*, by the way, refers to the atomic nucleus in Vedic Physics, lending more evidence to support this theory. Moreover, physicist Frank "Tony" Smith has illustrated how the pattern shown in the Reece Harvey Matrix and Clifford Algebra chart repeats itself consistently and infinitely in nature, in a separate chapter below.

Finally, the shape of the Vedic Square lends itself to the formation of torii and then later to the Hopf Fibration. In turn, the Hopf Fibration resembles the Hyper - Circles the author discussed in a recent paper about Vedic Nuclear Physics published on the Vixra server. In a series of Vixra papers published in 2013, the author described the importance of the Hopf Fibration in the emergence of visible matter from the state of Dark matter.

This paper attempts to establish a series of isomorphic relationships: from the Vastu Purusha to the Matrix / Clifford Chart, from Vedic Deities to Clifford Algebras; from Chinese Trigrams to Clifford Algebras via the Clifford Clock, and from Trigrams to the amino acids of the DNA helix. While the paper may prove unsuccessful, the exercise of doing so may help to align the model correctly.

For example, whoever assigned Clifford Algebra values to Trigrams for the Clifford Algebras seems to have reasoning which fails to match the model. Perhaps the Clifford Clock assignments could be changed to better match reality. This paper makes a brief attempt to correct these values.

Pascal's Triangle was never really Pascal's, he merely combined the state of knowledge surrounding the triangle, which had been discovered or presented by Indian mathematicians concerned with combinatorics, as Wiki states. The Indians were interested in combinatorics and so devised the triangle, which contains the Fibonacci numbers as will be shown below. This is yet another example of Europeans taking credit for a key math concept which originated in India, from Vedic Physics. We live in a combinatoric universe, but western mathematicians still fail to grasp the fact.

# Vastu Purusha The Vedic Square

**Central palace No. 5 has this number determined by the Five Elements at the Periphery. As Dark Matter extrudes through the central hole, this causes tension in the periphery, which consists of five separate fields of five different elements: Fire, Wood, Water, Earth, Metal or Aether, depending upon whether the scheme is Vedic or Chinese. This tension sets up a rotational movement: as matter extrudes from the central hole, each particle is stamped in turn with the quality of one of the Five Elements.**

**Hypothesis:** the matrix chart and the Clifford Algebra chart consist of a natural numerical order: this is not simply conceived by humans as a heuristic device. Indeed, in an email to the author, physicist Frank “Tony” Smith illustrated how the pattern endlessly repeats itself ad infinitum throughout the universe.

**Having established the veracity of the 9 x 9 Vastu Purusha and its 32 deities, is it possible to draw isomorphic relationships between the matrix algebras, Clifford Algebras and ancient Vedic deities? This paper attempts to do so.**

## North

$M_{14}(R)$ $CL(0,8)$	$M_4(R)$ $\oplus M_4(R)$	$M_4(R)$	$M_4(C)$	$M_2(H)$	$H \oplus H$ $CL(0,3)$	$H$	$C$	$R$
$M_{14}(R) \oplus M_{14}(R)$	$M_{16}(R)$ $CL(1,9)$	$M_8(C)$	$M_4(H)$	$M_2(H)$ $\oplus M_2(H)$ $CL(1,4)$	$M_2(H)$ $CL(1,3)$	$M_2(C)$	$M_2(R)$	$R \oplus R$
$M_{32}(R)$	$M_{24}(C)$	$M_8(H)$ $CL(2,8)$	$M_4(H)$	$M_4(H)$	$M_4(C)$	$M_4(R)$	$M_2(R)$ $\oplus M_2(R)$	$M_2(R)$
$M_{32}(C)$	$M_{32}(H)$	$M_6(H) \oplus M_6(H)$	$M_6(H)$	$M_6(C)$	$M_6(R)$ $CL(3,5)$	$M_4(C)$	$M_2(R)$ $\oplus M_2(R)$	$M_2(C)$
$M_{32}(H)$	$M_{24}(H) \oplus M_{24}(H)$	$M_{16}(H)$	$M_{32}(R)$	$M_{16}(R)$	$M_8(R) \oplus M_8(R)$	$M_4(R)$ $CL(4,4)$	$M_4(C)$	$M_2(H)$
$M_{32}(H) \oplus M_{32}(H)$	$M_{24}(H)$	$M_{64}(C)$	$M_{32}(R) \oplus M_{32}(R)$	$M_{32}(R) \oplus M_{32}(R)$	$M_{16}(R)$	$M_8(C)$	$M_4(H)$	$M_2(H) \oplus M_2(H)$
$M_{256}(H)$	$M_{128}(R) \oplus M_{128}(R)$	$M_{128}(R)$	$M_{64}(C)$	$M_{64}(C)$	$M_{16}(C)$	$M_8(H)$	$M_4(H) \oplus M_4(H)$	$M_4(H)$
$M_{256}(R)$	$M_{128}(R) \oplus M_{128}(R)$	$M_{128}(R)$	$M_{64}(C)$	$M_{32}(H)$	$M_{16}(H)$	$M_8(H) \oplus M_8(H)$	$M_8(H)$	$M_8(C)$
$M_{256}(R) \oplus M_{256}(R)$	$M_{128}(R) \oplus M_{128}(R)$	$M_{128}(R)$	$M_{64}(C)$	$M_{32}(H)$	$M_{16}(H)$ $M_{16}(H)$	$M_{16}(H)$	$M_{16}(R)$	$M_{16}(C)$

## South

## List of 32 Vedic Deities and Corresponding Matrix Algebras

	Name	Clifford Algebra	Alternative Name	Matrix Algebra	Square No.
1	Isana	Real		Real	1
2		Complex		Complex	2
3	Aditi	Quaternion	Hamiltonian	Quaternion	3 अदिति
4					4
5	Kubera			$M_2(H)$	5
6					6
7					7
8					8
9	Vaya			$M_{14}(R)$ CL (0,8)	9
10	Indra			RR	10
11					11
12					12
13					13
14					
15					
16					
17					
18					

<b>19</b>					
<b>20</b>					
<b>21</b>					
<b>22</b>					
<b>23</b>					
<b>24</b>					
<b>25</b>					
<b>26</b>					
<b>27</b>					<b>76</b>
<b>28</b>					<b>77</b>
<b>29</b>					<b>78</b>
<b>30</b>					<b>79</b>
<b>31</b>					<b>80</b>
<b>32</b>	M <sub>256</sub> (R)	<b>Nimitti</b>			<b>81</b>

# Eight Gods of Eight Directions

N	Kubera, Chandra, Soma	Wealth	Moon	Demoness
NE	Isana	Aether	Shiva	
E	Indra			
SE	Agni	Fire		
S	Yama	Death		
SW	Nirutti	Earth		
W	Varuna	Water		
NW	Vayu	Wind		
C	Brahma	Origin		

# Gods of the Rig Veda

In the [Rigveda](#) it is stated that there are 33 [deities](#) associated with sky (dyu), earth (prithvi) and the middle realm (antariksha), though a larger number of deities are mentioned in the text.<sup>[1]</sup> There are 1028 hymns in the Rigveda, most of them dedicated to specific deities.

[Indra](#), a heroic god, slayer of [Vrtra](#) and destroyer of the [Vala](#), liberator of the cows and the rivers; [Agni](#) the sacrificial fire and messenger of the gods; and [Soma](#) the ritual drink dedicated to Indra are the most prominent deities.

Invoked in groups are the [Vishvedevas](#) (the "all-gods"), the [Maruts](#), violent storm gods in Indra's train and the [Ashvins](#), the twin horsemen.

There are two major groups of gods, the [Devas](#) and the [Asuras](#). Unlike in later Vedic texts and in [Hinduism](#), the Asuras are not yet demonized, [Mitra](#) and [Varuna](#) being their most prominent members. [Aditi](#) is the mother both of Agni and of the [Adityas](#) or Asuras, led by Mitra and Varuna, with [Aryaman](#), [Bhaga](#), [Ansa](#) and [Daksha](#).

[Surya](#) is the personification of the [Sun](#), but [Savitr](#), Vivavant, the [Ashvins](#) and the [Rbhus](#), semi-divine craftsmen, also have aspects of [solar deities](#). Other natural phenomena deified include [Vayu](#), (the wind), [Dyaus](#) and [Prithivi](#) (Heaven and Earth), Dyaus continuing [Dyeus](#), the chief god of the [Proto-Indo-European religion](#), and [Ushas](#) (the dawn), the most prominent [goddess](#) of the Rigveda, and [Apas](#) (the waters).

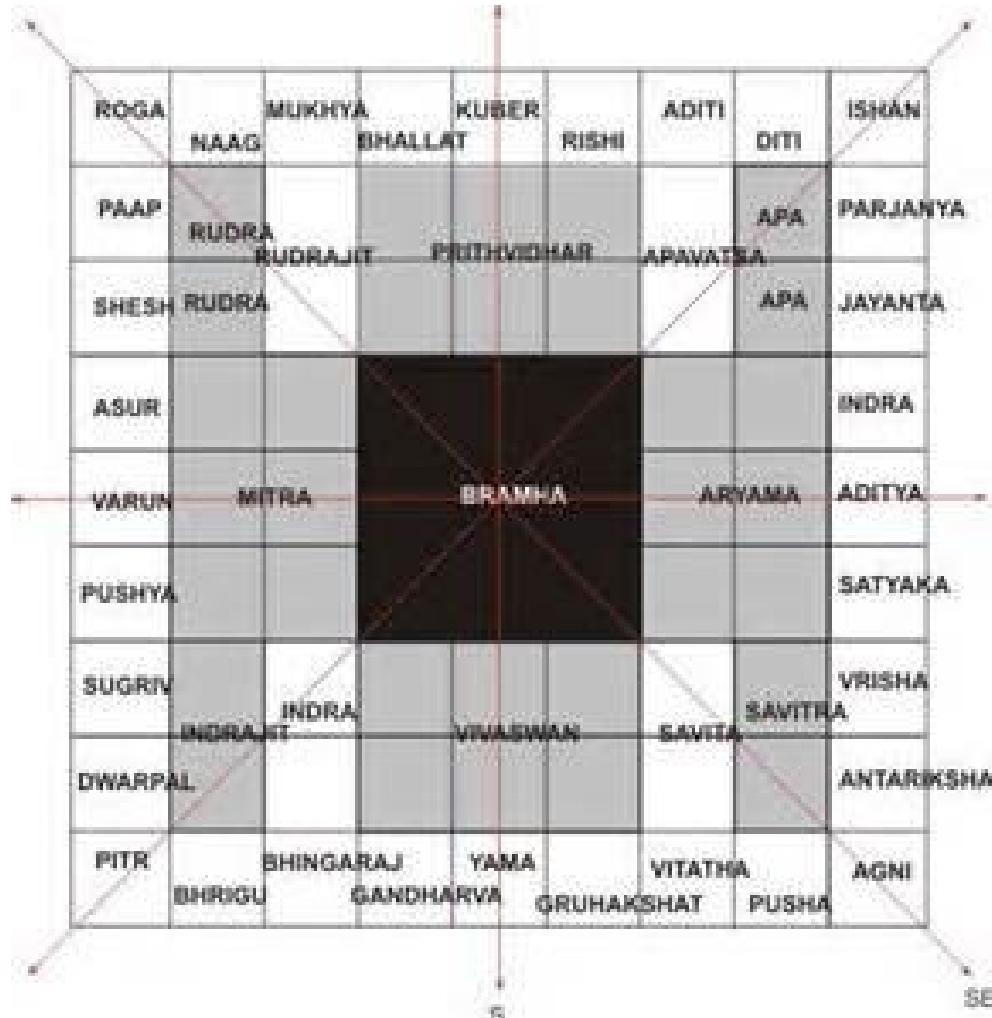
[Rivers](#) play an important role, deified as goddesses, most prominently the [Sapta Sindhu](#) and the [Sarasvati River](#).

[Yama](#) is the first ancestor, also [worshipped](#) as a deity, and the god of the [underworld](#) and death.

[Vishnu](#) and [Rudra](#), the prominent deities of later [Hinduism](#) (Rudra being an early form of [Shiva](#)) are present as marginal gods.

The names of Indra, Mitra, Varuna and the Nasatyas are also attested in a [Mitanni](#) treaty, suggesting that some of the religion of the Mitannis was very close to that of the Rigveda.

In the [Vedas](#), [Aditi](#) ([Sanskrit](#): अदिति "limitless")<sup>[1]</sup> is mother of the gods (*devamatar*) from whose cosmic matrix the heavenly bodies were born. As celestial mother of every existing form and being, the synthesis of all things, she is associated with space (*akasa*) and with mystic speech (*Vāc*). She may be seen as a feminized form of [Brahma](#) and associated with the primal substance (*mulaprakriti*) in [Vedanta](#). She is mentioned nearly 80 times in the [Rigveda](#): the verse "Daksha sprang from Aditi and Aditi from Daksha" is seen by [Theosophists](#) as a reference to "the eternal cyclic re-birth of the same divine Essence"<sup>[2]</sup> and divine wisdom.<sup>[3]</sup> In contrast, the [Puranas](#), such as the [Shiva Purana](#) and the [Bhagavata Purana](#), suggest that Aditi is wife of [sage Kashyap](#) and gave birth to the [Adityas](#) such as [Indra](#), [Surya](#), and also [Yamana](#).



Eight Vasus, eleven Rudras, twelve Adityas, Indra, and Prajapati

Aditi is said to be the mother of the great god [Indra](#), the mother of kings ([Mandala 2.27](#)) and the mother of gods ([Mandala 1.113.19](#)). In the Vedas, Aditi is Devmatar (mother of the celestial gods) as from and in her cosmic matrix all the heavenly bodies were born. She is preeminently the mother of 12 [Adityas](#) whose names include [Vivasvān](#), [Aryamā](#), [Pūsā](#), [Tvastā](#), [Savitā](#), [Bhaga](#), [Dhātā](#), [Vidhātā](#), [Varuna](#), [Mitra](#), [Śatru](#), and [Urukrama](#) (Vishnu was born as Urukrama, the son of Nabhi and Meru.)<sup>[5]</sup> She is also is the mother of the [Vamana](#) avatar of [Vishnu](#). Accordingly, Vishnu was born as the son of Aditi in the month of [Shravana](#) (fifth month of the [Hindu Calendar](#), also called [Avani](#)) under the star Shravana. Many auspicious signs appeared in the heavens, foretelling the good fortune of this child.

In the Rigveda, Adhithe is one of most important figures of all. As a mothering presence, Aditi is often asked to guard the one who petitions her ([Mandala 1.106.7](#); [Mandala 8.18.6](#)) or to provide him or her with wealth, safety, and abundance ([Mandala 10.100](#); [1.94.15](#)).

In [Hinduism](#), **Ādityas** (Sanskrit: आदित्य, pronounced [əd̪it̪ja]), meaning "of Aditi", refers to the offspring of [Aditi](#). In Hinduism, Aditya is used in the singular to mean the Sun God, [Surya](#). [Bhagavata Purana](#)<sup>[1]</sup> enlists total 12 Adityas as twelve Sun-gods. In each month of the year, it is a different Aditya

(Sun God) who shines. All these 12 Adityas are the opulent expansions of Lord [Vishnu](#) in the form of Sun-God.<sup>[2]</sup>

In the [Rigveda](#), the Ādityas are the seven celestial deities, sons of Āditi,

1. [Varuna](#)
2. [Mitra](#)
3. [Aryaman](#)
4. [Bhaga](#)
5. [Anśa](#) or [Amśa](#)
6. [Dhatri](#)
7. [Indra](#)

The eighth Āditya ([Mārtanda](#)) was rejected by Aditi, leaving seven sons. In the [Yajurveda](#) (Taittiriya Samhita), their number is given as eight, and the last one is believed to be [Vivasvān](#). Hymn LXXII of Rig Veda, Book 10, also confirms that there are eight Adityas, the eighth one being [Mārtanda](#), who is later revived back as Vivasvān.<sup>[4]</sup>

"So with her **Seven Sons** Aditi went forth to meet the earlier age. She brought Mārtanda thitherward to spring to life and die again."

The rigvedic Ādityas are Asuras, a class of gods in the Rigveda and are distinct from other groups such as the [Maruts](#), the [Rbhus](#) or the [Viśve-devāḥ](#) (although Mitra and Varuna are associated with the latter).<sup>[5]</sup>

[Indra](#), also known as Śakra in the Vedas, is the leader of the [Devas](#) or gods and the lord of [Svargaloka](#) or heaven in the [Hindu](#) religion. He is the god of rain and thunderstorms.<sup>[1]</sup> He wields a lightning thunderbolt known as [vajra](#) and rides on a [white elephant](#) known as [Airavata](#). Indra is the supreme [deity](#) and is the twin brother of [Agni](#) and is also mentioned as an [Āditya](#), son of [Aditi](#). His home is situated on [Mount Meru](#) in the heaven.<sup>[1]</sup>

The Rig-Veda frequently refers to him as [Śakra](#): the mighty-one. In the Vedic period, the number of gods was assumed to be thirty-three and Indra was their lord. (Some early post Rigvedic texts such as the Khilas and the late Vedic Brihad-Aranyaka Upanishad enumerates the gods as the eight Vasus, the eleven [Rudras](#), the twelve Adityas, Indra, and Prajapati). As lord of the Vasus, Indra was also referred to as [Vāsava](#).

[Vedic](#) Mitra is the patron divinity of contracts and meetings. He is a prominent deity of the [Rigveda](#) distinguished by a relationship to [Varuna](#), the protector of [ṛta](#). Together with Varuna, he counted among the chief [Adityas](#), a group of deities with social functions. They are the supreme keepers of order and gods of the [law](#). The next two in importance are [Aryaman](#) (who guards guest friendship and bridal exchange) and [Bhaga](#) (share in bounty, good luck).

Varuna and [Mitra](#) are the gods of the blood [oath](#) and tribal contracts, often twinned as [Mitra-Varuna](#) (a [dvandva](#) compound). In the [Vedic](#) hymns, Mitra is often invoked together with [Var-](#)

[una](#), as **Mitra-Varuna**. In some of their aspects, Varuna is lord of the cosmic rhythm of the celestial spheres, while Mitra brings forth the light at dawn, which was covered by Varuna. Mitra together with Varuna is the most prominent deity and the chief of the [Adityas](#) in the [Rigveda](#). Though being Asuras, Mitra and Varuna are also addressed as [devas](#) in [Rigveda](#) (e.g., [RV 7.60.12](#)), and in the only hymn dedicated to Mitra, he is referred to as a [deva](#) (*mitrasya...devasya*) in [RV 3.59.6](#).

The pairing with Varuna, a god unknown in Iranian religion, is very strong already in the Rigveda, which has few hymns where Mitra is mentioned without Varuna. [RV 3.59](#) is the only hymn dedicated to Mitra exclusively, where he is lauded as a god following *rta*, order and stability and of observances (2b, *vrata*), the sustainer of mankind (6a), said also of [Indra](#) in 3.37.4c) and of all gods (8c, [devān vishvān](#)).

3.59.1 *Mitra, when speaking, stirreth men to labour: Mitra sustaineth both the earth and heaven.*

*Mitra beholdeth men with eyes that close not. To Mitra bring, with holy oil, oblation.*  
(trans. Griffith)

Rigvedic hymns to Mitra-Varuna are [RV 1.136, 137, 151-153](#), [RV 5.62-72](#), [RV 6.67](#), [RV 7.60-66](#), [RV 8.25](#) and [RV 10.132](#).

Where Mitra appears not paired with Varuna, it is often for the purpose of comparison, where other gods are lauded as being "like Mitra", without the hymn being addressed to Mitra himself ([Indra](#) 1.129.10, 10.22.1-2 etc.; [Agni](#) 1.38.13 etc.; [Soma](#) 1.91.3; [Vishnu](#) 1.156.1).

In the late Vedic [Shatapatha Brahmana](#), Mitra-varuna is analyzed as "the Counsel and the Power" — Mitra being the priesthood ([Purohita](#)), Varuna the royal power ([Rājān](#)). As [Joseph Campbell](#) remarked, "Both are said to have a thousand eyes. Both are active foreground aspects of the light or solar force at play in time. Both renew the world by their deed."

**Rudras** are forms and followers of the god [Rudra-Shiva](#) and make eleven of the [Thirty-three gods](#) in the [Hindu](#) pantheon.<sup>[1]</sup> They are at times identified with the [Maruts](#) – sons of Rudra; while at other times, considered distinct from them. The [Ramayana](#) tells they are eleven of the 33 children of the sage [Kashyapa](#) and his wife [Aditi](#), along with the 12 [Adityas](#), 8 [Vasus](#) and 2 [Ashvins](#), constituting the [Thirty-three gods](#).<sup>[2]</sup> The [Vamana Purana](#) describes the Rudras as the sons of Kashyapa and Aditi.<sup>[3]</sup> The [Matsya Purana](#) notes that [Surabhi](#) – the mother of all cows and the "cow of plenty" – was the consort of Brahma and their union produced the eleven Rudras. Here they are named Nirriti, Shambhu, Aparajita Mrigavyadha, Kapardi, Dahana, Khara, Ahirabradhya, Kapali, Pingala and Senani – the foremost.<sup>[4]</sup> The [Harivamsa](#), an appendix of the [Mahabharata](#), makes Kashyapa and Surabhi – here, portrayed as his wife – the parents of the Rudras.<sup>[3][5]</sup> In another instance in the Mahabharata, it is [Dharma](#) (possibly identified with [Yama](#)) who is the father of the Rudras and the Maruts.<sup>[1]</sup>

The [Vishnu Purana](#) narrates that Rudra – here identified with [Shiva](#) – was born from the anger of the creator-god [Brahma](#). The furious Rudra was in [Ardhanari](#) form, half his body was male and other half female. He divided himself into two: the male and female. The male form then split itself into eleven, forming the eleven Rudras. Some of them were white and gentle; while others were dark and fierce. They are called Manyu, Manu, Mahmasa, Mahan, Siva, Rrudhvaja, Ugraretas, Bhava, Kama, Vamadeva and Dhrtavrata. From the woman were born the eleven Rudranis who became wives of the Rudras. They are Dhi, Vrtti, Usana, Urna, Niyuta, Sarpis, Ila, Ambika, Iravatl, Sudha and Diksa. Brahma allotted to the Rudras the eleven positions of the heart and the five [sensory organs](#), the five

organs of action and the mind.<sup>[3][2]</sup> Other Puranas call them Aja, [Ekapada](#) (Ekapat), Ahirbudhnya, Tvasta, Rudra, Hara, Sambhu, Tryambaka, Aparajita, Isana and Tribhuvana.<sup>[3][2]</sup>

In one instance in the epic [Mahabharata](#), the Rudras are eleven in number and are named Mrgavadha, Sarpa, [Nirriti](#), Ajaikapad, Ahi Budhnya, Pinakin, Dahana, Ishvara, Kapalin, Sthanu and Bhaga. While Kapalin is described the foremost of Rudras here,<sup>[1]</sup> in the [Bhagavad Gita](#) – a discourse by the god [Krishna](#) in the epic – it is Sankara who is considered the greatest of the Rudras.<sup>[6]</sup> Both Kapalin and Sankara are epithets of Shiva.<sup>[1]</sup> In another instance, they are described as sons of [Tvastr](#) and named: Vishvarupa, Ajaikapad, Ahi Budhnya, Virupaksa, Raivata, Hara, Bahurupa, Tryambaka, Savitra, Jayanta and Pinakin.<sup>[1]</sup> While usually the Rudras are described to eleven, in one instance in the *Mahabharata*; they are said to be eleven thousand and surrounding Shiva.<sup>[3][1]</sup> The eleven groups of hundred are named: Ajaikapad, Ahi Budhnya, Pinakin, Rta, Pitrrupa, Tryamabaka, Maheshvara, Vrsakapi, Sambhu, Havana and Ishvara.<sup>[1]</sup>

The [Matsya Purana](#) mentions the ferocious eleven Rudras – named Kapali, Pingala, Bhima, Virupaksa, Vilohita, Ajesha, Shasana, Shasta, Shambhu, Chanda and Dhruva – aiding the god [Vishnu](#) in his fight against the demons. They wear lion-skins, matted-hair and serpents around their necks. They have yellow throats, hold tridents and skulls and have the crescent moon on their foreheads. Together headed by Kapali, they slay the elephant demon [Gajasura](#).<sup>[4]</sup>

In [Vedic mythology](#), Rudras are described as loyal companions of Rudra, who later was identified with [Shiva](#). They are considered as friends, messengers and aspects of Rudra. They are fearful in nature. The [Satapatha Brahmana](#) mentions that Rudra is the prince, while Rudras are his subjects. They are considered as attendants of Shiva in later mythology.<sup>[3]</sup>

The [Rig Veda](#) makes the Rudras the gods of the middle world, situated between earth and sky. As wind-gods, the Rudras represent the life-breath.<sup>[3]</sup> In the [Brihadaranyaka Upanishad](#), the Rudras are associated with the ten [vital energies](#) (*rudra-prana*) in the body and the eleventh being the [Ātman](#) (the soul).<sup>[3]</sup>

The Rudras are said to preside over the second stage of creation and the intermediary stage of life. They govern the second ritual of sacrifice, the mid-day offering and the second stage of life – from the 24th to the 68 year of life. The [Chandogya Upanishad](#) prescribes that the Rudras be propitiated in case of sickness in this period and further says that they on departing the body become the cause of tears, the meaning of the name Rudra being the "ones who make cry".<sup>[3]</sup> The [Brihadaranyaka Upanishad](#) explicitly states the fact that since the Rudras leaving the body – causing death – makes people cry, they are Rudras.<sup>[3]</sup>

The *Mahabharata* describes the Rudras as companions of [Indra](#), servants of Shiva and his son [Skanda](#) and companions of Yama, who is surrounded by them. They have immense power, wear golden necklaces and are "like lighting-illuminated clouds".<sup>[1]</sup> The [Bhagavata Purana](#) prescribes the worship of the Rudras to gain virile power.<sup>[3]</sup>

In [Hinduism](#), [Prajapati](#) ([Sanskrit](#): प्रजापति (IAST: *prajā-pati*)) "lord of creatures" is a group [Hindu deity](#) presiding over procreation, and protector of life. Vedic commentators also identify him

with the creator referred to in the<sup>[1]</sup> [Nasadiya Sukta](#). According to later beliefs in the post-Vedic Era, the Prajapatis were elected democratically. Lord Vishnu was first elected democratically/unanimously as Prajapati (in the North of Aryavarta or Bharta) by all the Rishis and subjects of that era and sat on the throne of Prajapati. Thereafter, Lord Brāhma was elected as Prajapati (in the west of Aryavrat or Bharta), after which Lord Shankar (in the South of Aryavrat or Bharta) or Rudras were elected as Prajapatis. The throne of Prajapati succeeded further and there were about 26 Prajapatis, as mentioned in the Vedas.

Prajapati is a Vedic deity presiding over procreation, and the protector of life. He appears as a creator deity or supreme god [vishvakarman](#) above the other Vedic deities in RV 10 and in Brahmana literature. Vedic commentators also identify him with the creator referred to in the Nasadiya Sukta.

In later times, he is identified with Vishnu, Shiva, with the personifications of Time, Fire, the Sun, etc. He is also identified with various mythical progenitors, especially (Manu Smṛti 1.34) the ten lords of created beings first created by Brahmā, the Prajapatis Marichi, Atri, Angiras, Pulastya, Pulaha, Kratu, Vasishtha, Prachetas or Daksha, Bhrigu, Nārada.

The Mahabharata mentions, in the words of celestial sage Narada, 14 Prajapatis (lit:care-takers of the Praja) Hiranyagarbha is the source of the creation of the Universe or the manifested cosmos in Indian philosophy, it finds mention in one hymn of the Rigveda (RV 10.121), known as the 'Hiranyagarbha sukta' and presents an important glimpse of the emerging monism, or even monotheism, in the later Vedic period, along with the Nasadiya sukta suggesting a single creator deity predating all other gods (verse 8: yó devéṣv ádhi devá éka âsīt, Griffith: "He is the God of gods, and none beside him."), in the hymn identified as Prajapati.

The Upanishads calls it the Soul of the Universe or Brahman, and elaborates that Hiranyagarbha floated around in emptiness and the darkness of the non-existence for about a year, and then broke into two halves which formed the Swarga and the Prithvi. In classical Puranic Hinduism, Hiranyagarbha is a name of Brahma, so called because he was born from a golden egg (Manusmṛti 1.9), while the Mahabharata calls it the Manifest.

Śrīmad Bhāgavatam 8.8.16 cites [Vishvakarman](#) as the leader of the prajāpatis, the sons of Lord Brahmā who generate progeny.<sup>[2]</sup> The eleven lords of created beings first created by [Brahmā](#), which are the **Prajapatis**:

1. [Vishvakarman](#)<sup>[3]</sup>
2. [Marichi](#)
3. [Atri](#)
4. [Angiras](#)
5. [Pulastya](#),
6. Pulaha,
7. Kratu,
8. [Vasishtha](#)
9. Prachetas or [Daksha](#)

10. [Bhrigu](#)

11. [Nārada](#)

The [Mahabharata](#) mentions, in the words of celestial sage [Narada](#), 14 *Prajapatis* (lit:care-takers of the *Praja*) excluding [Vishvakarman](#) namely:

1. [Daksha](#),

2. Prachetas,

3. Pulaha,

4. [Marichi](#),

5. [Kasyapa](#),

6. [Bhrigu](#),

7. [Atri](#),

8. [Vasistha](#),

9. [Gautama](#),

10. [Angiras](#),

11. [Pulastyā](#),

12. Kratu,

13. [Prahlada](#) and

14. Kardama

They are the caretakers of the fourteen worlds - seven *lokas* and seven *talas*.<sup>[4]</sup>

In [Vedic religion](#), **Varuna** ([Sanskrit](#) *Varuṇa* वरुण, [Malay](#): *Baruna*) or **Waruna**, is a god of the [water](#) and of the [celestial ocean](#), as well as a god of [law](#) of the [underwater](#) world. A [Makara](#) is his mount. In [Hindu mythology](#), Varuna continued to be considered the [god](#) of all forms of the [water element](#), particularly the [oceans](#). As chief of the [Adityas](#), Varuna has aspects of a [solar deity](#) though, when opposed to [Mitra \(Vedic term for Surya\)](#), he is rather associated with the night, and Mitra with the daylight. As the most prominent [Deva](#), however, he is mostly concerned with moral and societal affairs than being a deification of nature. Together with Mitra—originally 'agreement' (between tribes) personified—being master of [ṛtā](#), he is the supreme keeper of order and god of the [law](#). The word [ṛtā](#), order, is also translated as "season".

Varuna and Mitra are the gods of the societal affairs including the [oath](#), and are often twinned **Mitra-Varuna** (a [dvandva](#) compound). Varuna is also twinned with [Indra](#) in the Rigveda, as *Indra-Varuna* (when both cooperate at New Year in re-establishing order <sup>[1]</sup>).

The [Rigveda](#) and [Atharvaveda](#)<sup>[2]</sup> portrays Varuna as omniscient, catching liars in his snares. The stars are his thousand-eyed spies, watching every movement of men.

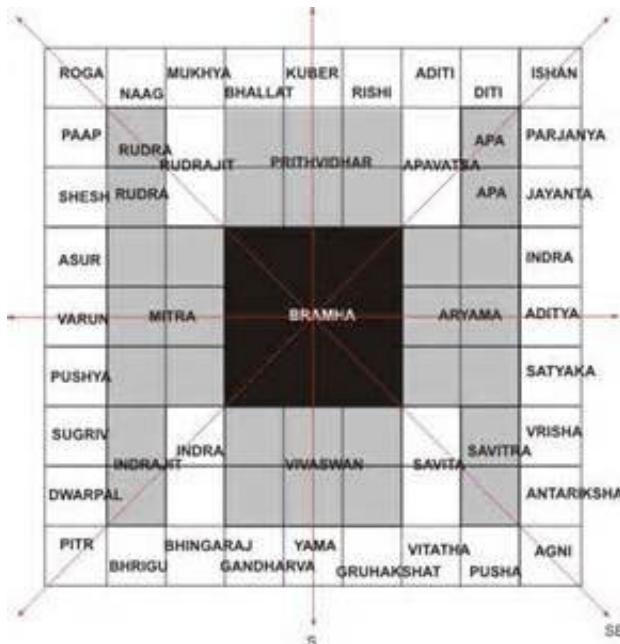
In the [Rigveda](#), [Indra](#), chief of the [Devas](#), is about six times more prominent than Varuna, who is mentioned 341 times. This may misrepresent the actual importance of Varuna in early Vedic society due to the focus of the Rigveda on [fire](#) and [Soma](#) ritual, Soma being closely associated with Indra; Varuna with his omniscience and omnipotence in the affairs of men has many aspects of a supreme deity. The daily [Sandhyavandanam](#) ritual of a *dvija* addresses Varuna in this aspect in its evening routine, asking him to forgive all sins, while Indra receives no mention.

Both Mitra and Varuna are classified as [Asuras](#) in the Rig veda (e.g. [RV 5.63.3](#)), although they are also addressed as [Devas](#) as well (e.g. [RV 7.60.12](#)), possibly indicating the beginning of the negative connotations carried by Asura in later times.



# 81 Squares of the Vastu Purusha

Roga	Naag	Mukhya	Bhallat	Kuber	Rishi	Aditi	Diti	Ishan
Paap	Rudra	Rudrajit	Prith Vidhar	Prith Vidhar	Prith Vidhar	Apa Vatsa	Apa	Parjanya
Shesh	Rudra	Rudrajit	Prith Vidhar	Prith Vidhar	Prith Vidhar	Apa Vatsa	Apa	Jayanta
Asur	Mitra	Mitra	Brahma	Brahma	Brahma	Aryama	Aryama	Indra
Varuna	Mitra	Mitra	Brahma	Brahma	Brahma	Aryama	Aryama	Aditya
वरुण								आदित्य
Pushya	Mitra	Mitra	Brahma	Brahma	Brahma	Aryama	Aryama	Satyaka
Sugriv	Indrant	Indra	Viva Swan	Viva Swan	Viva Swan	Savita	Savitra	Vrisha
Dwarpal	Indrant	Indra	Viva Swan	Viva Swan	Viva Swan	Savita	Savitra	Antarksha
Pitr	Bhrigu	Bhingaraj	Gand Harva	Yama	Gru Hakshat	Vitatha	Pusha	Agni



Eight Vasus, eleven [Rudras](#), twelve Adityas, Indra, and Prajapati and PrithVidhar

32 deities + 9 Brahma squares in the central Brahmathan

= 41 squares + 40 “extra” squares taken by one god; the four sides of Brahma are taken up by 6 squares each of Aryama, VivaSwan; Mitra;

=  $4 \times 6 = 24 - 4 = 20 + 41 = 61$  Then gods on corners of Brahma take up two, =  $69 + 12 = 81$

# Matrix Algebras

F. Reece Harvey has written a great textbook which describes Matrix Algebras and from whence the Matrix Algebra chart originates. South Florida University describes Matrix Algebras in this way:

## Definitions

"A matrix is an n-by-k rectangle of numbers or symbols that stand for numbers" (Pedhazur, 1997, p. 983). The size of the matrix is called its order, and it is denoted by rows and columns. By convention, rows are always mentioned first. So a matrix of order 3 by 2 called **A** might look like this:

$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 2 & 2 \end{bmatrix}$$

A matrix called **B** of order 4 by 4 might look like this:

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & .5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ .5 & 0 & 0 & 1 \end{bmatrix}$$

By convention, matrices in text are printed in **bold face**.

Elements (entries) of the matrix are referred to by the name of the matrix in lower case with a given row and column (again, row comes first). For example,  $a_{31} = 2$ ,  $b_{22}=1$ . In general,  $a_{ij}$  means the element of **A** in the  $i$ th row and  $j$ th column. By convention, elements are printed in *italics*.

A **transpose** of a matrix is obtained by exchanging rows and columns, so that the first row becomes the first column, and so on. The transpose of a matrix is denoted with a single quote and called prime. For example  $\mathbf{A}'$  (A prime) is:

$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\mathbf{A}' = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

Note that  $\mathbf{A}'$  is not just  $\mathbf{A}$  "tipped over" on its side (if so, we would see the first column as 1 3 instead of 3 1). It's as if cards or boards with numbers on them for each row were pulled 1 by 1 and placed in order for the transpose. The transpose of  $\mathbf{B}$  is:

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & .5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ .5 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{B}' = \begin{bmatrix} 1 & 0 & 0 & .5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ .5 & 0 & 0 & 1 \end{bmatrix}$$

(With some matrices, the transpose equals the original matrix.)

If  $n = k$ , the number of rows equals the number of columns, and the matrix is **square**. A square matrix can be **symmetric** or **asymmetric**. A symmetric matrix has the property that elements above and below the main diagonal are the same such that  $\text{element}(i,j) = \text{element}(j,i)$ , as in our matrix  $\mathbf{B}$ . (The main or principal diagonal in matrix  $\mathbf{B}$  is composed of elements all equal to 1.) With a square, symmetric matrix, the transpose of the matrix is the original matrix. A correlation matrix will always be a square, symmetric matrix so the transpose will equal the original.

A **column vector** is an n-by-1 matrix of numbers. For example:

<b>b =</b>	.4
	.5
	.2
	.1

So, **b** is a column vector. A **row vector** is a 1-by-k matrix of numbers. For example,

<b>b' =</b>	.4	.5.	2.	.1
-------------	----	-----	----	----

So, **b'** is a row vector. Note that **b'** is the transpose of **b**. By convention, vectors are printed as lower case bold face letters, and row vectors are represented as the transpose of column vectors.

A **diagonal matrix** is a square, symmetric matrix that has zeros everywhere except on the main diagonal. For example:

<b>C =</b>	12	0	0
	0	10	0
	0	0	5

**C** is a diagonal matrix.

A particularly important diagonal matrix is called the identity matrix, **I**. This diagonal matrix has 1s on the main diagonal.

<b>I =</b>	1	0	0
	0	1	0
	0	0	1

**I** is an identity matrix. It happens that a correlation matrix in which all variables are orthogonal is an identity matrix.

A **scalar** is a matrix with a single element. For example

<b>d =</b>	12
------------	----

**d** is a scalar.

## Matrix Operations

### Addition and Subtraction

Matrices can be added and subtracted if and only if they are of the same order (identical in the number of rows and columns). Matrices upon which an operation is permissible are said to **conform** to the operation.

We are blessed by the fact that matrix addition and subtraction merely means to add or subtract the respective elements of the two matrices.

### Multiplication

Unlike matrix addition and subtraction, matrix multiplication is not a straightforward extension of ordinary multiplication. Matrix multiplication involves both multiplying and adding elements. If we multiply a row vector by a column vector, we obtain a scalar.

To get it, we first multiply corresponding elements, and then add them.

The result of multiplying two such vectors is called a scalar product. Scalar products have many statistical applications. For example, the sum of a variable can be found by placing that variable in a column vector and pre - multiplying it by row vector made of 1s.

# Clifford Algebras

(Wikipedia) In [mathematics](#), **Clifford algebras** are a type of [associative algebra](#). As [K-algebras](#), they generalize the [real numbers](#), [complex numbers](#), [quaternions](#) and several other [hypercomplex number](#) systems.<sup>[1][2]</sup> The theory of Clifford algebras is intimately connected with the theory of [quadratic forms](#) and [orthogonal transformations](#). Clifford algebras have important applications in a variety of fields including [geometry](#), [theoretical physics](#) and [digital image processing](#). They are named after the English geometer [William Kingdon Clifford](#).

A Clifford algebra is a [unital associative algebra](#) that contains and is generated by a [vector space](#)  $V$  over a [field](#)  $K$ , where  $V$  is equipped with a [quadratic form](#)  $Q$ . The Clifford algebra  $C\ell(V, Q)$  is the "freest" algebra generated by  $V$  subject to the condition<sup>[4]</sup>

$$v^2 = Q(v)1 \text{ for all } v \in V,$$

where the product on the left is that of the algebra, and the 1 is its [multiplicative identity](#).

The definition of a Clifford algebra endows it with more structure than a "bare" [K-algebra](#): specifically it has a designated or privileged subspace that is [isomorphic](#) to  $V$ . Such a subspace cannot in general be uniquely determined given only a  $K$ -algebra isomorphic to the Clifford algebra.

If the [characteristic](#) of the ground [field](#)  $K$  is not 2, then one can rewrite this fundamental identity in the form

$$uv + vu = 2\langle u, v \rangle 1 \text{ for all } u, v \in V,$$

where

$$\langle u, v \rangle = \frac{1}{2} (Q(u + v) - Q(u) - Q(v))$$

is the [symmetric bilinear form](#) associated with  $Q$ , via the [polarization identity](#). The idea of being the "freest" or "most general" algebra subject to this identity can be formally expressed through the notion of a [universal property](#), as done [below](#).

Quadratic forms and Clifford algebras in [characteristic](#) 2 form an exceptional case. In particular, if  $\text{char}(K) = 2$  it is not true that a quadratic form determines a symmetric bilinear form, or that every quadratic form admits an orthogonal basis. Many of the statements in this article include the condition that the characteristic is not 2, and are false if this condition is removed.

Let  $V$  be a [vector space](#) over a [field](#)  $K$ , and let  $Q: V \rightarrow K$  be a [quadratic form](#) on  $V$ . In most cases of interest the field  $K$  is either the field of [real numbers](#)  $\mathbf{R}$ , or the field of [complex numbers](#)  $\mathbf{C}$ , or a [finite field](#).

A Clifford algebra  $C\ell(V, Q)$  is a [unital associative algebra](#) over  $K$  together with a [linear map](#)  $i: V \rightarrow C\ell(V, Q)$  satisfying  $i(v)^2 = Q(v)1$  for all  $v \in V$ , defined by the following [universal property](#): given any associative algebra  $A$  over  $K$  and any linear map  $j: V \rightarrow A$  such that

$$j(v)^2 = Q(v)1_A \text{ for all } v \in V$$

(where  $1_A$  denotes the multiplicative identity of  $A$ ), there is a unique [algebra homomorphism](#)  $f: C\ell(V, Q) \rightarrow A$  such that the following diagram [commutes](#) (i.e. such that  $f \circ i = j$ ):

$$\begin{array}{ccc} V & \xrightarrow{i} & C\ell(V, Q) \\ & \searrow j & \downarrow f \\ & & A \end{array}$$

Working with a symmetric [bilinear form](#)  $\langle \cdot, \cdot \rangle$  instead of  $Q$  (in characteristic not 2), the requirement on  $j$  is

$$j(v)j(w) + j(w)j(v) = 2\langle v, w \rangle 1_A \quad \text{for all } v, w \in V.$$

A Clifford algebra as described above always exists and can be constructed as follows: start with the most general algebra that contains  $V$ , namely the [tensor algebra](#)  $T(V)$ , and then enforce the fundamental identity by taking a suitable [quotient](#). In our case we want to take the [two-sided ideal](#)  $I_Q$  in  $T(V)$  generated by all elements of the form

$$v \otimes v - Q(v)1 \text{ for all } v \in V$$

and define  $C\ell(V, Q)$  as the quotient algebra

$$C\ell(V, Q) = T(V)/I_Q.$$

The ring product inherited by this quotient is sometimes referred to as the **Clifford product**<sup>[5]</sup> to differentiate it from the inner and outer products.

It is then straightforward to show that  $C\ell(V, Q)$  contains  $V$  and satisfies the above universal property, so that  $C\ell$  is unique up to a unique isomorphism; thus one speaks of "the" Clifford algebra  $C\ell(V, Q)$ . It also follows from this construction that  $i$  is [injective](#). One usually drops the  $i$  and considers  $V$  as a [linear subspace](#) of  $C\ell(V, Q)$ .

The universal characterization of the Clifford algebra shows that the construction of  $C\ell(V, Q)$  is [functorial](#) in nature. Namely,  $C\ell$  can be considered as a [functor](#) from the [category](#) of vector spaces with quadratic forms (whose [morphisms](#) are linear maps preserving the quadratic form) to the category of associative algebras. The universal property guarantees that linear maps between vector spaces (preserving the quadratic form) extend uniquely to [algebra homomorphisms](#) between the associated Clifford algebras.

## Basis and dimension

If the [dimension](#) of  $V$  is  $n$  and  $\{e_1, \dots, e_n\}$  is a [basis](#) of  $V$ , then the set

$$\{e_{i_1}e_{i_2}\cdots e_{i_k} \mid 1 \leq i_1 < i_2 < \cdots < i_k \leq n \text{ and } 0 \leq k \leq n\}$$

is a basis for  $C\ell(V, Q)$ . The empty product ( $k = 0$ ) is defined as the multiplicative [identity element](#). For each value of  $k$  there are [n choose k](#) basis elements, so the total dimension of the Clifford algebra is

$$\dim C\ell(V, Q) = \sum_{k=0}^n \binom{n}{k} = 2^n.$$

Since  $V$  comes equipped with a quadratic form, there is a set of privileged bases for  $V$ : the [orthogonal](#) ones. An [orthogonal basis](#) is one such that

$$\langle e_i, e_j \rangle = 0 \quad i \neq j.$$

where  $\langle \cdot, \cdot \rangle$  is the symmetric bilinear form associated to  $Q$ . The fundamental Clifford identity implies that for an orthogonal basis

$$e_i e_j = -e_j e_i \quad i \neq j.$$

This makes manipulation of orthogonal basis vectors quite simple. Given a product  $e_{i_1}e_{i_2}\cdots e_{i_k}$  of *distinct* orthogonal basis vectors of  $V$ , one can put them into standard order while including an overall sign determined by the number of [pairwise swaps](#) needed to do so (i.e. the [signature](#) of the ordering [permutation](#)).

## Examples: real and complex Clifford algebras

The most important Clifford algebras are those over [real](#) and [complex](#) vector spaces equipped with [nondegenerate quadratic forms](#).

It turns out that every one of the algebras  $C\ell_{p,q}(\mathbf{R})$  and  $C\ell_n(\mathbf{C})$  is isomorphic to  $A$  or  $A \oplus A$ , where  $A$  is a [full matrix ring](#) with entries from  $\mathbf{R}$ ,  $\mathbf{C}$ , or  $\mathbf{H}$ . For a complete classification of these algebras see [classification of Clifford algebras](#).

### Real numbers

The geometric interpretation of real Clifford algebras is known as [geometric algebra](#).

Every nondegenerate quadratic form on a finite-dimensional real vector space is equivalent to the standard diagonal form:

$$Q(v) = v_1^2 + \cdots + v_p^2 - v_{p+1}^2 - \cdots - v_{p+q}^2$$

where  $n = p + q$  is the dimension of the vector space. The pair of integers  $(p, q)$  is called the [signature](#) of the quadratic form. The real vector space with this quadratic form is often denoted  $\mathbf{R}^{p, q}$ . The Clifford algebra on  $\mathbf{R}^{p, q}$  is denoted  $C\ell_{p, q}(\mathbf{R})$ . The symbol  $C\ell_n(\mathbf{R})$  means either  $C\ell_{n,0}(\mathbf{R})$  or  $C\ell_{0,n}(\mathbf{R})$  depending on whether the author prefers positive definite or negative definite spaces.

A standard [orthonormal basis](#)  $\{e_i\}$  for  $\mathbf{R}^{p,q}$  consists of  $n = p + q$  mutually orthogonal vectors,  $p$  of which have norm  $+1$  and  $q$  of which have norm  $-1$ . The algebra  $C\ell_{p,q}(\mathbf{R})$  will therefore have  $p$  vectors that square to  $+1$  and  $q$  vectors that square to  $-1$ .

Note that  $C\ell_{0,0}(\mathbf{R})$  is naturally isomorphic to  $\mathbf{R}$  since there are no nonzero vectors.  $C\ell_{0,1}(\mathbf{R})$  is a two-dimensional algebra generated by a single vector  $e_1$  that squares to  $-1$ , and therefore is isomorphic to  $\mathbf{C}$ , the field of [complex numbers](#). The algebra  $C\ell_{0,2}(\mathbf{R})$  is a four-dimensional algebra spanned by  $\{1, e_1, e_2, e_1e_2\}$ . The latter three elements square to  $-1$  and all anti-commute, and so the algebra is isomorphic to the [quaternions](#)  $\mathbf{H}$ .  $C\ell_{0,3}(\mathbf{R})$  is an 8-dimensional algebra isomorphic to the [direct sum](#)  $\mathbf{H} \oplus \mathbf{H}$  called [split-biquaternions](#).

## Complex Numbers

One can study Clifford algebras on complex vector spaces. Every non-degenerate quadratic form on a complex vector space is equivalent to the standard diagonal form

$$Q(z) = z_1^2 + z_2^2 + \cdots + z_n^2$$

where  $n = \dim V$ , up to isomorphism so there is only one non-degenerate Clifford algebra for each dimension  $n$ . We will denote the Clifford algebra on  $\mathbf{C}^n$  with the standard quadratic form by  $C\ell_n(\mathbf{C})$ .

The first few cases are not hard to compute. One finds that

$$C\ell_0(\mathbf{C}) \cong \mathbf{C}, \text{ the } \text{complex numbers}$$

$$C\ell_1(\mathbf{C}) \cong \mathbf{C} \oplus \mathbf{C}, \text{ the } \text{bicomplex numbers}$$

$$C\ell_2(\mathbf{C}) \cong M(2, \mathbf{C}), \text{ the } \text{biquaternions}$$

where  $M(n, \mathbf{C})$  denotes the algebra of  $n \times n$  matrices over  $\mathbf{C}$ .

## Examples: constructing quaternions

In this section, Hamilton's [quaternions](#) are constructed as the even sub algebra of the Clifford algebra  $C\ell_{0,3}(\mathbf{R})$ .

Let the vector space  $V$  be real three dimensional space  $\mathbf{R}^3$ , and the quadratic form  $Q$  be derived from the usual Euclidean metric. Then, for  $\mathbf{v}, \mathbf{w}$  in  $\mathbf{R}^3$  we have the quadratic form, or dot product,

$$\mathbf{v} \cdot \mathbf{w} = v_1 w_1 + v_2 w_2 + v_3 w_3.$$

Now introduce the Clifford product of vectors  $\mathbf{v}$  and  $\mathbf{w}$  given by

$$\mathbf{v}\mathbf{w} + \mathbf{w}\mathbf{v} = -2(\mathbf{v} \cdot \mathbf{w}).$$

This formulation uses the negative sign so the correspondence with [quaternions](#) is easily shown.

Denote a set of orthogonal unit vectors of  $\mathbf{R}^3$  as  $\mathbf{e}_1$ ,  $\mathbf{e}_2$ , and  $\mathbf{e}_3$ , then the Clifford product yields the relations

$$\mathbf{e}_2\mathbf{e}_3 = -\mathbf{e}_3\mathbf{e}_2, \quad \mathbf{e}_3\mathbf{e}_1 = -\mathbf{e}_1\mathbf{e}_3, \quad \mathbf{e}_1\mathbf{e}_2 = -\mathbf{e}_2\mathbf{e}_1,$$

and

$$\mathbf{e}_1^2 = \mathbf{e}_2^2 = \mathbf{e}_3^2 = -1.$$

The general element of the Clifford algebra  $C\ell_{0,3}(\mathbf{R})$  is given by

$$A = a_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3 + a_4\mathbf{e}_2\mathbf{e}_3 + a_5\mathbf{e}_3\mathbf{e}_1 + a_6\mathbf{e}_1\mathbf{e}_2 + a_7\mathbf{e}_1\mathbf{e}_2\mathbf{e}_3.$$

The linear combination of the even grade elements of  $C\ell_{0,3}(\mathbf{R})$  defines the even sub algebra  $C\ell^0_{0,3}(\mathbf{R})$  with the general element

$$Q = q_0 + q_1\mathbf{e}_2\mathbf{e}_3 + q_2\mathbf{e}_3\mathbf{e}_1 + q_3\mathbf{e}_1\mathbf{e}_2.$$

The basis elements can be identified with the quaternion basis elements  $i, j, k$  as

$$i = \mathbf{e}_2\mathbf{e}_3, \quad j = \mathbf{e}_3\mathbf{e}_1, \quad k = \mathbf{e}_1\mathbf{e}_2,$$

which shows that the even sub algebra  $C\ell^0_{0,3}(\mathbf{R})$  is Hamilton's real [quaternion](#) algebra.

To see this, compute

$$i^2 = (\mathbf{e}_2\mathbf{e}_3)^2 = \mathbf{e}_2\mathbf{e}_3\mathbf{e}_2\mathbf{e}_3 = -\mathbf{e}_2\mathbf{e}_2\mathbf{e}_3\mathbf{e}_3 = -1,$$

and

$$ij = \mathbf{e}_2\mathbf{e}_3\mathbf{e}_3\mathbf{e}_1 = -\mathbf{e}_2\mathbf{e}_1 = \mathbf{e}_1\mathbf{e}_2 = k.$$

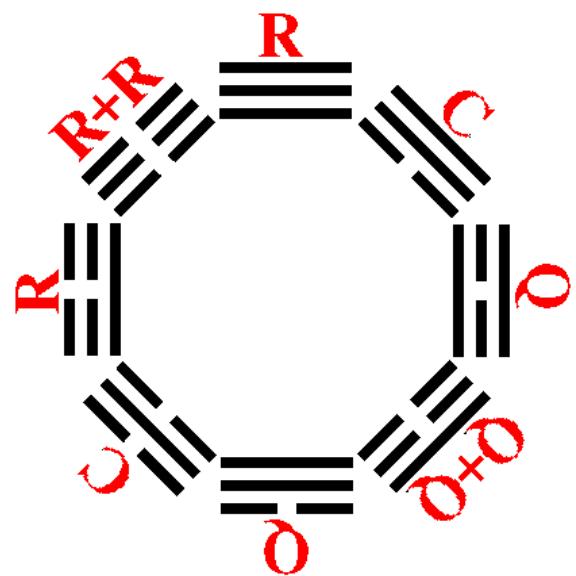
Finally,

$$ijk = \mathbf{e}_2\mathbf{e}_3\mathbf{e}_3\mathbf{e}_1\mathbf{e}_1\mathbf{e}_2 = -1.$$

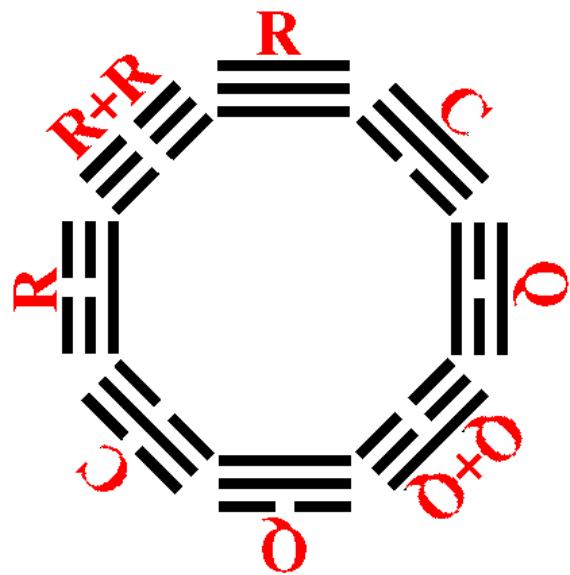
## Clifford Clock and the 9 x 9 Vastu Purusha

North

$M_{14}(R)$				$M_2(H)$	H	H	C	R
CL (0,8)					H			
$M_{32}(H)$				$M_{16}(R)$				$M_2(H)$
$M_{256}(R)$				$M_{32}(H)$				$M_{16}(C)$



# Clifford Clock and 9 x 9 Square Matrix Algebras



1	≡	R						
2	☰	C						
3	☷	Q						
4	☳	Q+Q						
5	☷	Q						
6	☰	C						
7	☷	R						
8	☳	R+R						

≡ ≡ ☰ ☱ ☲ ☳ ☴ ☵

The author matches the Matrix / Clifford Algebra Chart superimposed over the 9 x 9 Vastu Purusha chart and then employed in coordination with the heuristic Clifford Clock of Tony Smith, based on the spinorial clock of Andrez Trautman and P. Budinovich.

Straightforward application of the clock shows R for Real Numbers corresponding with Isana, the Northeast, Earth and the Qian Metal Trigram. This creates problems or conflicts when attempting to include the Chinese order of metaphysics, since Qian Metal is not an Earth element. Alternatively, we could re-assign the trigrams to match the Vastu Shasta, in which case R and Isana would match the Kun 2 Earth Trigram. In this version of the Clifford Clock, then, we would have R + R under this rubric. More likely, the assignments need to shift one place to the right to match the Vastu Purusha chart.

	Algebra	Chinese	Family	Meanings
1	R	Qian	Father	Creative
2	C	Xun		Gentle
3	Q (H)	Li		Summer, south, Fire
4	Q + Q	Gen		Keeping Still
5	Q	Dui		Autumn
6	C	Kan		Winter, north, Water
7	R	Zhen		Spring, east
8	R + R	Kun	Mother	Receptive, southeast

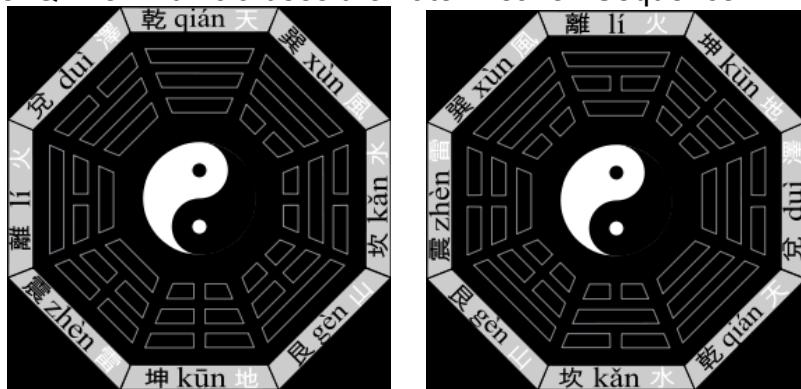
☰ ☷ ☵ ☷ ☷ ☷ ☷ ☷).

The 3 x 3 Magic Square of Qi Men Dun Jia uses this array:

4	9	2
3	5	7
8	1	6

	Algebra	Chinese	Family	Meanings
2	R	Kun	Mother	Receptive
7	C	Dui		Gentle
6	Q (H)	Qian	Father	Summer, south, Fire
1	Q + Q	Kan		Keeping Still
8	Q	Gen		Autumn
3	C	Zhen		Winter, north, Water
4	R	Xun		Spring, east
9	R + R	Li	Mother	Receptive, southeast

Chinese metaphysics organizes the eight trigrams by either the Early Heaven or Later Heaven sequences. Qi Men Dun Jia uses the Later Heaven Sequence.



# Qi Men Dun Jia Magic Square and Brahma

Do these share an isomorphic relationship?

$$\mathbf{Brahma} = M_{16}(R)$$

$M_6(R)$	$M_6(C)$	$M_6(Q)$
$M_8(R)$ $M_8(R)$	$M_{16}(R)$	$M_{16}(C)$
$M_{16}(R)$	$M_{16}(R)$ $M_{16}(R)$	$M_{32}(R)$

# Pascal Triangle of Clifford Algebras

(Wikipedia)

Of all matrix ring types mentioned, there is only one type shared between complex and real algebras: the type  $\mathbf{C}(2^m)$ .

For example,  $\text{Cl}_2(\mathbf{C})$  and  $\text{Cl}_{3,0}(\mathbf{R})$  are determined as  $\mathbf{C}(2)$ . There is a difference in the classifying isomorphisms used.

Since  $\text{Cl}_2(\mathbf{C})$  algebra is isomorphic via a  $\mathbf{C}$ -linear map (which is necessarily  $\mathbf{R}$ -linear), and  $\text{Cl}_{3,0}(\mathbf{R})$  algebra is isomorphic via an  $\mathbf{R}$ -linear map,

Then:  $\text{Cl}_2(\mathbf{C})$  and  $\text{Cl}_{3,0}(\mathbf{R})$  are  $\mathbf{R}$ -algebra isomorphic.

A table of this classification for  $p + q \leq 8$  follows: Here  $p + q$  runs vertically and  $p - q$  runs horizontally (e.g. the algebra  $\text{Cl}_{1,3}(\mathbf{R}) \cong M_2(\mathbf{H})$  is found in row 4, column -2).

8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8
0								$\mathbf{R}$								
1								$\mathbf{R}^2$	$\mathbf{C}$							
2							$M_2(\mathbf{R})$	$M_2(\mathbf{R})$	$\mathbf{H}$							
3						$M_2(\mathbf{C})$	$M_2^2(\mathbf{R})$	$M_2(\mathbf{C})$	$\mathbf{H}^2$							
4					$M_2(\mathbf{H})$	$M_4(\mathbf{R})$	$M_4(\mathbf{R})$	$M_2(\mathbf{H})$	$M_2(\mathbf{H})$							
5				$M_2^2(\mathbf{H})$	$M_4(\mathbf{C})$	$M_4^2(\mathbf{R})$	$M_4(\mathbf{C})$	$M_2^2(\mathbf{H})$	$M_4(\mathbf{C})$							
6			$M_4(\mathbf{H})$	$M_4(\mathbf{H})$	$M_8(\mathbf{R})$	$M_8(\mathbf{R})$	$M_4(\mathbf{H})$	$M_4(\mathbf{H})$	$M_8(\mathbf{R})$							
7		$M_8(\mathbf{C})$	$M_4^2(\mathbf{H})$	$M_8(\mathbf{C})$	$M_8^2(\mathbf{R})$	$M_8(\mathbf{C})$	$M_8(\mathbf{C})$	$M_8^2(\mathbf{H})$	$M_8(\mathbf{C})$							
8	$M_{16}(\mathbf{R})$	$M_8(\mathbf{H})$	$M_8(\mathbf{H})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R})$	$M_8(\mathbf{H})$	$M_8(\mathbf{H})$	$M_8(\mathbf{H})$	$M_{16}(\mathbf{R})$	$M_8(\mathbf{H})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R})$		
$\omega^2$	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+

## Symmetries

There is a tangled web of symmetries and relationships in the above table.

$$\begin{aligned} \text{Cl}_{p+1,q+1}(\mathbf{R}) &= M_2(\text{Cl}_{p,q}(\mathbf{R})) \\ \text{Cl}_{p+4,q}(\mathbf{R}) &= \text{Cl}_{p,q+4}(\mathbf{R}) \end{aligned}$$

Going over 4 spots in any row yields an identical algebra.

From these Bott periodicity follows:

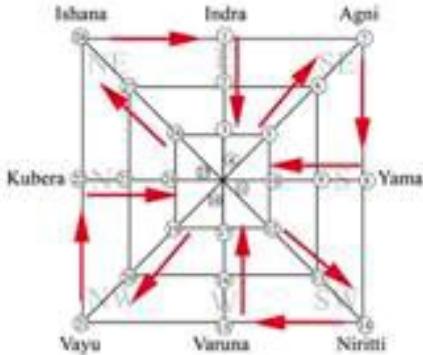
$$\text{Cl}_{p+8,q}(\mathbf{R}) = \text{Cl}_{p+4,q+4}(\mathbf{R}) = M_2^4(\text{Cl}_{p,q}(\mathbf{R})).$$

If the signature satisfies  $p - q \equiv 1 \pmod{4}$  then

$$\text{Cl}_{p+k,q}(\mathbf{R}) = \text{Cl}_{p,q+k}(\mathbf{R}).$$

(The table is symmetric about columns with signature 1, 5, -3, -7, and so forth.) Thus if the signature satisfies  $p - q \equiv 1 \pmod{4}$ ,

$$\text{Cl}_{p+k,q}(\mathbf{R}) = \text{Cl}_{p,q+k}(\mathbf{R}) = \text{Cl}_{p-k+k,q+k}(\mathbf{R}) = M_{2^k}(\text{Cl}_{p-k,q}(\mathbf{R})) = M_{2^k}(\text{Cl}_{p,q-k}(\mathbf{R})).$$



## The 9 x 9 Vastu Purusha

The red arrows above may mark an Energy Path or Hamiltonian Path or Clifford Path.

	<b>Location</b>	<b>Clifford Algebra</b>	<b>Direction</b>	<b>Meaning</b>
1	Brahma Center	$M_{16}(R)$	Brahmanasthan	
2	Ishana	R	North East	Real Numbers
3	Indra	$M_2(H)$	East	Power
4	Brahma Center	$M_{16}(R)$		
5	Agni	$M_{16}(C)$	South East	Fire
6	Yama	$M_{32}(H)$	South	Death
7	Brahma Center	$M_{16}(R)$		
8	Nirimi	$M_{256}(R)$	South West	
9	Varuna	$M_{32}(H)$	West	Wandering
10	Brahma Center	$M_{16}(R)$		
11	Vaya	$M_{14}(R)$ CL (0,8)	North West	Wind
12	Kubera	$M_2(H)$	North	Wealth
13	Brahma Center	$M_{16}(R)$		

An illustration on the Hare Krsna website (above) illustrates a definite path through the Vedic Square. The chart above details the number and places of directional turns the path takes, from the Brahmanasthan (center) all the way to each major god of the eight directions before returning to the Brahmanasthan, 13 turns later.

The essential question here is whether this chart is valid: is it proper to match  $M_{32}(H)$  with the West and thus with Wandering? Does CL (0,8) truly represent the North West and Vaya?

In short, is it possible to quantify the ancient gods into modern numbers? Do the gods and Five Elements bear mathematical relationships to one another? The Ancient Egyptians evidently believed so, and if the coded systems in Vedic Literature prove valid, then so did the ancient Hindu people of the Vedas.

	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8		
0									<b>R</b>										
1									<b>R</b> <sup>2</sup>		<b>C</b>								
2									<b>M</b> <sub>2</sub> ( <b>R</b> )		<b>M</b> <sub>2</sub> ( <b>R</b> )		<b>H</b>						
3									<b>M</b> <sub>2</sub> ( <b>C</b> )		<b>M</b> <sub>2</sub> <sup>2</sup> ( <b>R</b> )		<b>M</b> <sub>2</sub> ( <b>C</b> )		<b>H</b> <sup>2</sup>				
4									<b>M</b> <sub>2</sub> ( <b>H</b> )		<b>M</b> <sub>4</sub> ( <b>R</b> )		<b>M</b> <sub>4</sub> ( <b>R</b> )		<b>M</b> <sub>2</sub> ( <b>H</b> )		<b>M</b> <sub>2</sub> ( <b>H</b> )		
5									<b>M</b> <sub>2</sub> <sup>2</sup> ( <b>H</b> )		<b>M</b> <sub>4</sub> ( <b>C</b> )		<b>M</b> <sub>4</sub> <sup>2</sup> ( <b>R</b> )		<b>M</b> <sub>4</sub> ( <b>C</b> )		<b>M</b> <sub>2</sub> <sup>2</sup> ( <b>H</b> )		<b>M</b> <sub>4</sub> ( <b>C</b> )
6									<b>M</b> <sub>4</sub> ( <b>H</b> )		<b>M</b> <sub>4</sub> ( <b>H</b> )		<b>M</b> <sub>8</sub> ( <b>R</b> )		<b>M</b> <sub>4</sub> ( <b>H</b> )		<b>M</b> <sub>4</sub> ( <b>H</b> )		<b>M</b> <sub>8</sub> ( <b>R</b> )
7									<b>M</b> <sub>8</sub> ( <b>C</b> )		<b>M</b> <sub>4</sub> <sup>2</sup> ( <b>H</b> )		<b>M</b> <sub>8</sub> ( <b>C</b> )		<b>M</b> <sub>8</sub> <sup>2</sup> ( <b>R</b> )		<b>M</b> <sub>8</sub> ( <b>C</b> )		<b>M</b> <sub>8</sub> <sup>2</sup> ( <b>R</b> )
8	<b>M</b> <sub>16</sub> ( <b>R</b> )	<b>M</b> <sub>8</sub> ( <b>H</b> )	<b>M</b> <sub>8</sub> ( <b>H</b> )	<b>M</b> <sub>8</sub> ( <b>H</b> )		<b>M</b> <sub>16</sub> ( <b>R</b> )		<b>M</b> <sub>16</sub> ( <b>R</b> )		<b>M</b> <sub>8</sub> ( <b>H</b> )		<b>M</b> <sub>8</sub> ( <b>H</b> )		<b>M</b> <sub>16</sub> ( <b>R</b> )		<b>M</b> <sub>16</sub> ( <b>R</b> )			
<b>w</b> <sup>2</sup>	+	-	-	+	+	-	-	+	+	+	-	-	+	+	-	-	+		

<b>M</b> <sub>14</sub> ( <b>R</b> ) CL (0,8)					<b>M</b> <sub>2</sub> ( <b>H</b> )	<b>H</b> <b>H</b>	<b>H</b>	<b>C</b>	<b>R</b>
<b>M</b> <sub>32</sub> ( <b>H</b> )				<b>M</b> <sub>16</sub> ( <b>R</b> )				<b>M</b> <sub>2</sub> ( <b>H</b> )	
<b>M</b> <sub>256</sub> ( <b>R</b> )				<b>M</b> <sub>32</sub> ( <b>H</b> )				<b>M</b> <sub>16</sub> ( <b>C</b> )	

There is a tangled web of symmetries and relationships in the above table.

$$\begin{aligned} C\ell_{p+1,q+1}(\mathbf{R}) &= \mathbf{M}_2(C\ell_{p,q}(\mathbf{R})) \\ C\ell_{p+4,q}(\mathbf{R}) &= C\ell_{p,q+4}(\mathbf{R}) \end{aligned}$$

Going over 4 spots in any row yields an identical algebra.

From these Bott periodicity follows:

$$C\ell_{p+8,q}(\mathbf{R}) = C\ell_{p+4,q+4}(\mathbf{R}) = M_{2^4}(C\ell_{p,q}(\mathbf{R})).$$

If the signature satisfies  $p - q \equiv 1 \pmod{4}$  then

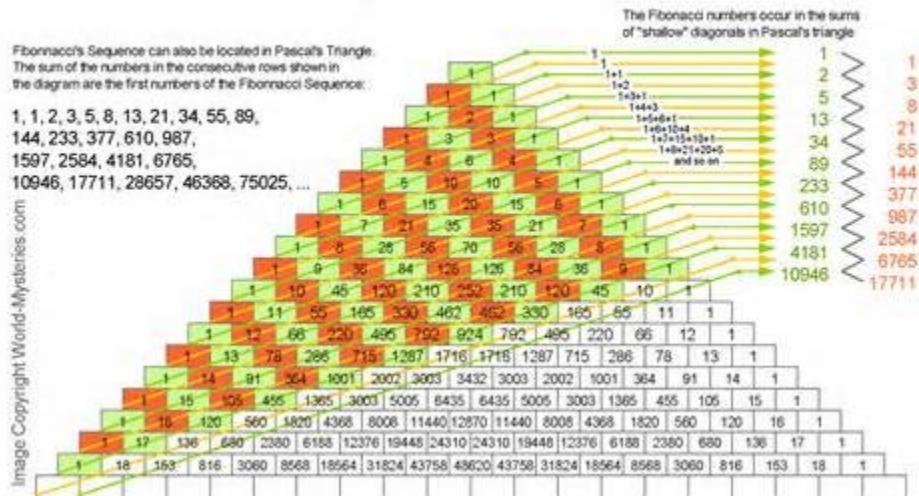
$$C\ell_{p+k,q}(\mathbf{R}) = C\ell_{p,q+k}(\mathbf{R}).$$

(The table is symmetric about columns with signature 1, 5, -3, -7, and so forth.) Thus if the signature satisfies  $p - q \equiv 1 \pmod{4}$ ,

$$C\ell_{p+k,q}(\mathbf{R}) = C\ell_{p,q+k}(\mathbf{R}) = C\ell_{p-k+k,q+k}(\mathbf{R}) = M_{2^k}(C\ell_{p-k,q}(\mathbf{R})) = M_{2^k}(C\ell_{p,q-k}(\mathbf{R})).$$

# Pascal Triangle and Fibonacci Numbers

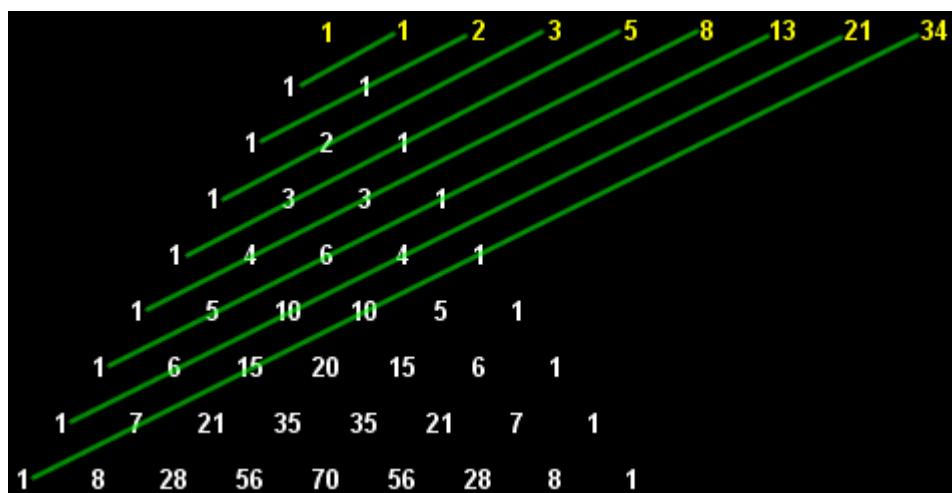
## Pascal's Triangle and Fibonacci Numbers



**The Fibonacci Series is found in Pascal's Triangle**

Pascal's Triangle, developed by the French Mathematician Blaise Pascal, is formed by starting with an apex of 1. Every number below in the triangle is the sum of the two numbers diagonally above it to the left and the right, with positions outside the triangle counting as zero.

The numbers on diagonals of the triangle add to the Fibonacci series, as shown below.

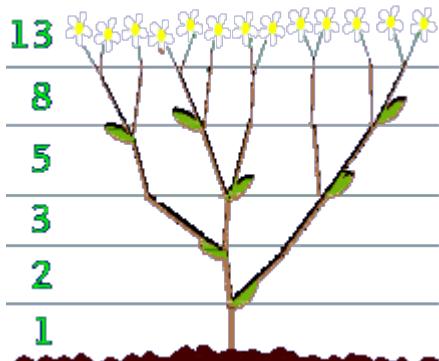


Pascal's triangle has many unusual properties and a variety of uses:

- Horizontal rows add to powers of 2 (i.e., 1, 2, 4, 8, 16, etc.)
  - The horizontal rows represent powers of 11 (1, 11, 121, 1331, etc.)
  - Adding any two successive numbers in the diagonal 1-3-6-10-15-21-28... results in a perfect square (1, 4, 9, 16, etc.)

- It can be used to find combinations in probability problems (if, for instance, you pick any two of five items, the number of possible combinations is 10, found by looking in the second place of the fifth row. Do not count the 1's.)
- When the first number to the right of the 1 in any row is a prime number, all numbers in that row are divisible by that prime number

## Fibonacci Numbers and the Pascal Triangle



In the book, **Liber abaci** ( meaning *Book of the Abacus* or *Book of Calculating* ), the practical arithmetic problem is presented : A pair of rabbits is put in a limited area. This pair of rabbits produces another pair each month. If the rabbits do not die, the question is : How many pairs of rabbits there would be ? The answer from the book is this sequence of numbers :

1 1 2 3 5 8 13 21 34 55 .....

The series of numbers was named " Fibonacci numbers" by Edouard Lucas ( 1842-1899 ).

Lucas invented numerous significant applications of these. The Fibonacci sequence is a recursive sequence where the first two values are 1 and each successive term is obtained by adding together the two previous terms. The definition of the Fibonacci series is :

$$F(n+1) = F(n-1) + F(n) , \quad \text{if } n>1 \text{ and } F(0) = 0, F(1) = 1$$

By adding diagonal numbers of the Pascal Triangle Fibonacci sequence can be obtained :

			1		1
		1	1		1
	1	2	1		2
1	3	3	1		3
1	4	6	4	1	5
1	5	10	10	5	8
1	6	15	20	15	13
1	7	21	35	35	21

It can be presumed where on his journeys Fibonacci met this sequence if "rows" in double Pascal's Triangle are summed :

1		1					
1	1	2					
1	1	1	3				
1	1	2	1	5			
1	1	3	2	1	8		
1	1	4	3	3	1	13	
1	1	5	4	6	3	1	21
.....	.....	.....	.....	.....	.....	.....	.....

For the sake of playfulness among numbers and areas let us point out the connection between the arithmetical triangle and Fibonacci numbers. The Pascal Triangle is shown as follows and the numbers in rows are summed:

1		1		
1		1		
1	1	2		
1	2	3		
1	3	1	5	
1	4	3	8	
1	5	6	1	13
.....	.....	.....	.....	.....

R. Knott found the Fibonacci numbers appearing as sums of "rows" in Pascal's Triangle. By drawing Pascal's Triangle with all the rows moved over by 1 place, we have a clearer arrangement which shows the Fibonacci numbers as sums of columns :

	0	1	2	3	4	5	6	7	8	9
0	1									
1		1	1							
2			1	2	1					
3				1	3	3	1			
4					1	4	6	4	1	
5						1	5	10	10	5
6							1	6	15	20
7								1	7	21
8									1	8
9										1
	1	1	2	3	5	8	13	21	34	55

Here is the alternative form of Pascal's Triangle, with the *double rows* re-aligned as columns and the sums of the new columns are the Fibonacci numbers :

	0	1	2	3	4	5	6	7	8	9
0	1									
1		1								
2		1	1							
3			1	1						
4				1	2	1				
5					1	2	1			
6						1	3	3	1	
7							1	3	3	1
8							1	4	6	4
9								1	4	6
	1	2	3	5	8	.	.	.	.	.

There is a reciprocal connection between Fibonacci numbers and arithmetical triangle. There are also numerous recursive relations for the Fibonacci numbers :

$$\begin{aligned}
 F(n+1) &= 1*F(n) + 1*F(n-1) \\
 F(n+2) &= 1*F(n) + 2*F(n-1) + 1*F(n-2) \\
 F(n+3) &= 1*F(n) + 3*F(n-1) + 3*F(n-2) + 1*F(n-3) \\
 F(n+4) &= 1*F(n) + 4*F(n-1) + 6*F(n-2) + 4*F(n-3) + 1*F(n-4) \\
 F(n+5) &= 1*F(n) + 5*F(n-1) + 10*F(n-2) + 10*F(n-3) + 5*F(n-4) + 1*F(n-5) \\
 F(n+6) &= 1*F(n) + 6*F(n-1) + 15*F(n-2) + 20*F(n-3) + 15*F(n-4) + 6*F(n-5) + 1*F(n-6) \\
 \dots &\quad \dots \quad \dots
 \end{aligned}$$

It is obvious that the structure of Pascal's Triangle is built in these recursive relations, which certainly indicates the existing connection between the numbers of Pascal's Triangle and Fibonacci numbers .

The French mathematician Edouard Lucas found a similar series :

$$1, 3, 4, 7, 11, 18, 29, 47 \dots$$

The Fibonacci rule of adding the latest two to get the next is kept, but here we start from 2 and 1 (in this order) instead of 0 and 1 for the (ordinary) Fibonacci numbers :

$$L(n+1) = L(n-1) + L(n), \quad \text{if } n>1 \text{ and } L(0) = 2, L(1) = 1$$

Let us point out the connection between the arithmetical triangle and Lucas numbers. The Pascal Triangle is shown as follows and the numbers in rows are summed:

1										1
1	1	1								3
1	1	1	1							4
1	1	1	2	1	1					7
1	1	1	3	2	2	1				11
1	1	1	4	3	3	3	1	1		18
1	1	1	5	4	4	6	3	3	1	29

.....

Also, there is the alternative form of Pascal's Triangle, with the double rows re-aligned as columns and the sums of the new columns are the Lucas numbers :

	0	1	2	3	4	5	6	7	8	9
0			1							
1		1	1							
2				1	1					
3				1	2	1				
4					1	2	1			
5					1	3	3	1		
6						1	3	3	1	
7						1	4	6	4	1
8							1	4	6	4
9							1	5	10	10
.	1	3	4	7	11	.	.	.	.	.

Here is shewn is a reciprocal connection between Lucas numbers and arithmetical triangle:

$$L(n+1) = 1*L(n) + 1*L(n-1)$$

$$L(n+2) = 1*L(n) + 2*L(n-1) + 1*L(n-2)$$

$$L(n+3) = 1*L(n) + 3*L(n-1) + 3*L(n-2) + 1*L(n-3)$$

$$L(n+4) = 1*L(n) + 4*L(n-1) + 6*L(n-2) + 4*L(n-3) + 1*L(n-4)$$

$$L(n+5) = 1*L(n) + 5*L(n-1) + 10*L(n-2) + 10*L(n-3) + 5*L(n-4) + 1*L(n-5)$$

$$L(n+6) = 1*L(n) + 6*L(n-1) + 15*L(n-2) + 20*L(n-3) + 15*L(n-4) + 6*L(n-5) + 1*L(n-6)$$

.....

# Pascal's Triangle - Negative Row Numbers

Pascal's Triangle can be extended to negative row numbers.

First write the triangle in the following form:

$$m = 0 \ m = 1 \ m = 2 \ m = 3 \ m = 4 \ m = 5 \dots$$

$n = 0$	1	0	0	0	0	0	...
$n = 1$	1	1	0	0	0	0	...
$n = 2$	1	2	1	0	0	0	...
$n = 3$	1	3	3	1	0	0	...
$n = 4$	1	4	6	4	1	0	...

Next, extend the column of 1s upwards:

$$m = 0 \ m = 1 \ m = 2 \ m = 3 \ m = 4 \ m = 5 \dots$$

$n = -4$	1						...
$n = -3$	1						...
$n = -2$	1						...
$n = -1$	1						...
$n = 0$	1	0	0	0	0	0	...
$n = 1$	1	1	0	0	0	0	...
$n = 2$	1	2	1	0	0	0	...
$n = 3$	1	3	3	1	0	0	...
$n = 4$	1	4	6	4	1	0	...

Now the rule:

$$\binom{n}{m} = \binom{n-1}{m-1} + \binom{n-1}{m}$$

can be rearranged to:

$$\binom{n-1}{m} = \binom{n}{m} - \binom{n-1}{m-1}$$

which allows calculation of the other entries for negative rows:

$$m = 0 \ m = 1 \ m = 2 \ m = 3 \ m = 4 \ m = 5 \dots$$

$n = -4$	1	-4	10	-20	35	-56	...
$n = -3$	1	-3	6	-10	15	-21	...
$n = -2$	1	-2	3	-4	5	-6	...
$n = -1$	1	-1	1	-1	1	-1	...
$n = 0$	1	0	0	0	0	0	...
$n = 1$	1	1	0	0	0	0	...
$n = 2$	1	2	1	0	0	0	...
$n = 3$	1	3	3	1	0	0	...
$n = 4$	1	4	6	4	1	0	...

This extension preserves the property that the values in the  $m$ th column viewed as a function of  $n$  are fit by an order  $m$  polynomial, namely

$$\binom{n}{m} = \frac{1}{m!} \prod_{k=0}^{m-1} (n - k) = \frac{1}{m!} \prod_{k=1}^m (n - k + 1)$$

This extension also preserves the property that the values in the  $n$ th row correspond to the coefficients of  $(1 + x)^n$ :

$$(1 + x)^n = \sum_{k=0}^{\infty} \binom{n}{k} x^k \quad |x| < 1$$

For example:

$$(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots \quad |x| < 1$$

When viewed as a series, the rows of negative  $n$  diverge. However, they are still [Abel summable](#), which summation gives the standard values of  $2^n$ . (In fact, the  $n = -1$  row results in [Grandi's series](#) which "sums" to  $1/2$ , and the  $n = -2$  row results in [another well-known series](#) which has an Abel sum of  $1/4$ .)

# Clifford Extensions

by Frank Tony Smith

Here is how to extend the Real Clifford 8-Periodicity Structure:

Start with the diagram of  $\text{Cl}(p,q)$  where p and q go from 0 through 8:

$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{C})$	$M_{16}(\mathbf{H})$	$M_{16}(\mathbf{H}) \oplus M_{16}(\mathbf{H})$	$M_{32}(\mathbf{H})$	$M_{64}(\mathbf{C})$	$M_{128}(\mathbf{R})$	$M_{128}(\mathbf{R}) \oplus M_{128}(\mathbf{R})$	$M_{128}(\mathbf{R})$
$M_8(\mathbf{C})$	$M_8(\mathbf{H})$	$M_8(\mathbf{H}) \oplus M_8(\mathbf{H})$	$M_{16}(\mathbf{H})$	$M_{32}(\mathbf{C})$	$M_{64}(\mathbf{R})$	$M_{64}(\mathbf{R}) \oplus M_{64}(\mathbf{R})$	$M_{128}(\mathbf{R})$	$M_{128}(\mathbf{C})$
$M_8(\mathbf{H})$	$M_8(\mathbf{H}) \oplus M_8(\mathbf{H})$	$M_8(\mathbf{H})$	$M_{16}(\mathbf{C})$	$M_{32}(\mathbf{R})$	$M_{32}(\mathbf{R}) \oplus M_{32}(\mathbf{R})$	$M_{64}(\mathbf{R})$	$M_{64}(\mathbf{C})$	$M_{64}(\mathbf{H})$
$M_2(\mathbf{H}) \oplus M_2(\mathbf{H})$	$M_4(\mathbf{H})$	$M_4(\mathbf{C})$	$M_{16}(\mathbf{R})$	$M_{32}(\mathbf{R}) \oplus M_{32}(\mathbf{R})$	$M_{64}(\mathbf{R})$	$M_{128}(\mathbf{C})$	$M_{128}(\mathbf{H}) \oplus M_{128}(\mathbf{H})$	$M_{128}(\mathbf{H})$
$M_2(\mathbf{H})$	$M_4(\mathbf{C})$	$M_8(\mathbf{R})$	$M_8(\mathbf{R}) \oplus M_8(\mathbf{R})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{C})$	$M_{16}(\mathbf{H})$	$M_{16}(\mathbf{H}) \oplus M_{16}(\mathbf{H})$	$M_{32}(\mathbf{H})$
$M_2(\mathbf{C})$	$M_4(\mathbf{R})$	$M_8(\mathbf{R}) \oplus M_8(\mathbf{R})$	$M_8(\mathbf{R})$	$M_8(\mathbf{C})$	$M_8(\mathbf{H})$	$M_8(\mathbf{H}) \oplus M_8(\mathbf{H})$	$M_{16}(\mathbf{H})$	$M_{32}(\mathbf{C})$
$M_2(\mathbf{R})$	$M_4(\mathbf{R}) \oplus M_4(\mathbf{R})$	$M_8(\mathbf{R})$	$M_8(\mathbf{C})$	$M_8(\mathbf{H})$	$M_8(\mathbf{H}) \oplus M_8(\mathbf{H})$	$M_8(\mathbf{H})$	$M_{16}(\mathbf{C})$	$M_{32}(\mathbf{R})$
$\mathbf{R} \oplus \mathbf{R}$	$M_4(\mathbf{R})$	$M_8(\mathbf{C})$	$M_8(\mathbf{H})$	$M_8(\mathbf{H}) \oplus M_8(\mathbf{H})$	$M_8(\mathbf{H})$	$M_8(\mathbf{C})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R}) \oplus M_{16}(\mathbf{R})$
$\mathbf{R}$	$\mathbf{C}$	$\mathbf{H}$	$\mathbf{H} \oplus \mathbf{H}$	$M_8(\mathbf{H})$	$M_8(\mathbf{C})$	$M_8(\mathbf{R})$	$M_8(\mathbf{R}) \oplus M_8(\mathbf{R})$	$M_{16}(\mathbf{R})$

Then separate out the periodic 8x8 part where p and q go from 1 through 8: the real (green) and complex (white) and quaternion (yellow) types continue in a consistent pattern after

$M_{16}(\mathbb{R})$	$M_{16}(\mathbb{C})$	$M_{16}(\mathbb{H})$	$M_{16}(\mathbb{H}) \oplus M_{16}(\mathbb{H})$	$M_{32}(\mathbb{H})$	$M_{64}(\mathbb{C})$	$M_{128}(\mathbb{R})$	$M_{128}(\mathbb{R}) \oplus M_{128}(\mathbb{R})$	$M_{256}(\mathbb{R})$
$M_8(\mathbb{C})$	$M_8(\mathbb{H})$	$M_8(\mathbb{H}) \oplus M_8(\mathbb{H})$	$M_{16}(\mathbb{H})$	$M_{32}(\mathbb{C})$	$M_{64}(\mathbb{R})$	$M_{64}(\mathbb{R}) \oplus M_{64}(\mathbb{R})$	$M_{128}(\mathbb{R})$	$M_{128}(\mathbb{C})$
$M_4(\mathbb{H})$	$M_4(\mathbb{H}) \oplus M_4(\mathbb{H})$	$M_4(\mathbb{H})$	$M_{16}(\mathbb{C})$	$M_{32}(\mathbb{R})$	$M_{32}(\mathbb{R}) \oplus M_{32}(\mathbb{R})$	$M_{64}(\mathbb{R})$	$M_{64}(\mathbb{C})$	$M_{64}(\mathbb{H})$
$M_2(\mathbb{H}) \oplus M_2(\mathbb{H})$	$M_4(\mathbb{H})$	$M_4(\mathbb{C})$	$M_{32}(\mathbb{R})$	$M_{32}(\mathbb{R}) \oplus M_{32}(\mathbb{R})$	$M_{32}(\mathbb{R})$	$M_{32}(\mathbb{C})$	$M_{32}(\mathbb{H})$	$M_{32}(\mathbb{H}) \oplus M_{32}(\mathbb{H})$
$M_2(\mathbb{H})$	$M_4(\mathbb{C})$	$M_8(\mathbb{R})$	$M_8(\mathbb{R}) \oplus M_8(\mathbb{R})$	$M_{16}(\mathbb{R})$	$M_{16}(\mathbb{C})$	$M_{16}(\mathbb{H})$	$M_{16}(\mathbb{H}) \oplus M_{16}(\mathbb{H})$	$M_{32}(\mathbb{H})$
$M_2(\mathbb{C})$	$M_8(\mathbb{R})$	$M_8(\mathbb{R}) \oplus M_8(\mathbb{R})$	$M_8(\mathbb{R})$	$M_8(\mathbb{C})$	$M_8(\mathbb{H})$	$M_8(\mathbb{H}) \oplus M_8(\mathbb{H})$	$M_{16}(\mathbb{H})$	$M_{32}(\mathbb{C})$
$M_2(\mathbb{R})$	$M_8(\mathbb{R}) \oplus M_8(\mathbb{R})$	$M_8(\mathbb{R})$	$M_8(\mathbb{C})$	$M_8(\mathbb{H})$	$M_8(\mathbb{H}) \oplus M_8(\mathbb{H})$	$M_8(\mathbb{H})$	$M_{16}(\mathbb{C})$	$M_{32}(\mathbb{R})$
$\mathbb{R} \oplus \mathbb{R}$	$M_2(\mathbb{R})$	$M_2(\mathbb{C})$	$M_2(\mathbb{H})$	$M_2(\mathbb{H}) \oplus M_2(\mathbb{H})$	$M_4(\mathbb{H})$	$M_8(\mathbb{C})$	$M_{16}(\mathbb{R})$	$M_{16}(\mathbb{R}) \oplus M_{16}(\mathbb{R})$
$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{H} \oplus \mathbb{H}$	$M_2(\mathbb{H})$	$M_4(\mathbb{C})$	$M_8(\mathbb{R})$	$M_8(\mathbb{R}) \oplus M_8(\mathbb{R})$	$M_{16}(\mathbb{R})$

extension

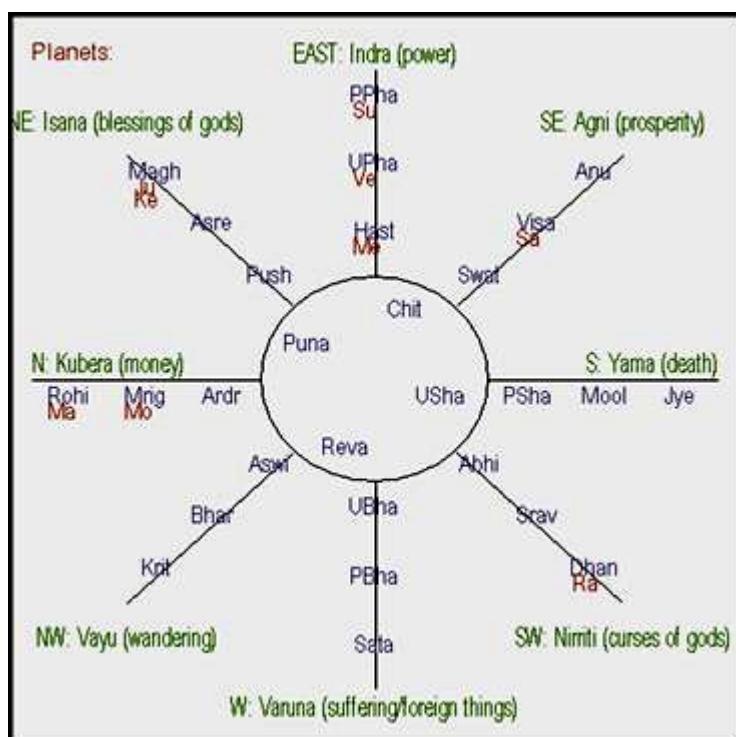
$M_4(R)$	$M_{16}(C)$	$M_{16}(H)$	$M_{16}(H) \oplus M_{16}(H)$	$M_{32}(H)$	$M_{64}(C)$	$M_{128}(R) \oplus M_{256}(R)$	$M_{256}(R)$	$M_{512}(R)$
$M_4(C)$	$M_4(H)$	$M_4(H) \oplus M_4(H)$	$M_4(H)$	$M_{32}(C)$	$M_4(R)$	$M_{128}(R) \oplus M_{256}(R)$	$M_{256}(R)$	$M_{512}(C)$
$M_4(H)$	$M_4(H) \oplus M_4(H)$	$M_4(H)$	$M_{16}(C)$	$M_{128}(R) \oplus M_{256}(R)$	$M_4(R)$	$M_{128}(R)$	$M_{256}(C)$	$M_{512}(C)$
$M_2(H) \oplus M_2(H)$	$M_4(H)$	$M_4(C)$	$M_{16}(R) \oplus M_{32}(R)$	$M_{32}(C)$	$M_{32}(H)$	$M_{32}(H) \oplus M_{32}(H)$	$M_{32}(C)$	$M_{32}(C)$
$M_2(H)$	$M_4(C)$	$M_4(H) \oplus M_4(H)$	$M_{16}(R) \oplus M_{32}(R)$	$M_{16}(C)$	$M_{16}(H) \oplus M_{32}(H)$	$M_{32}(H)$	$M_{32}(H) \oplus M_{32}(H)$	$M_{32}(C)$
$M_1(C)$	$M_4(R)$	$M_4(R) \oplus M_4(R)$	$M_4(C)$	$M_4(H)$	$M_4(H) \oplus M_{16}(H)$	$M_{16}(R)$	$M_{32}(C)$	$M_{32}(C)$
$M_1(H)$	$M_4(R) \oplus M_4(R)$	$M_4(H) \oplus M_4(H)$	$M_4(C)$	$M_4(H) \oplus M_4(H)$	$M_4(H)$	$M_4(H) \oplus M_{16}(H)$	$M_{16}(R)$	$M_{32}(R)$
$R \oplus R$	$M_4(R)$	$M_2(C)$	$M_2(H) \oplus M_2(H)$	$M_4(H)$	$M_8(C)$	$M_{16}(R) \oplus M_{32}(R)$	$M_{32}(R) \oplus M_{64}(R)$	$M_{64}(R)$
$M_{16}(R)$	$M_{16}(C)$	$M_{16}(H)$	$M_{16}(H) \oplus M_{16}(H)$	$M_{32}(H)$	$M_{64}(C)$	$M_{128}(R) \oplus M_{256}(R)$	$M_{256}(R)$	$M_{512}(R)$
$M_8(C)$	$M_8(H)$	$M_8(H) \oplus M_8(H)$	$M_8(H)$	$M_{32}(C)$	$M_8(R)$	$M_{128}(R) \oplus M_{256}(R)$	$M_{256}(R)$	$M_{512}(C)$
$M_8(H)$	$M_8(H) \oplus M_8(H)$	$M_8(H)$	$M_{16}(C)$	$M_8(R) \oplus M_{16}(R)$	$M_{16}(R)$	$M_{128}(R) \oplus M_{256}(R)$	$M_{256}(R)$	$M_{512}(C)$
$M_4(H) \oplus M_4(H)$	$M_4(H)$	$M_4(H)$	$M_{16}(C)$	$M_{128}(R) \oplus M_{256}(R)$	$M_4(R)$	$M_{128}(R)$	$M_{256}(C)$	$M_{512}(C)$
$M_2(H) \oplus M_2(H)$	$M_4(H)$	$M_4(C)$	$M_{16}(R) \oplus M_{32}(R)$	$M_{32}(C)$	$M_{32}(H)$	$M_{32}(H) \oplus M_{32}(H)$	$M_{32}(C)$	$M_{32}(C)$
$M_2(H)$	$M_4(C)$	$M_4(H) \oplus M_4(H)$	$M_{16}(R) \oplus M_{32}(R)$	$M_{16}(C)$	$M_{16}(H) \oplus M_{32}(H)$	$M_{32}(H)$	$M_{32}(H) \oplus M_{32}(H)$	$M_{32}(C)$
$M_1(C)$	$M_4(R)$	$M_4(R) \oplus M_4(R)$	$M_4(C)$	$M_4(H)$	$M_4(H) \oplus M_{16}(H)$	$M_{16}(R)$	$M_{32}(C)$	$M_{32}(C)$
$M_1(H)$	$M_4(R) \oplus M_4(R)$	$M_4(H) \oplus M_4(H)$	$M_4(C)$	$M_4(H) \oplus M_4(H)$	$M_4(H)$	$M_4(H) \oplus M_{16}(H)$	$M_{16}(R)$	$M_{32}(R)$
$R \oplus R$	$M_4(R)$	$M_2(C)$	$M_2(H) \oplus M_2(H)$	$M_4(H)$	$M_8(C)$	$M_{16}(R) \oplus M_{32}(R)$	$M_{32}(R) \oplus M_{64}(R)$	$M_{64}(R)$
$M_{16}(R)$	$M_{16}(C)$	$M_{16}(H)$	$M_{16}(H) \oplus M_{16}(H)$	$M_{32}(H)$	$M_{64}(C)$	$M_{128}(R) \oplus M_{256}(R)$	$M_{256}(R)$	$M_{512}(R)$
$M_8(C)$	$M_8(H)$	$M_8(H) \oplus M_8(H)$	$M_8(H)$	$M_{32}(C)$	$M_8(R)$	$M_{128}(R) \oplus M_{256}(R)$	$M_{256}(R)$	$M_{512}(C)$
$M_8(H)$	$M_8(H) \oplus M_8(H)$	$M_8(H)$	$M_{16}(C)$	$M_8(R) \oplus M_{16}(R)$	$M_{16}(R)$	$M_{128}(R) \oplus M_{256}(R)$	$M_{256}(R)$	$M_{512}(C)$
$M_4(H) \oplus M_4(H)$	$M_4(H)$	$M_4(H)$	$M_{16}(C)$	$M_{128}(R) \oplus M_{256}(R)$	$M_4(R)$	$M_{128}(R)$	$M_{256}(C)$	$M_{512}(C)$
$M_2(H) \oplus M_2(H)$	$M_4(H)$	$M_4(C)$	$M_{16}(R) \oplus M_{32}(R)$	$M_{32}(C)$	$M_{32}(H)$	$M_{32}(H) \oplus M_{32}(H)$	$M_{32}(C)$	$M_{32}(C)$
$M_2(H)$	$M_4(C)$	$M_4(H) \oplus M_4(H)$	$M_{16}(R) \oplus M_{32}(R)$	$M_{16}(C)$	$M_{16}(H) \oplus M_{32}(H)$	$M_{32}(H)$	$M_{32}(H) \oplus M_{32}(H)$	$M_{32}(C)$
$M_1(C)$	$M_4(R)$	$M_4(R) \oplus M_4(R)$	$M_4(C)$	$M_4(H)$	$M_4(H) \oplus M_{16}(H)$	$M_{16}(R)$	$M_{32}(C)$	$M_{32}(C)$
$M_1(H)$	$M_4(R) \oplus M_4(R)$	$M_4(H) \oplus M_4(H)$	$M_4(C)$	$M_4(H) \oplus M_4(H)$	$M_4(H)$	$M_4(H) \oplus M_{16}(H)$	$M_{16}(R)$	$M_{32}(R)$
$M_2(C)$	$M_4(R) \oplus M_4(R)$	$M_4(H) \oplus M_4(H)$	$M_4(C)$	$M_4(H) \oplus M_4(H)$	$M_4(H)$	$M_4(H) \oplus M_{16}(H)$	$M_{16}(R)$	$M_{32}(C)$
$M_2(R)$	$M_4(R) \oplus M_4(R)$	$M_4(H) \oplus M_4(H)$	$M_4(C)$	$M_4(H) \oplus M_4(H)$	$M_4(H)$	$M_4(H) \oplus M_{16}(H)$	$M_{16}(R)$	$M_{32}(R)$
$R \oplus R$	$M_4(R)$	$M_2(C)$	$M_2(H) \oplus M_2(H)$	$M_4(H)$	$M_8(C)$	$M_{16}(R) \oplus M_{32}(R)$	$M_{32}(R) \oplus M_{64}(R)$	$M_{64}(R)$
$R$	$C$	$H$	$H \oplus H$	$M_1(H)$	$M_4(C)$	$M_8(R) \oplus M_8(R)$	$M_{16}(R)$	$M_{32}(R)$
	$C$	$H$	$H \oplus H$	$M_1(H)$	$M_4(C)$	$M_8(R)$	$M_{16}(R) \oplus M_{16}(R)$	$M_{32}(R)$

Then extend the 8x8 part like a checkerboard and extend the 1x7 and 7x1 strips for p=0 and q=0  
The extension (at the stage of extending by 3 steps) looks like this:

As you can see  
the real (green) and complex (white) and quaternion (yellow) types continue in a consistent pattern after extension.

$M_{16}(\mathbb{R})$	$M_{16}(\mathbb{C})$	$M_{16}(\mathbb{H})$	$M_{16}(\mathbb{H}) \oplus M_{16}(\mathbb{H})$	$M_{32}(\mathbb{H})$	$M_{64}(\mathbb{C})$	$M_{128}(\mathbb{R})$	$M_{128}(\mathbb{R}) \oplus M_{128}(\mathbb{R})$	$M_{256}(\mathbb{R})$
$M_8(\mathbb{C})$	$M_8(\mathbb{H})$	$M_8(\mathbb{H}) \oplus M_8(\mathbb{H})$	$M_{16}(\mathbb{H})$	$M_{32}(\mathbb{C})$	$M_{64}(\mathbb{R})$	$M_{64}(\mathbb{R}) \oplus M_{64}(\mathbb{R})$	$M_{128}(\mathbb{R})$	$M_{128}(\mathbb{C})$
$M_4(\mathbb{H})$	$M_4(\mathbb{H}) \oplus M_4(\mathbb{H})$	$M_8(\mathbb{H})$	$M_{16}(\mathbb{C})$	$M_{32}(\mathbb{R})$	$M_{64}(\mathbb{R}) \oplus M_{64}(\mathbb{R})$	$M_{64}(\mathbb{R})$	$M_{128}(\mathbb{C})$	$M_{128}(\mathbb{H})$
$M_2(\mathbb{H}) \oplus M_2(\mathbb{H})$	$M_4(\mathbb{H})$	$M_4(\mathbb{C})$	$M_{16}(\mathbb{R})$	$M_{32}(\mathbb{R}) \oplus M_{32}(\mathbb{R})$	$M_{64}(\mathbb{R})$	$M_{128}(\mathbb{C})$	$M_{32}(\mathbb{H})$	$M_{32}(\mathbb{H}) \oplus M_{32}(\mathbb{H})$
$M_2(\mathbb{H})$	$M_4(\mathbb{C})$	$M_8(\mathbb{R})$	$M_8(\mathbb{R}) \oplus M_8(\mathbb{R})$	$M_{16}(\mathbb{H})$	$M_{16}(\mathbb{C})$	$M_{16}(\mathbb{H})$	$M_{16}(\mathbb{H}) \oplus M_{16}(\mathbb{H})$	$M_{32}(\mathbb{H})$
$M_2(\mathbb{C})$	$M_4(\mathbb{R})$	$M_4(\mathbb{R}) \oplus M_4(\mathbb{R})$	$M_8(\mathbb{R})$	$M_8(\mathbb{C})$	$M_8(\mathbb{H})$	$M_8(\mathbb{H}) \oplus M_8(\mathbb{H})$	$M_{16}(\mathbb{H})$	$M_{32}(\mathbb{C})$
$M_2(\mathbb{R})$	$M_2(\mathbb{R}) \oplus M_2(\mathbb{R})$	$M_4(\mathbb{R})$	$M_4(\mathbb{C})$	$M_4(\mathbb{H})$	$M_4(\mathbb{H}) \oplus M_4(\mathbb{H})$	$M_8(\mathbb{H})$	$M_{16}(\mathbb{C})$	$M_{32}(\mathbb{R})$
$\mathbb{R} \oplus \mathbb{R}$	$M_2(\mathbb{R})$	$M_2(\mathbb{C})$	$M_2(\mathbb{H})$	$M_2(\mathbb{H}) \oplus M_2(\mathbb{H})$	$M_4(\mathbb{H})$	$M_8(\mathbb{C})$	$M_{16}(\mathbb{R})$	$M_{32}(\mathbb{R}) \oplus M_{32}(\mathbb{R})$
$\mathbb{R}$	$\mathbb{C}$	$\mathbb{H}$	$\mathbb{H} \oplus \mathbb{H}$	$M_2(\mathbb{H})$	$M_4(\mathbb{C})$	$M_8(\mathbb{R})$	$M_8(\mathbb{R}) \oplus M_8(\mathbb{R})$	$M_{16}(\mathbb{R})$

# THE KALACHAKRA AND SCIENCE OF DIRECTIONS



The kalachakra or "Wheel of Time" chart in Vedanga Jyotish reflects the cosmic model of the universe. Every individual has his own universal kalachakra model according to his birth chart. All nine planets also take their place in the individual kalachakra chart depending upon which one of the 27 nakshatras they occupy. The central position is given to the Sun. It is necessary to construct the kalachakra around the Sun's position since it is the Sun who determines kala or time by his movement. Of the importance of the Sun in measuring time, Shrila Prabhupada writes in his introduction to Shrimad-Bhagavatam (5.22):

"The Sun-god, who controls the affairs of the entire universe, especially in regard to heat, light, seasonal changes and so on, is considered an expansion of Narayana. He represents the three Vedas—*Rig*, *Yajur* and *Sama*—and therefore he is known as *Trayimaya*, the form of Lord Narayana. Sometimes the Sun-god is also called *Surya Narayana*. The Sun-god has expanded himself in twelve divisions, and thus he controls the six seasonal changes and causes winter, summer, rain and so on."

Yogis and *karmis* following the *varnashrama* institution, who practice *hatha* or *ashtanga-yoga* or who perform *agnihotra* sacrifices, worship *Surya Narayana* for their own benefit. The demigod *Surya* is always in touch with the Supreme Personality of Godhead, Narayana. Residing in outer space, which is in the middle of the universe, between Bhuloka and Bhuvarloka, the Sun rotates through the time circle of the zodiac, represented by

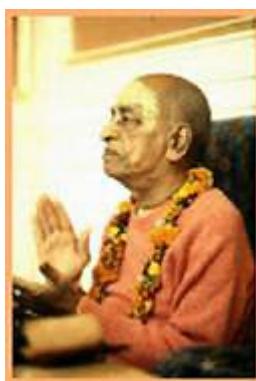
twelve *rashis*, or signs, and assumes different names according to the sign he is in.\* For the moon, every month is divided into two fortnights. Similarly, according to solar calculations, a month is equal to the time the Sun spends in one constellation; two months constitute one season, and there are twelve months in a year.

The entire area of the sky is divided into two halves, each representing an *ayana*, the course traversed by the Sun within a period of six months. The Sun travels sometimes slowly, sometimes swiftly and sometimes at a moderate speed. In this way he travels within the three worlds, consisting of the heavenly planets, the earthly planets and outer space. These orbits are referred to by great learned scholars by the names Samvatsara, Parivatsara, Idavatsara, Anuvatsara and Vatsara."

\*For the names and qualities of Surya Narayana in each of the twelve rashis, please refer to the May 2012 issue of The Astrological Newsletter, article (beginning from p. 10) entitled "Lord Surya Narayana: Controller of Time and Destiny" by Abhaya Mudra Dasi ([Issue 29, May 2012](#))

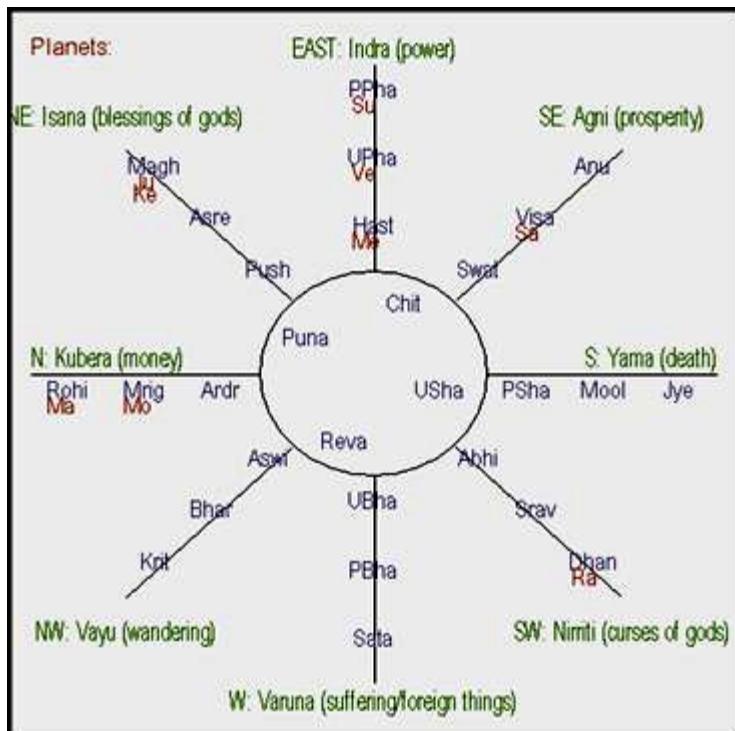
By examining this kalachakra chart based upon the Sun's location, an astrologer can determine certain trends during each particular period or dasha cycle in the life of any individual. Such forecasts of either favorable or unfavorable results rely upon the directions that the planets align within the kalachakra. Planets that fall in the section that is closer to the areas around the southern direction always offer more challenging experiences during their planetary periods. On the other hand, planets that shine to the north and its surrounding directions offer blessings and other good results.

#### **KALA CHAKRA IN THE CHART OF SHRILA PRABHUPADA**



Below is Srila Prabhupada's kalachakra chart. His Divine Grace single-handedly spread the sankirtana movement all over the world during the years of his planetary periods of Mercury and Ketu (according to the 120-year system of planetary cycles called vimshottari dasha). Srila Prabhupada's Mercury period ended around August 1971. In Srila Prabhupada's horoscope, exalted Mercury forms bhadra mahapurusha yoga in the 10th house karmastan. Note how in the kalachakra, Budha (Mercury) is powerfully positioned in the eastern direction of spiritual power alongside Sun and Venus. The eastern side of the kalachakra indicates that His Divine Grace has the blessings of the king of heaven lending additional power to the yoga.

Next, Ketu period followed the Budha dasha. In Shrila Prabhupada's chart, Ketu is on the North-east side of blessings that are given by the grace of Ishana (Shiva). Ketu is positioned in the east alongside the planet of dharma (Jupiter). Ketu sits in the kingly nakshatra of Magha, "the great one."



Kalachakra or "Wheel of Time" chart of Shrila Prabhupada

## UNDERSTANDING YOUR KALACHAKRA

When a weak or destructive planet is positioned in any of the eight directions, it is best to avoid a prolonged stay in this particular direction in one's residence. Remaining in those directions in the house or office can promote the effects associated with the weak or harmful planet. On the contrary, the person should spend more time in his favorable directions where good planets dwell. Following these simple guidelines can make a person more content, which for us devotees is another inducement to the practice of bhakti yoga.

As Vedangas, the sciences of jyotish and vastu are meant to help free us from negative effects of past karmas and thus become more inclined to engage in devotional service to Shri Krishna, the Supreme Personality of Godhead. Practicing these sciences for any other goal would contradict their ultimate purpose as adjuncts (angas or "limbs") to bhakti and only implicate the living entity in never-ending karmic reactions. For example as we have seen from the Krishna Book, Dvaraka was perfectly planned according to vastu which helped each fortunate Dvarakavasi to live in Krishna's city in perfect Krishna consciousness. This is the goal of vastu.

(from Hare Krsna website)

# Conclusion

Pascal's famous Triangle belongs to Mount Meru, as Frank "Tony" Smith points out on his monumental website. As the author has argued in a previous Vixra paper, many of the mathematical "discoveries" of the past 500 to 1000 years have simply been versions of Indian mathematics translated into Chinese, Persian, Greek, Arabic or Latin. Westerners are accustomed to thinking of their society as an improvement over some ancient rude society: in fact, these mathematical discoveries constitute nothing more than the crumbs of a far more advanced ancient society than contemporary civilization will ever know. Hindu civilization continues to preserve this for us, yet the task grows more difficult each year under Kali Yuga. Smith has done a great deal to re-build Mount Meru on his website and his papers, at the level of Clifford Algebras, and this is the proper direction for mathematical physics to take at this time.

This paper is in some ways incomplete and unfinished, and may require a later edition. Some of the charts begun here may have great heuristic importance. Whether or not one may substitute an ancient Indian god for a Clifford Algebra remains to be worked out, yet the suggestion begins here. Whether or not it remains mathematically possible to apply the laws of Pascal's Triangle to the Clifford Algebra Triangle constructed by Frank "Tony" Smith remains to be worked out as well. In principle, it appears that the rules which apply to one triangle ought to apply to the other. We live in a combinatorial universe and for this reason these questions deserve exploration and research.

The matching of the Vastu Purusha 9 x 9 grid with the Matrix / Clifford Algebra grid may yield benefits, especially in the area of Five Elements. Doctors of Chinese medicine work with the Five Elements each day, but lack intellectual theory for why they work. If mathematicians research the corresponding Clifford Algebras to each of the Five Elements, this could lead to new theory.

A further avenue research is the Hamiltonian Path or the Clifford Path suggested by the Hare Krsna paper and illustration. Does Prana / Qi / Ki move in these stated directions?

Much work remains to be done with the Clifford Clock to align the Trigrams properly with the Vastu Purusha: the present arrangement may not work adequately.

The author has written papers published on Vixra which theorize that the borders between the stable 8 x 8 Satva state of matter and the dynamic 9 x 9 Raja state of matter are probably marked with the Golden Section and Fibonacci Numbers. Moreover, the author has stated the importance of Fibonacci (Pisano) Periodicity and Bott Periodicity (described above) in the growth of matter in the universe. This paper has suggested a close relationship between Clifford Algebras and Fibonacci Numbers, linking these to Vedic deities. In ways which remain unclear to the author, somehow these concepts all fit in to a complete model of the development of matter into structures in the universe, and this deserves further research.

This paper presents the 9 x 9 grid of the Vastu Purusha and its assembly of deities, suggesting that they bear a numerical relationship in an analogous manner to the *Neter* of remotely ancient Egypt, described by Schwaller de Liebeicz. Rig Veda scholars may use the instances included in this paper to deconstruct the passages of the Rig Veda in search for the numerical codes embedded in the superficial stories about the gods. Christopher Minkowski has done so for the Magic Squares that Nilakantha discovered in the Rig Veda, and the sources this author has used for writing about Vedic Physics both rely upon secret decoded scientific text embedded within the Rig Veda and Vedic Literature.

As noted above, many of the words used to describe the lives of Vedic deities bear scientific meanings, specifically to Vedic Nuclear Physics, such as *purusha* and *loka*. Others have engendering relationships, such as Indra as mother. Some have matching or twin relationships, such as Mitra and Varuna. The author deeply suspects that the stories in the Rig Veda, the Puranas and the Bagavad Gita श्रीमद्भगवद्गीता contain code for describing nuclear physics – secrets which remain unknown in the west. Of course, this will prove an object of future research for the author.



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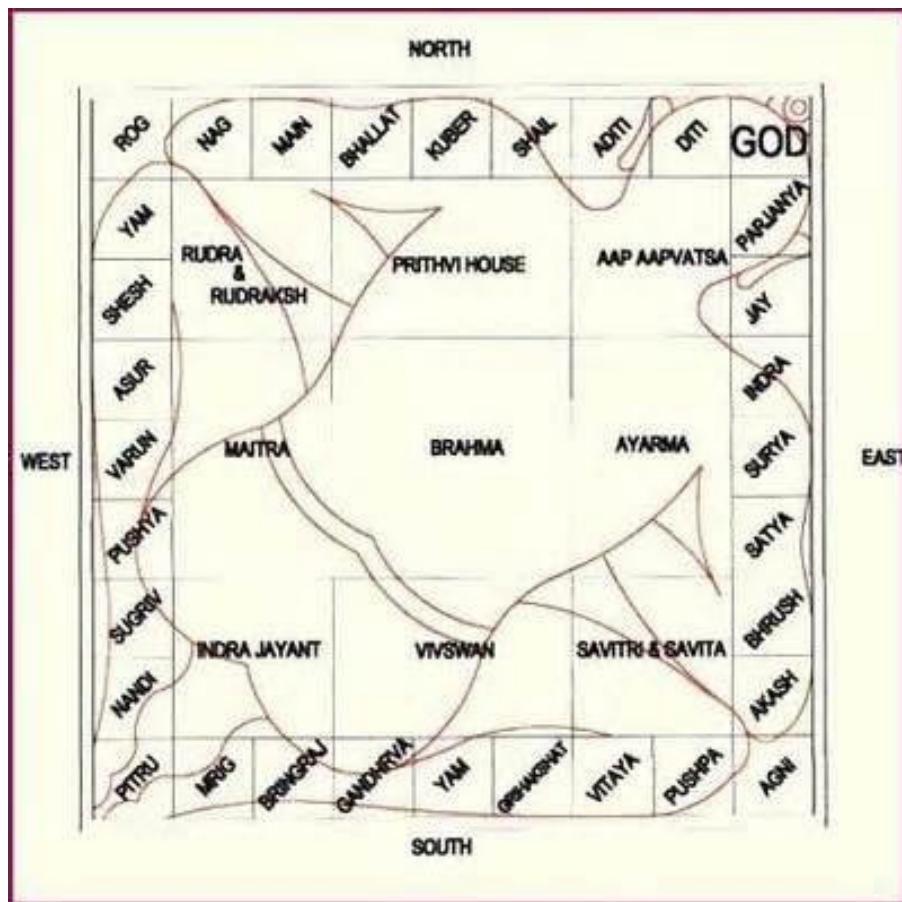
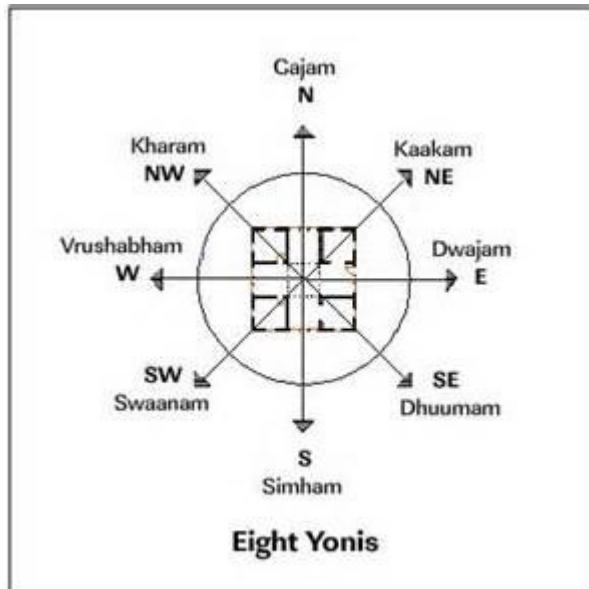
<http://www.goldenumber.net/pascals-triangle/>

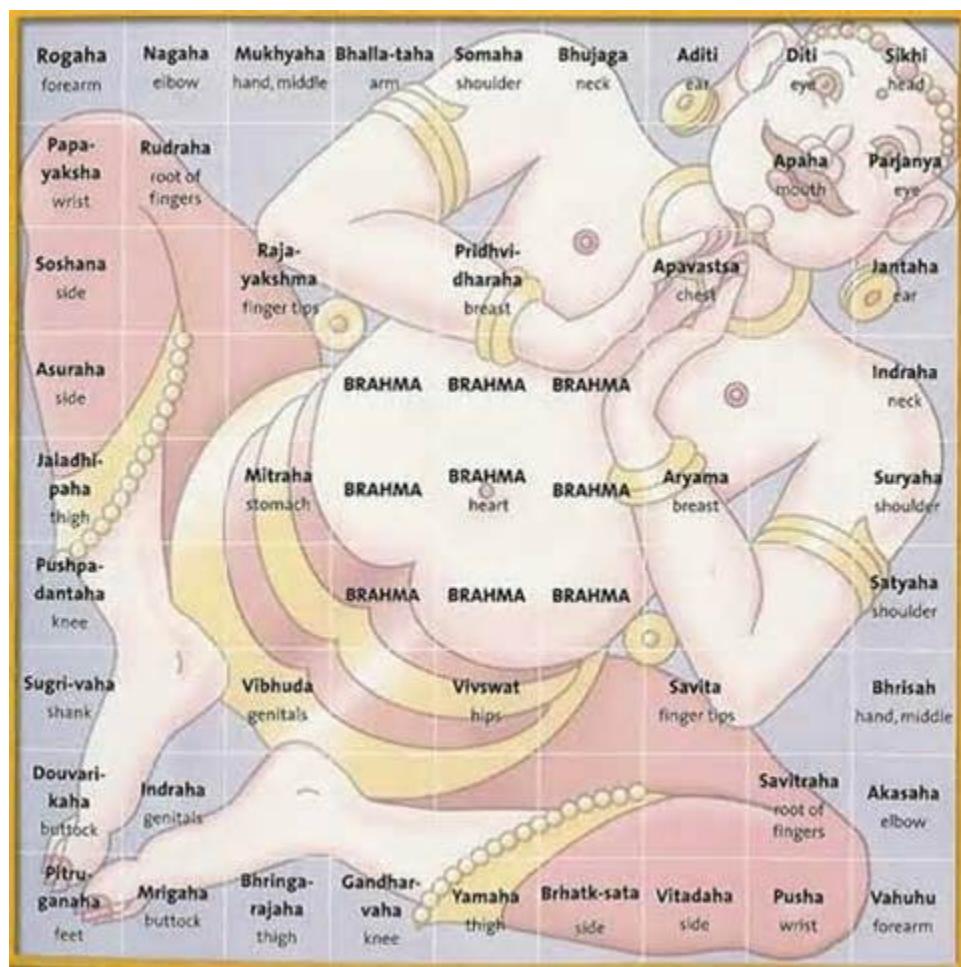
<http://www.harekrnsa.com/sun/editorials/09-13/editorials10737.htm>

NW		N			NE	
Store	Conjugated Living	Treasure	Medical Room	Pooja		
Place for Mourning				Multi-Purpose Room		
W Dinning		Centre (Free from Obstruction)		Bathroom E		
Study				Curd Turning		
Armoury or Maternity Room	Toilet	Sleeping	Chee Store	Cooking Fire		

# Appendix I

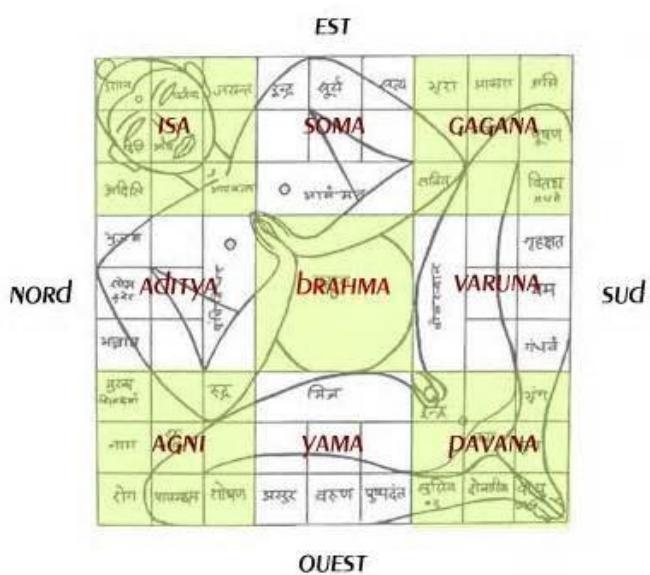
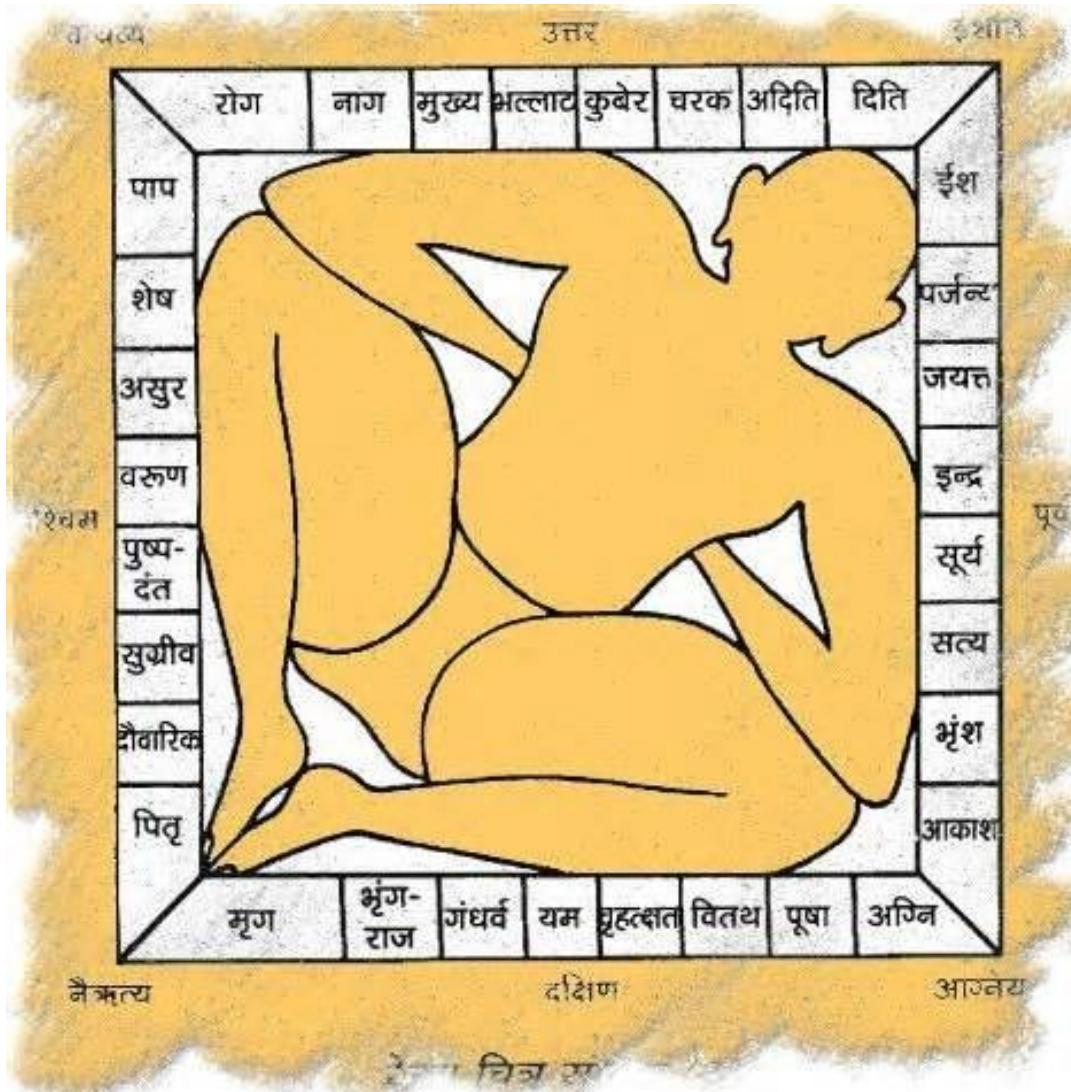
Internet graphics about the Vastu Purusha



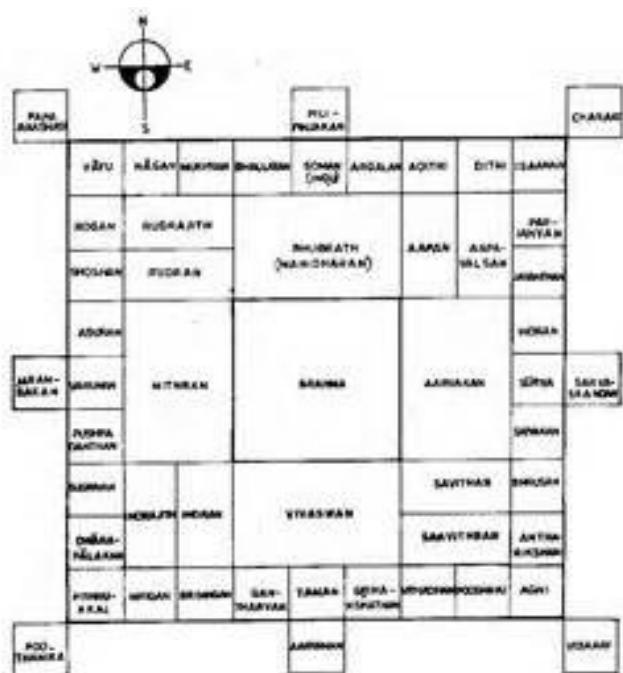
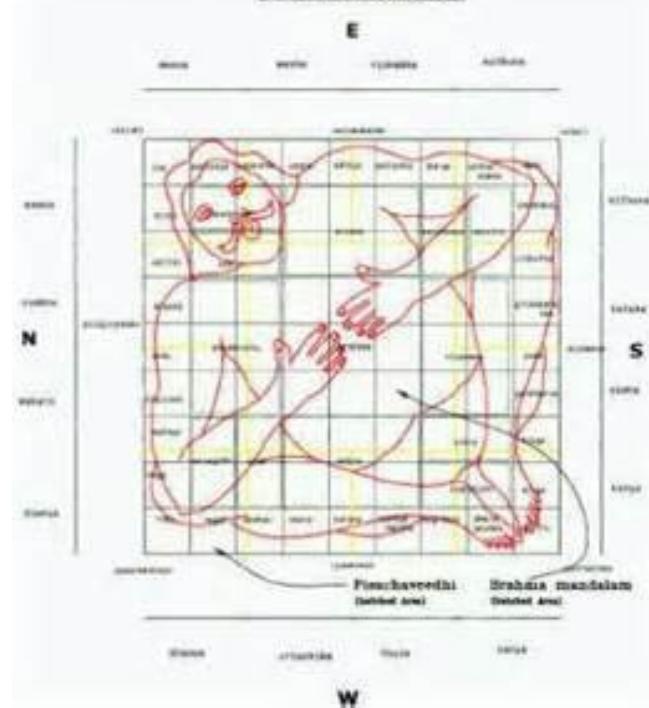


### VASTUPURUSHAMANDALA

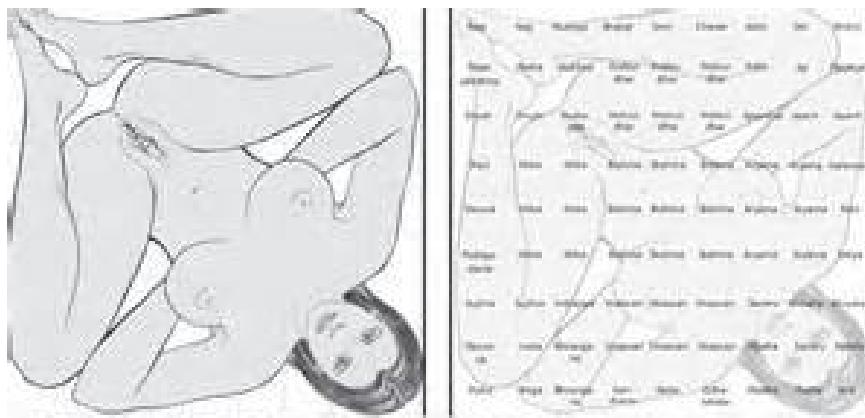
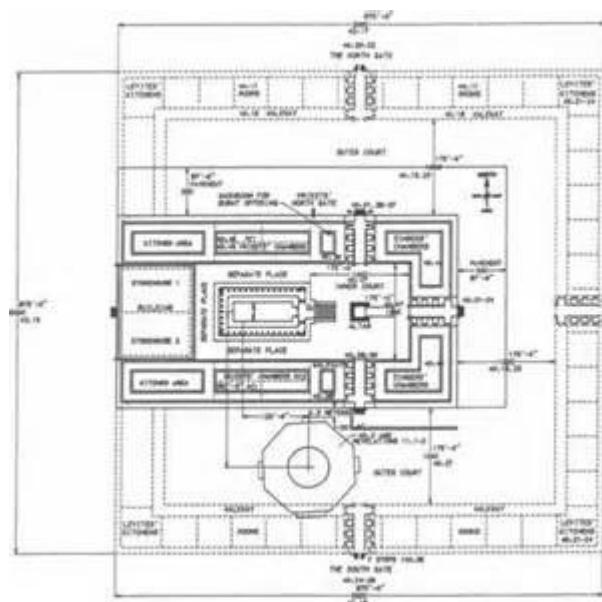
TAPASAKARNA		PRITHIVIDHARA								GHRANA	
ROGA	AHE	MUKHTA	BHALLATA	SOMA	BHUJAGA	ADITI	DITI	AGNI			
PAPA-YAKSHMAN	RUDRA						APA		PARJANYA		
SHOSHNA		RAJA-YAKSHMAN				APA-VATRA			JAYANTA		
ASURA			M			R			INDRA		
VARUNA			I			V			SURYA		
KUSUMA-DANTA			T			A			SATYA		
SUGRIVA		INDRA	R			M			BHRISHA		
DAU-VARIKA	JAYA		A			A		SAVITRA	ANTARIKSHA		
PITAKA	MRIGA	BHRINGA-RAJA	V	GAN-DHARVA	YAMA	H			ANILA		
			Y			I					
			Y			T					
			Y			A					
			Y			N					



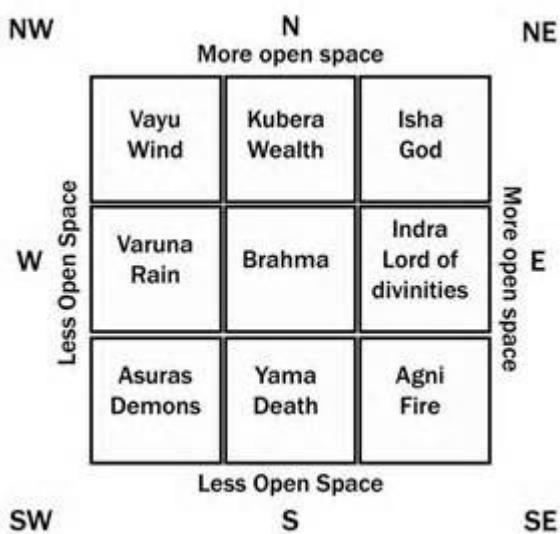
**VASTHUPURUSHAN**



**DIAGRAM 3**  
**VAASTHU-MANDALAM (81 Padas)**



VASTU PURUSHA MANDALA. Ondas purasas, no projecto para exemplar no centro metropolitano de Pampulha, representando os 27 bhupura, ou canto de vaidika, dentro de um mandala a forma de setor, cada canto correspondente a direção cardinal e sua divindade. Refere-se à figura para o da sala (casa), a qual fazendo uso desse conceito, integrado com os 9 casas, criada para locação urbana com cinco portas (ver Fig. 10), ilustrando a localização das fachadas e comportamento.



# Appendix II

Wiki on extending Pascal's Triangle to Negative Numbers

Another option for extending Pascal's triangle to negative rows comes from extending the *other* line of 1s:

$m = -4$	$m = -3$	$m = -2$	$m = -1$	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$\dots$
$n = -4$	1	0	0	0	0	0	0	0	0	$\dots$
$n = -3$		1	0	0	0	0	0	0	0	$\dots$
$n = -2$			1	0	0	0	0	0	0	$\dots$
$n = -1$				1	0	0	0	0	0	$\dots$
$n = 0$	0	0	0	1	0	0	0	0	0	$\dots$
$n = 1$	0	0	0	1	1	0	0	0	0	$\dots$
$n = 2$	0	0	0	1	2	1	0	0	0	$\dots$
$n = 3$	0	0	0	1	3	3	1	0	0	$\dots$
$n = 4$	0	0	0	1	4	6	4	1	0	$\dots$

Applying the same rule as before leads to

$m = -4$	$m = -3$	$m = -2$	$m = -1$	$m = 0$	$m = 1$	$m = 2$	$m = 3$	$m = 4$	$m = 5$	$\dots$
$n = -4$	1	0	0	0	0	0	0	0	0	$\dots$
$n = -3$	-3	1	0	0	0	0	0	0	0	$\dots$
$n = -2$	3	-2	1	0	0	0	0	0	0	$\dots$
$n = -1$	-1	1	-1	1	0	0	0	0	0	$\dots$
$n = 0$	0	0	0	1	0	0	0	0	0	$\dots$
$n = 1$	0	0	0	1	1	0	0	0	0	$\dots$
$n = 2$	0	0	0	1	2	1	0	0	0	$\dots$
$n = 3$	0	0	0	1	3	3	1	0	0	$\dots$
$n = 4$	0	0	0	1	4	6	4	1	0	$\dots$

Note that this extension has the properties: that just as

$$\exp \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & 2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 3 & \cdot & \cdot \\ \cdot & \cdot & \cdot & 4 & \cdot \end{pmatrix} = \begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot \\ 1 & 1 & \cdot & \cdot & \cdot \\ 1 & 2 & 1 & \cdot & \cdot \\ 1 & 3 & 3 & 1 & \cdot \\ 1 & 4 & 6 & 4 & 1 \end{pmatrix},$$

we have

$$\exp \begin{pmatrix} \cdot & \cdot \\ -4 & \cdot \\ \cdot & -3 & \cdot \\ \cdot & \cdot & -2 & \cdot \\ \cdot & \cdot & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 2 & \cdot & \cdot & \cdot \\ \cdot & 3 & \cdot & \cdot \\ \cdot & 4 & \cdot \end{pmatrix} = \begin{pmatrix} 1 & \cdot \\ -4 & 1 & \cdot \\ 6 & -3 & 1 & \cdot \\ -4 & 3 & -2 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & -1 & 1 & -1 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 & \cdot & \cdot \\ \cdot & 1 & 2 & 1 \\ \cdot & 1 & 3 \\ \cdot & 4 \end{pmatrix}$$

Just as summing along the lower-left to upper-right diagonals of the Pascal matrix yields the [Fibonacci numbers](#), this second type of extension still sums to the Fibonacci numbers for negative index.

Either of these extensions can be reached if we define

$$\binom{n}{k} = \frac{n!}{(n-k)!k!} \equiv \frac{\Gamma(n+1)}{\Gamma(n-k+1)\Gamma(k+1)}$$

and take certain limits of the [Gamma function](#),  $\Gamma(z)$ .

# Contact

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**Some men see things as they are and say *why?* I dream things that never were and say *why not?***

**Let's dedicate ourselves to what the Greeks wrote so many years ago:**

**to tame the savageness of man and make gentle the life of this world.**

**Robert Francis Kennedy**