

On the Instrumentation of an Oedometer for Hoop Strain Measurement

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Abstract. Oedometer cells are commonly used in geotechnical engineering laboratory to determine the one-dimensional deformation of soil samples. The cell is simply a short open-ended cylinder, capped with a pair of porous discs, for containing the soil. Load is applied on the sample with a lever arm mechanism, which transfers the load from weights placed on a hanger suspended away from the oedometer cell. In the laterally-confined test condition, simultaneous measurement of radial stress of the soil sample under vertical load is understandably desirable, to obtain a more comprehensive picture of the soil's behaviour at-rest. Direct measurement of the soil body is almost impossible in an oedometer cell, but radial stress can be indirectly gauged by instrumentation of the cell wall. This paper describes the design and building of an instrumented floating type oedometer cell, which concurrently measures vertical deformation as well as radial stress of stabilized soils. Based on fundamental hoop strain principles, 2 pairs of micro-strain foil gauges, perpendicularly arranged and affixed on opposite sides of the cell's outer wall, were connected in a Wheatstone full-bridge circuit for maximum voltage output. The design, construction and installation procedure as well as calibration methods are detailed in this paper to illustrate feasibility of the instrumentation adopted. The technique can be easily duplicated for similar rigid type cells and provide an economical means of monitoring hoop strain, and hence radial stress of soils under loading.

Keywords: instrumentation, hoop strain, radial stress, soil, oedometer

1 Introduction

Often due to the large lateral extent of applied loading, lateral deformation is negligible relative to the vertical displacement. Therefore compression of the soil is considered to be one-dimensional with zero lateral strain. This is known as the K_0 -condition. K_0 values are dependent on the soil microstructure or fabric, strength and stress history (Edil and Dhowian 1981). It was also reported that in unstabilized soil, K_0 is known to remain constant during virgin compression but increases during unloading. Hence, K_0 values tend to decrease when a soil sample is being loaded from an overconsolidated condition.

The measurement methods of lateral stress under K_0 loading conditions are categorised into two main groups, namely the rigid and flexible lateral boundary methods (Ting et al. 1994). The rigid lateral boundary method uses an oedometer type apparatus, providing the zero lateral strain condition, but usually allowing for undefined friction between the soil sample and the internal wall of the cell. In practice, very small lateral strain (e.g. microstrain) is actually allowed for the cell containing the sample to facilitate K_0 measurements. A full-bridge strain gauge circuit, with strain gauges mounted on the oedometer cell wall, has perhaps been most widely adopted (e.g. Newlin 1965, Edil and Dhowian 1981, Zhu et al. 1995). Brooker and Ireland (1965) and Singh et al. (1973) developed a null strain system, regulating hydraulic pressure at the back of a thin-walled oedometer ring to measure the lateral stresses. Abdelhamid and Krizek (1976) attached flush diaphragm transducers to a rigid oedometer cell for monitoring lateral stress changes. Also, Thomann and Hryciw (1990) used a horizontal loading piston with a load cell connection for lateral stress measurements in an oedometer. On the other hand, the flexible lateral boundary method utilises feedback systems to maintain the position of the boundaries, as in triaxial type equipment. The apparent advantage of this method is the absence of side friction, but the inherent disadvantage is that the best that can be achieved with the soil sample is zero mean lateral stress. With a flexible lateral boundary, Bishop (1958), Moore (1971), Menzies et al. (1977) incorporated various local lateral strain measurement devices in conventional triaxial apparatus, while regulating the cell pressure to achieve zero lateral strain conditions.

Very little research has been reported on K_0 values in stabilized soils. Zhu et al. (1995) studied the effect of artificial cementation on the lateral stress in sands, and showed that lateral stress decreases with higher cement content. Adopting a very similar fixed ring stainless steel strain-gauged oedometer cell, but with an almost doubled wall thickness of 1.5 mm, the authors successfully measured K_0 values for artificially cemented sands, even with a hoop strain in the region of 0.4 micro strain. Earlier, Edil and Dhowian (1981), using a strain-gauged stainless steel test tube for peat samples, quoted a maximum hoop strain of the order of 0.2 micro strain, which was claimed to be far too small to bring K_0 values down to active values as proposed by Terzaghi (1934) and Bishop (1958). In 1973, Andrawes and El-Sohby (1973) proved that K_0 was unaffected by hoop strains within the limit of 0.15 %, by interpolation from two constant stress ratio tests on dense glass ballotini.

With the aid of these past records and some calculated innovations, a segmented thin-walled oedometer cell has been successfully instrumented to monitor radial stress of stabilized soils. With the enhanced stiffness of the stabilized material, one of the biggest challenges of the attempt was to develop a system which could detect and capture the very small radial displacement (translated to radial stress) of the sample within the cell under repeated loading-unloading. The ensuing sections give a thorough account of the oedometer cell, instrumentation procedure and calibration methods.

2 Materials and Methodology

2.1 Oedometer Cell

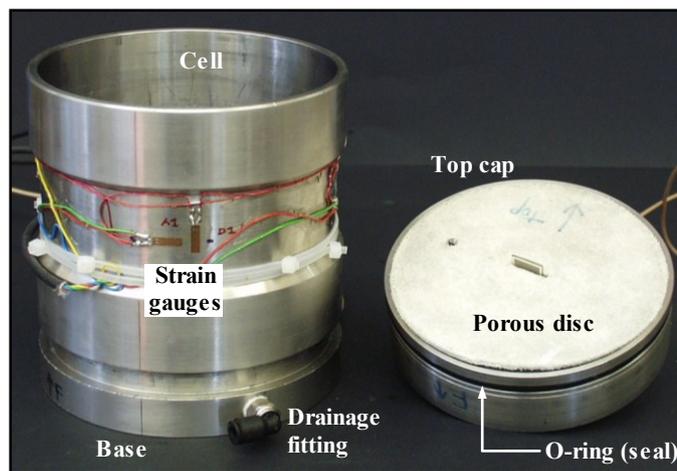


Fig. 1. The instrumented oedometer cell, top cap and base.

The oedometer cell was designed and built as a floating type. The stainless steel ring was machined to 100 mm diameter and 90 mm high with a thin-walled middle section 30 mm high (Fig. 1). ASTM Standards D2435-96 (2000) stipulated a minimum aspect ratio (diameter to height) of 2.5 for samples not less

than 50 mm in diameter and 12.5 mm in height. Considering that the cell was a floating type, the sample height was taken as half of the sample height (i.e. $0.5 \times 70 \text{ mm} = 35 \text{ mm}$) and the ratio obtained was 2.86, fulfilling the ASTM requirements. The cell was mounted on a conventional Wykeham Farrance oedometer frame with an LVDT (linear variable differential transformer) for automated monitoring of the vertical displacement. Drainage was allowed from both the top and bottom of the sample via the top cap and base of the cell. Porous discs were placed at the contact of the top cap and base with the sample. A burette was connected to the drainage tubes for monitoring volume change due to water expulsion. Both the top cap and base had o-rings for sealing the sample within the cell and the resulting frictional loss was found to be negligible under normal loading circumstances.

2.2 Instrumentation of the Cell: Strain Gauges

The cell had a total length of 90 mm which varied in thickness. The 30 mm high thin-walled section 0.8 mm thick was sandwiched between the two equal heights 3 mm thick sections (Fig. 2). The thickness of the thin-walled section was at the limit of what the milling machine available in the workshop could achieve without producing a cell with non-uniform all round thickness. The top and bottom thicker sections provided support and protection to the middle thin wall, especially for handling when

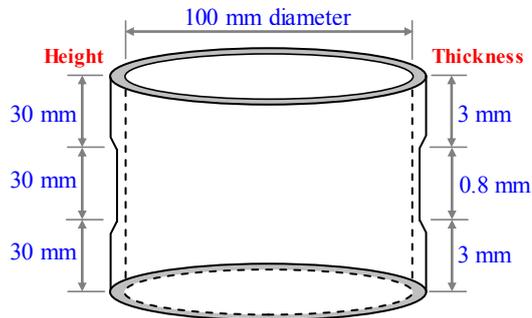


Fig. 2. Dimensions of the oedo-BE ring.

the cell was fully instrumented. A thin wall all through would also have subjected the cell to the risk of deformation and damage during compaction of soil sample within.

The thin-walled mid-height section of the cell was instrumented with two identical Wheatstone full-bridge strain gauge circuits, labelled as A1/A2 and A3/A4 (referring to the horizontal gauges in each circuit) to measure lateral stresses. The strain gauges were KYOWA foil

gauges, KYOWA KFG-5-120-C1-16, by Kyowa Electronic Instruments Co. Ltd. (Japan).

Each circuit consisted of two pairs of gauges placed on opposite sides of the cell, and the two gauges in each pair were arranged perpendicular to each other (Fig. 1). The vertical gauges, though subjected to far less strain compared to their horizontal counterparts, were still active. Hoop strain and vertical strain induced by lateral stress on the internal cell wall were detected by the strain gauges, which in turn generated an output via the circuits. The micro-strains generated a very small voltage output that was amplified 500 times by the Fylde FE-492-BBS bridge conditioner and FE-254-GA differential DC pre-amplifier prior to being logged by a Viglen Genie Professional 4Dx33 computer. With input of the corresponding calibration factors to the logging program, QuickLog PC by Strawberry Tree Inc., measurement of the lateral stress within the ring was achieved.

3 Results and Discussions

3.1 Hoop Strain

The hoop strain induced by an estimated maximum internal pressure of 462 kPa was 0.015 micro strain, equivalent to a change of 1.5 microns in the sample's diameter. Such a minute deviation from zero hoop strain should have a negligible effect on values of K_o , as shown by previous researchers (see section 1).

Using Jaky's (1944) expression for normally consolidated clay, the coefficient of lateral earth pressure at rest, K_o , for a typical clay sample was estimated.

Taking the angle of shearing resistance, $\phi' = 25^\circ$,

$$\begin{aligned} [K_o &= 1 - \sin \phi'] && \text{(Eq. 1)} \\ \Rightarrow K_o &= 1 - \sin (25^\circ) \\ \mathbf{K_o} &= \mathbf{0.577} \end{aligned}$$

At maximum vertical stress, $\sigma_v = 800$ kPa, the internal pressure or lateral stress, σ_h was calculated as follows:

$$\begin{aligned} [K_o &= \sigma_h / \sigma_v'] && \text{(Eq. 2)} \\ \Rightarrow 0.577 &= \sigma_h / 800 \\ \mathbf{\sigma_h} &= \mathbf{462 \text{ kPa}} \end{aligned}$$

Based on the maximum internal pressure of 462 kPa, the hoop stress and corresponding hoop strain were obtained using the following equation for a thin-walled cylinder (e.g. Roark and Young 1975).

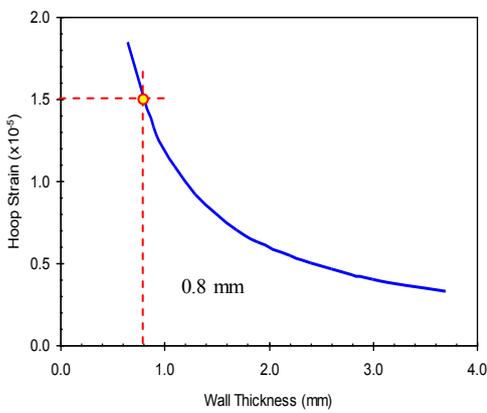


Fig. 3. Hoop strain – wall thickness plot.

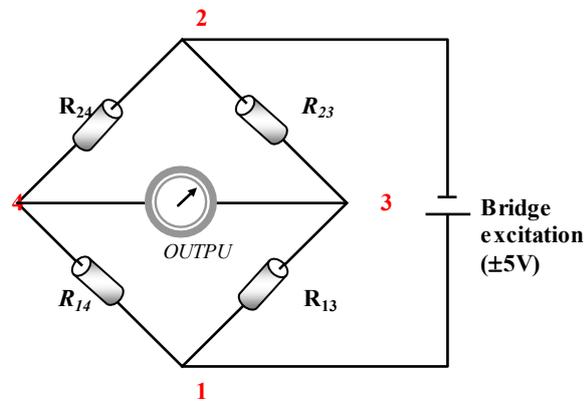


Fig. 4. Wheatstone full-bridge circuit for the strain gauges.

$$\begin{aligned} [\sigma_h &= \sigma_c t / r], && \text{(Eq. 3)} \\ \text{where } \sigma_h &= \text{internal pressure or lateral stress} \\ &= 462 \text{ kPa} \\ \sigma_c &= \text{hoop stress} \\ t &= \text{wall thickness} \\ r &= \text{radius of cylinder} \end{aligned}$$

$$\Rightarrow 462 \text{ kPa} = \frac{\sigma_c (0.0008 \text{ m})}{(0.05 \text{ m})}$$

$$\sigma_c = 28875 \text{ kPa}$$

where
Type 316)

$$[E = \sigma_c / \epsilon_c] \quad (\text{Eq. 4})$$

$$E = \text{Young's modulus (stainless steel)}$$

$$= 193 \text{ MPa (Callister 1991)}$$

$$\epsilon_c = \text{hoop strain}$$

$$\Rightarrow 193 \times 10^6 \text{ kPa} = 28875 \text{ kPa} / \epsilon_c$$

$$\epsilon_c = 1.496 \times 10^{-4}$$

$$\epsilon_c = 1.496 \times 10^{-2} \%$$

A plot of the hoop strain induced by σ_h of 462 kPa against various wall thicknesses is shown in Fig. 3. It was important to have a wall that was as thin as was practically possible, yet sufficiently robust to endure repeated use. Indeed, the ring was re-gauged after four initial tests due to one of the circuits breaking down; comparison of the calibration factors before and after the first four tests clearly indicated a problematic circuit. Although the other circuit was unimpaired, the tight clearance between the wiring and adjoining protective coating made it impossible to strip the malfunctioning circuit off without the risk of ripping off the other.

3.2 Strain Gauge Output

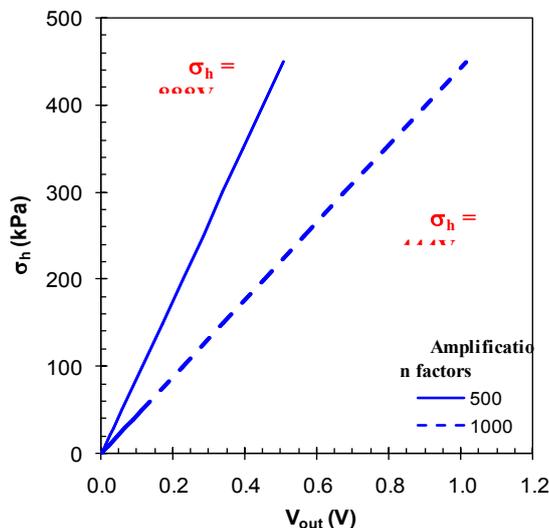


Fig. 5. Predicted calibration chart with amplification factors of 500 and 1000.

The gauges were organized in a Wheatstone full-bridge circuit to achieve maximum voltage output (Fig. 4). Two similar circuits were mounted on the ring, with a total of eight strain gauges. This was primarily to ensure a backup should either of the circuits malfunction. Each circuit consisted of four gauges, with a pair on opposite sides of the ring. In each pair, the gauges were arranged

perpendicular to one another: the horizontally orientated gauge measured the hoop strain while its vertical counterpart registered the vertical strain.

The calculations were essentially based on Ohm's law, as briefly explained below.

Ohm's Law: The electrical current in any conductor is proportional to the potential difference (voltage) between its ends, with all other factors remaining constant.

$$[V = IR] \quad \text{(Eq. 5)}$$

where

V	=	potential difference =	input voltage (V_{in})
I	=	current	
R	=	resistance	

Referring to Fig. 4 and taking Poisson's ratio, $\nu = 0.3$ (Callister 1991) for the ring, estimated outputs for both arms of the full-bridge circuit (i.e. 2-3-1 and 2-4-1) were as follow:

Horizontal gauges:

$$R_{24} = R + \delta R$$

$$R_{14} = R - 0.3\delta R$$

Vertical gauges:

$$R_{13} = R + \delta R$$

$$R_{23} = R - 0.3\delta R$$

$$I_{241} = V / (R_{24} + R_{14})$$

$$\Rightarrow I_{241} = V_{in} / (R + \delta R) + (R - 0.3\delta R)$$

$$I_{241} = V_{in} / (2R + 0.7\delta R)$$

$$V_{24} = I_{241} R_{24}$$

$$\Rightarrow V_{24} = V_{in} (R + \delta R) / (2R + 0.7\delta R)$$

$$V_{24} = (V_{in}R + V_{in}\delta R) / (2R + 0.7\delta R)$$

$$I_{231} = V / (R_{23} + R_{13})$$

$$\Rightarrow I_{231} = V_{in} / (R - 0.3\delta R) + (R + \delta R)$$

$$I_{231} = V_{in} / (2R + 0.7\delta R)$$

$$V_{23} = I_{231} R_{23}$$

$$\Rightarrow V_{23} = V_{in} (R - 0.3\delta R) / (2R + 0.7\delta R)$$

$$\begin{aligned} V_{23} &= (V_{in}R - 0.3V_{in}\delta R) / (2R + 0.7\delta R) \\ V_{out} &= \pm V_{23} - V_{24} \\ \Rightarrow V_{out} &= \pm [(V_{in}R - 0.3V_{in}\delta R) / (2R + 0.7\delta R)] - \\ &\quad [(V_{in}R + V_{in}\delta R) / (2R + 0.7\delta R)] \\ V_{out} &= \pm (1.3V_{in}\delta R) / (2R + 0.7\delta R) \end{aligned}$$

For negligibly small values of δR , $(2R + 0.7\delta R) \approx 2R$.

$$\Rightarrow V_{out} = \pm (1.3V_{in}\delta R) / 2R$$

$$\begin{aligned} \text{Gauge factor, } GF &= 2.14 \quad (\text{From manufacturer}) \\ [GF &= \square \delta R / R] / \epsilon \quad (\text{Eq. 6}) \\ \therefore V_{out} &= \pm 0.65 V_{in} GF \epsilon \end{aligned}$$

For V_{in} of ± 5 V and an amplification factor of 1000, as used for the ‘original’ circuits, the output voltage was estimated as follows:

$$\begin{aligned} V_{out} &= \pm 0.65 (5) (2.14) (1.496 \times 10^{-4}) \times (1000) \\ V_{out} &= \pm 1.04 \text{ V} \end{aligned}$$

This in turn produced a calibration factor of 444 kN/m²/V (see calibration chart in Fig. 5).

As for the ‘new’ circuits, V_{in} remained ± 5 V but the amplification factor used was 500, hence the output voltage was halved:

$$\begin{aligned} V_{out} &= \pm 0.65 (5) (2.14) (1.496 \times 10^{-4}) \times (500) \\ V_{out} &= \pm 0.52 \text{ V} \end{aligned}$$

The calibration factor was therefore doubled to 888 kN/m²/V, as shown in Fig. 5. In comparison with the actual calibration factors from both circuits, these predicted factors were consistently 4.5 to 11.5 % lower. Some plausible explanations were discussed in section 3.3.

3.3 Calibration

The strain gauge circuits were calibrated with a Budenberg dead weight tester using pressurized air in the range of 0 to 400 kPa, in three consecutive ascending and

descending cycles. The cell was mounted on the loading frame as in a normal test set-up, with the lever arm clamped down to the frame body. The bottom drainage access was sealed with a plug, while the top drainage access was connected to the calibrator via a plastic tube.

The pressure in the cell, as provided by a pressurized nitrogen gas cylinder, was brought up in increments of 10 kPa from 0 to 50 kPa, followed by increments of 50 kPa for the remaining pressures and vice versa for the descending order. Voltage outputs of the circuits were automatically amplified and logged by a computer.

The estimation of the lateral or hoop strain, and hence the output voltage, was based on the assumption of internal pressure acting on a thin-walled cylinder, as discussed in [section 3.2](#). However the predicted calibration factors were lower than the actual values by 4.5 to 11.5 % ([Table 1](#)), as obtained from calibration carried out after each test. This difference could be attributed to several factors:

- a. There were changes in thickness (from 0.8 mm to 3.0 mm) over the length of the ring, i.e. the ring was not uniform as assumed in the estimations.
- b. The connecting wires probably increased resistances within the circuit, hence lowering the outputs and increasing the calibration factors.
- c. The thin layer of cement between the gauge and the ring wall might have affected the sensitivity of the gauges.
- d. Material properties (i.e. Young's modulus and Poisson's ratio) for the stainless steel were assumed, as no test information was available.

Only two calibrations were carried out for the original circuits, before and after the four initial tests. Following the problem that developed, as a precautionary measure with the new circuits, calibrations were carried out before and after almost every test to ensure the integrity of both circuits. From the small differences in the successive calibration factors (the largest difference being approximately 4.5 % for both circuits), it is apparent that errors caused by changes in the calibration factors would have been relatively small.

Table 1. Calibration factors for strain gauge circuits

Circuits	Test	Calibration Factors (kN/m ² /V)	
		Circuit A1/A2	Circuit A3/A4
ORIGINAL	1		
	2	494.18	467.80
	3		
	4	473.12	474.55
NEW	1		
	2	933.78	971.07
	3	943.85	942.62
	4	969.95	943.83

5	929.55	985.92
6	942.66	959.02
7	937.83	961.23

4 Conclusions and Recommendations

- A floating-type oedometer cell was instrumented with strain gauges in a Wheatstone full-bridge circuit to measure hoop strain, translated to radial stress, of the soil sample contained within.
- The calibration factors, as determined from the initial calibration, were found to be $\square_h = 440V_{out}$ and $880V_{out}$ respectively, depending on the amplification factor used, i.e. 1000 or 500.
- Subsequent calibration of the circuits showed them to be robust and reliable in measuring the hoop strain, hence radial stress, where difference between the predicted or calculated and actual calibration factors was no greater than 12 %.
- As a precautionary step, it is advisable to conduct calibration of the strain gauge circuits regularly after each test, to ensure that reliable output is obtained.

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