The Hubble constant H_0 is not constant, but proportional to the density of free electrons

Herbert Weidner[a]

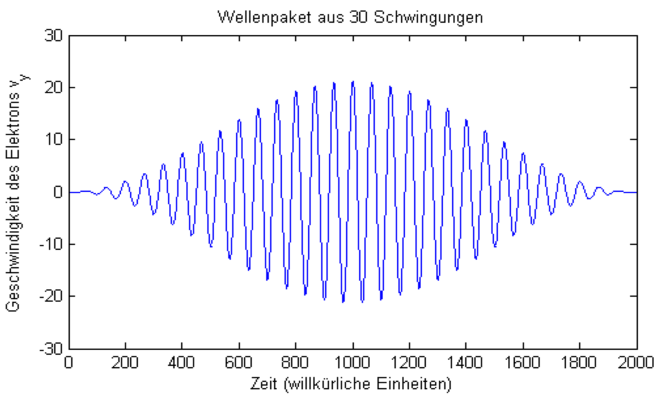
Abstract: If electromagnetic radiation is transmitted from *A* to *B*, the total received amplitude is calculated from the sum of secondary waves of the Fresnel zones. Whenever the electromagnetic wave packet crosses a thin plasma, the free electrons are accelerated and therefore radiate undirected energy which is taken from the wave packet. In dense plasma, this frequency reduction has already been proven[12]. The wave packet does not change its direction, but its frequency will be reduced. The Hubble constant is replaced by $H_0 = c\, w\, n_e$. The correct relationship between distance and redshift is $D = z/(w\, n_e)$. The redshift is no scattering effect and does *not* depend on ω and λ. The measured values of dispersion measure (radio astronomy) and redshift (optical astronomy) depend on each other: $z = w \cdot DM$

Keywords: envelope, wave packet, secondary wave, redshift, unbound electrons, dispersion measure

Introduction

J. J. Thomson assumed that the scattering of light by electrons[1] is a linear process. Under the then possible measurement accuracy the wavelength remained constant. That's not quite right, because the electron is accelerated and therefore radiates[8] energy. Strictly speaking, the term "scattering" is wrong because this implies a change in direction. If an electron is accelerated by linearly polarized light, it can not store energy but radiates transverse to the direction of acceleration like a dipole antenna. There is no preferred direction, so there is no recoil, and afterwards, the electron is at rest again[14]. The direction of light remains unchanged, but it loses a tiny amount of energy[12]. Only when the light encounters many unbound electrons on its way from a great distance, the energy loss is obvious.

Model of the envelope

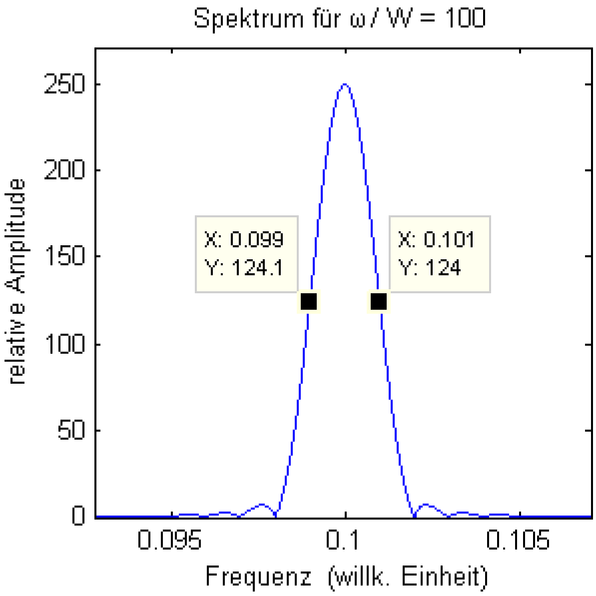


In textbooks, an electromagnetic wave is usually described by the formula $E = E_{max} \cdot \cos(\omega t)$ with $-\infty < t < \infty$ without mentioning that this representation is valid only for infinitely high energy content. The energy of a real wave is always finite and therefore the wave must be limited in time, have a beginning and an end. The wave can not produce an infinite number of infinitely extended wave fronts, as is often assumed in order to simplify the mathematical description. A meaningful discussion must be based on a wave packet of finite duration, whose envelope is continuous and outside a certain interval assumes the value zero. For lines in the optical spectral range, the exact shape of the envelope is unknown. There are

a) 8. Mai 2014, email: herbertweidner@gmx.de

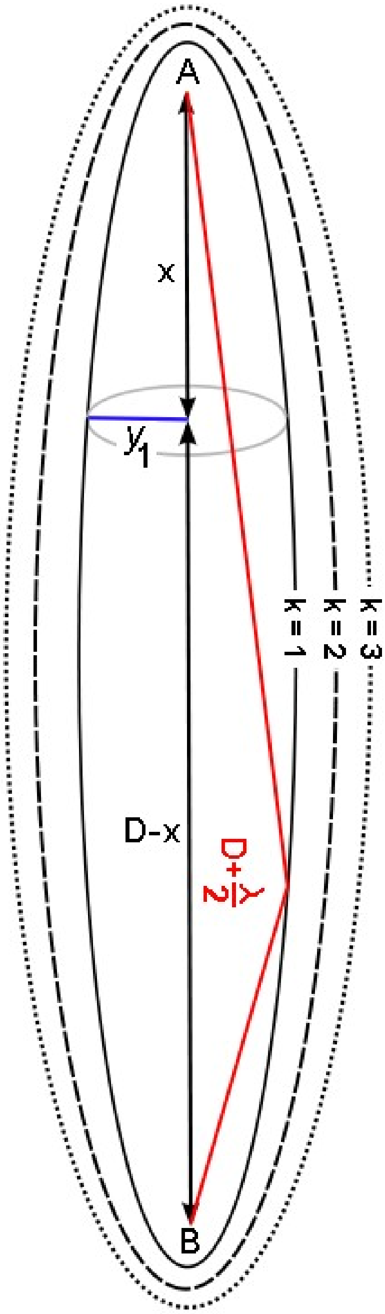
many possible shapes, i use the following: During the time period $0 \le W \cdot t < 2\pi$, the formula $E = \frac{1 - \cos(W\,t)}{2} \cdot E_{max} \cdot \cos(\omega t)$ describes the electric field strength. To produce a slow modulation, the pre-factor must satisfy the condition $W \ll \omega$. The wave packet described is shown in the picture above. Each modulation of a wave produces a certain amount of bandwidth, which can also be measured. For example, the natural line width of the sodium D-line is about 10 MHz, and the wave packet generated lasts about 10^7 cycles, which corresponds to a length of 6 m. The limiting case $W \rightarrow 0$ describes a constant-amplitude wave with infinite extent and is not discussed here.

Each modulation generates so-called sideband frequencies in the vicinity of ω, their amplitudes decrease generally with increasing frequency separation. The sideband frequencies occupy a frequency range which is called natural line width. Numerical tests show that the exact shape of the envelope does not affect the results of this study if the shape is sufficiently smooth and contains no discontinuities. The FWHM bandwidth is $\Delta\omega = 2\,W$ and the line width is $\Delta\lambda \approx \frac{4\pi\,W\,c}{\omega^2} = \frac{\lambda^2 W}{c\,\pi}$. Hereinafter only waveforms are considered, which consist of at least 100 oscillations, so $W \ll \omega$ is ensured. Those assumptions are true for most of the spectral lines.



Once a free, unpaired electron falls into the sphere of influence of the wave packet, it is accelerated by the electric field component during the period $0 \le W \cdot t < 2\pi$. Before and after the electron is at rest. Because the wave packet moves with the speed of light, it has finite length, the coherence length $L \approx \frac{2\pi\,c}{W}$. If the wave moves in the dispersion-free space, the coherence length remains unchanged and there is no wave packet spreading. For virtually all spectral lines in the visible light region, the coherence length is shorter than 10 m and therefore the electron is affected by the wave packet only for the duration of $\Delta t = \frac{L}{c} \approx 33\,ns$. For simplicity, it is assumed that during this short period, no impacts of other plasma particles disturb the unbound electrons, and that the positions of the positive ions of the plasma do not change perceptibly.

The Fresnel zones



In radio-technical terms, the observation of astronomical objects is a point-to-point connection, whose transmission quality is also affected by objects far away from the line of sight. Fresnel had the idea that each light source generates spherical waves that make each space point to the starting point of a new elementary wave. Adding up these at the destination *B* with correct phases and amplitudes, we obtain the received amplitude. Depending on the location of the spatial point *P*, the total path *A-P-B* is always a detour compared to the shortest distance $\overline{AB} = D$. Depending on the length of the detour, the elementary wave starting at *P* leads to constructive or destructive interference at the receiving point *B*. To enable a mathematical description, the line of sight is enveloped by three-dimensional boundaries, which are defined by $\overline{AP} + \overline{PB} = D + \frac{k \cdot \lambda}{2}$ with $k \in 1, 2, 3, ..$. These boundaries are ellipsoids around the line of sight as a symmetry axis, with focal points *A* and *B*.

- The innermost, first Fresnel zone is enclosed by the envelope surface k = 1. All elementary waves arising inside interfere constructively in *B*, because the phase shift is between 0 and π (compared with the shortest path). In that zone the main part of the energy is transfered. If the reception of all other elementary waves is prevented by a suitable pinhole, the amplitude at *B* is doubled. In contrast, when only the reception from the first Fresnel zone is prevented, the received amplitude does not change.
- For all elementary waves from the second Fresnel zone between the interfaces k = 1 and k = 2, the phase shift is between π and 2π (compared to the shortest path). Because they tend to compensate the elementary waves from the first zone due to destructive interference, in a zone plate they are suppressed by an annulus.
- The third Fresnel zone between the interfaces k=2 and k=3 follows. The outgoing elementary waves from here have phase shifts between 2π and 3π and enhance the overall amplitude in *B* by constructive interference.
- The energy contributions from far outboard shells decrease slowly and the elementary waves of adjacent shells largely compensate in pairs.

If the Fresnel ellipsoids are cut at a distance x from the light source transversely to the axis of symmetry and are marked according to the phases (constructive or destructive), a central circle with surrounding concentric circular rings is obtained like a Fresnel zone plate[2]. In sufficient distance from *A* and *B*, the radii of the respective limits are calculated to $y_k = \frac{1}{D}\sqrt{k\,\lambda\,D\,x\,(D-x)}$ with $k \in 1, 2, 3, ..$. When the light source emits radiation with a large coherence length, very many Fresnel zones contribute to the total intensity at the receiving point *B*. The largest radius of every zone is located in the middle of the distance star-earth and has the value $R_k = 0.5 \cdot \sqrt{k\,\lambda\,D}$. For remote objects enormous values are reached:

- if pulsar pulses are measured (f = 430 MHz), the innermost zone has the diameter $2 \cdot R_1(PSR\,B0531+21) = 6.9 \cdot 10^9 m$
- observing the nearest quasar with visible light, the diameter of the first Fresnel zone has about the same size $2 \cdot R_1(3C273) = 3.6 \cdot 10^9 m$.

In the laboratory, the size of the test setup is generally less than the coherence length $L \approx \frac{2\pi c}{W}$ of the radiation, whereas the opposite is true with the optical instruments of astronomy. Here, the distance between *A* and *B* exceeds by far the coherence length of the measured electromagnetic waves, and therefore, only those (inner) Fresnel zone should be considered, whose detour $k \cdot \lambda/2$ is smaller than the coherence length. Only *k* Fresnel zones contribute to the total energy at point *B*, with $1 \le k \le \frac{2\omega}{W} = k_{max}$. Elementary waves departing from further outward Fresnel zones arrive too late at the receiver and can not influence the amplitude at point *B*.

Although the diameters of the Fresnel zones still appear as large, the experience from the construction of optical devices enforces their consideration: Distant galaxies can not be mapped arbitrarily sharp because of the Airy disk, which is generated from the opening of the telescope[3]. If this is to be the sole cause of the blur, the aperture at *any* position of the remaining light path must be so large, that *everywhere* – even at half the galaxy distance – the Fresnel number[4] $F \gg 1$ is achieved. This is equivalent with the condition $R_k^2(max) \gg D\lambda$ or $k_{max} \gg 1$. It is not enough to consider only the first Fresnel zone or even narrow down to the immediate vicinity of the line of sight. This would lead to very pronounced diffraction effects due to $F \ll 1$.

Any matter within the Fresnel zone influences the received signal at point *B*. This can lead to increase in energy when one hides unwanted elementary waves[5] or to light phenomena in unexpected places[6] or, as explained below, to a frequency reduction.

The volume of the inner *k* Fresnel zones is $V_k = \frac{4\pi}{3}\frac{D}{2}R_k^2 = \frac{D^2 \lambda k \pi}{6} = \frac{k \pi^2 D^2 c}{3\omega}$ and is, remarkably, not proportional to D^3, as one might expect. Even if the volume contains no stars, it is filled with the extremely thin, transparent plasma of intergalactic space (IGM). Its average electron density is estimated[7] to be $n_e \approx 0.27/m^3$, but it must be reckoned with enormous deviations, since this estimate is based on questionable values of the ΛCDM model. In the interstellar medium of a galaxy probably values up to $n_e \approx 10^5/m^3$ can be found[8].

Energy loss by accelerated electrons

Each unbound electron in the Fresnel zones is accelerated by the electric field of the wave packet and emits the absorbed energy like a dipole antenna *omnidirectional* (torus-shaped radiation pattern) and immediately[9]. Because (in case of linearly polarized light) the emission occurs perpendicular to the direction of the oscillation of the electron and is of rotational symmetry, the electron experiences no recoil. The direction of the momentum of the wave packet remains unchanged and the image of a distant galaxy is not blurred by the energy loss. The process has nothing to do with the Compton effect[10], because the converted energy is much less than 0.001 eV. Only at much higher energies, asymmetric momentum transfer can occur, which can lead to a change in direction of the wave packet.

A free electron can not absorb a photon and then emit another photon in any direction, since Electrons have no internal structure. But an electron may be accelerated, causing the radiation of energy[11]. For kinetic energy, there is no known minimum amount of energy. Therefore, each unbound electron can absorb an arbitrarily small amount of energy (for example $h\,f/100000$) from the wave packet passing by and radiate that amount omnidirectional. Only a tiny fraction of that tiny amount is emitted in the same direction as the wave packet moves and may be neglected. The wave packet loses this amount, its energy drops to $h\,f_{after}=\frac{99999}{100000}h\,f_{before}$ and the frequency decreases slightly. This reduction is barely detectable with a single free electron. Since a dense plasma contains many electrons, the frequency reduction has already been proven[12].

The gigantic Fresnel zones of astronomy contain much more free electrons than a small volume of plasma in the laboratory and can reduce the energy and thus the frequency of the wave packet considerably stronger. This reduction of frequency is gradual, while the wave packet moves from the light source *A* to the detection location *B* where the difference appears as redshift.

With comparatively very low frequencies of microwave transmissions, the energy $h\,f$ of each wave packet is so small that even with low transmitting power, the Fresnel zones are completely filled with many overlapping wave packets. At high frequencies, particularly at low intensities, the few wave packets are separable.

According to Planck's radiation law[13], the surface of each black body emits only discrete energy quanta $h\cdot f$. To calculate the total energy loss between *A* and *B*, which is caused by all electrons within the inner fresnel zones up to $1\le k\le k_{max}$, it is assumed that a single wave packet with the initial energy $h\cdot f$ leaves the source *A*. The radiation loss per electron is dependent on the energy density at the location of that electron. Since the energy density near $x\approx D/2$ is considerably smaller than around *A* and *B*, the path of a wave packet is followed more accurately.

For all astronomical problems, $y_k\ll D$ is met. Therefore, it is assumed for simplicity that the wave packet is a circular cylinder having the thickness *L* (= coherence length) and the cross-sectional area $F=\pi\,y_k^2(max)$ and flies with the speed of light in parallel to the line of sight through the Fresnel zones. Each cylinder has the volume $V=L\,F=\frac{8\pi^3 c^2 x(D-x)}{D W^2}$ and contains $N=n_e V$ unbound electrons, the electron density is n_e . At low redshift can be assumed for simplicity that the energy of the wave packet equals $h\cdot f$ at any position between *A* and *B*. At high redshift, the energy reduction is to consider along the way. For $z\ll 1$, the mean energy density (energy / volume) *U* and the power density (power / area) *S* are valid for all the electrons of the cylinder:

$$U=\frac{h\,f}{V}=\frac{h\,\omega\,W^2 D}{16\,\pi^4 c^2 x(D-x)}=\frac{S}{c}$$

Each unbound, accelerated electron takes from the wave packet as much energy as its scattering cross-section corresponds to[14]:

$$A_y=\frac{q_e^4\mu_0^2}{8\,m_e^2 W}\frac{h\,\omega\,W^2 D}{16\,\pi^4 c\,x(D-x)}=\frac{q_e^4\mu_0^2 h\,\omega\,W\,D}{128\,m_e^2\pi^4 c\,x(D-x)}$$

If n_e has the same value everywhere, an integration along the entire path ist very simple and supplies the total energy that all the electrons contained in the Fresnel zone take away from the wave packet and emit non-directional and mostly sideways:

$$A_{y,Fresnel}=\int_0^D n_e A_y F\,dx=\frac{q_e^4 \mu_0^2 h\omega}{32 m_e^2 \pi^2} n_e \cdot D=h\cdot\Delta f$$

The wave packet emitted by the source *A* accelerates very many electrons in the Fresnel zones, and thereby loses energy. The receiver *B* measures a frequency reduction by Δf. Again: No unbound electron radiates energy asymmetrical and the wave packet does not change direction.

There is no reason, why the coherence of the radiation from the source should be destroyed.

The Energy loss is Redshift

If a wave packet with the initial energy $h\cdot f$ leaves the source *A*, it reaches the destination *B* with a lower energy $h\cdot(f-\Delta f)$. The relation $2\pi\,\Delta f=\Delta\omega$ allows the calculation of the redshift[15] z.

$$z=\frac{\omega}{\omega-\Delta\omega}-1=\frac{1}{\frac{16 m_e^2\pi}{q_e^4\mu_0^2 n_e D}-1}$$

Uniting the constant factors $w=\frac{q_e^4\mu_0^2}{16\pi m_e^2}=2.48\cdot10^{-29}m^2$, the result can be written more compact:

$$D=\frac{z}{w n_e(z+1)}\approx\frac{z}{w n_e} \quad \text{with} \quad z\ll1$$

This relationship between redshift, density of free electrons and distance was established by classical electrodynamics and contains no arbitrary variable, which can be changed to obtain a desired result. No, that's not quite right: You may vary the number of Fresnel zones that are included in the calculation, because the envelope of the wave packet is not right square. Nevertheless, this formula satisfies some astronomical observations:

- z does not depend on ω and λ
- z does not depend on the coherence length or intensity of the wave packet
- at small distances ($w n_e D\ll1$) z is proportional to the distance *D*

The predictions

It is not always great art to invent a theory that "explains" well-known results. Good theories can be seen as to whether they predict at least one verifiable connection that was previously unknown. Here is a selection:

- There is a simple relation between redshift and distance and the average density of free electrons (and some natural constants). This can be probably verified on laboratory scale.
- The Hubble “constant“ is not constant and physically unfounded and therefore unsuitable as a scale factor for the distance *D*. The decisive factor is the density of unbound electrons in an astonishingly voluminous environment of the “line of sight”.
- In redshift surveys also striking jumps or plateaus of redshift are measured which have so

far been interpreted as cosmic voids, walls and filaments. Maybe unusually strong or weak ionized gas clouds along the line of sight, which can not be directly observed, generate strong nonlinearities and pretend distance jumps, assuming constant electron density. Therefore, the ideas of the large-scale structure of the universe[16] must be fully revised.

- The mean density of free electrons is generated by the surrounding stars and is considerably higher in galaxies than outside. Therefore, it is not sufficient to only measure the redshift. The true distance also depends on the distance of the "line of sight" from galaxies or clusters of galaxies, because a portion of the Fresnel zones passes through areas with greatly enlarged electron density.
- Objects with the same redshift can have very different true distances when the light passes through different degrees of ionized regions. Conversely, the redshifts of objects of the same true distance, but different direction can be clearly distinguished from each other.
- This could help to explain the puzzling "fingers of God" in redshift space galaxy clusters or the speculative redshift quantization[17].
- The electron density of the ISM is calculated from the dispersion in pulsar timing[18]. The term *column density* used in this case must be discussed in more detail in view of the relatively large Fresnel zones. The naive idea that tiny photons fly like bullets along a line of sight (line-of-sight propagation) and are influenced only by the immediate vicinity, is diametrically opposed to the influenceability of a wave packet in the very voluminous Fresnel zones.

Determination of the Hubble-“Constant“ H_0

All astronomical observations are based on measurements of electromagnetic waves, which contain no indication, from which distance they originate. Most information on distances in astronomy are estimates, because there are hardly methods to determine them exactly[19]. The only physically sound and reliable measurement method *parallax*[20] works only in our immediate vicinity to a maximum of 1600 light years[21], which is far less than the diameter of our galaxy. This is so little that the Hubble constant (allegedly) does not affect their constituents. It only acts outside and in fact even with the next companions of our Galaxy, the Magellanic Clouds. Their distance[22] is not doubted and the latest measurements[23] of the redshift of the LMC with the Spitzer Space Telescope yield

$$H_0 \approx 74.3 \frac{km}{s \cdot Mpc} = 2.41 \cdot 10^{-18} \frac{1}{s} \quad .$$

Astronomers believe that this value is universally suitable to determine distances more than 10,000 times as long using the formula $c\,z = H_0 D$. A check by competing methods is excluded. Remarkably, we achieve a completely different result for H_0 with a much larger extragalactic object of our neighborhood, the Andromeda Galaxy, in which even the sign is wrong. As everywhere in science – you have to have a bit of tact to choose the "right" object to be measured so that the result does not deviate too much from the expected value.

The problem are those H_0 -values containing estimated values of a cosmological model. All, for example, the ΛCDM model[24], have arbitrary assumptions such as inflation and numerous "knobs" such as the "cosmological constant", with which almost any desired result can be adjusted without physical justification.

Some comparisons with known data

The Comparison of the above approximation $z = w\, n_e D$ for small distances with the Hubble formula $c\, z = H_0 D$ yields $c\, w\, n_e = H_0$. If you use the currently accepted value of the Hubble "constant", the calculation of the average electron density between our position and LMC yields $n_{Earth-LMC} = \frac{H_0}{c\, w} \approx \frac{324}{m^3}$. This value is significantly lower than the value $17000/m^3$ that has been determined by means of Pulsar timing within our galaxy[7], but larger than $0.27/m^3$ that is thought far out[6]. This value, however, was determined from WMAP data and therefore is based on estimates of the ΛCDM model.

The quasar 3C273 has the redshift 0.158. The combination of this value with the distance 2.44 billion light years (the Hubble formula delivers) gives a value of $n_e = 240/m^3$. The average true density of free electrons along this distance is unknown.

Concluding Remarks

The entire derivation is based on classical physics. It is assumed that the wave packet leaves the source with the initial energy $h \cdot f$. However, the value of the auxiliary variable *h* does not appear in the result and does not affect the derivation. Any other value would yield the same result.

The essential basis of the derivation is the fact that the wave packet is both temporally and spatially limited. One can not expect a meaningful result with infinitely extended wave fronts that will last forever. However, the exact knowledge of the coherence length is not necessary, it does not affect the result of the formula.

For astronomers, an experimental confirmation of the formula $z = w\, n_e D$ would be of paramount importance, even if an accurate analysis should show that the factor $w = \frac{q_e^4 \mu_0^2}{16 \pi\, m_e^2} = 2.48 \cdot 10^{-29}\, m^2$ must be corrected. Then astronomers would have independent and accurate methods to measure the density of unbound electrons between the earth and pulsars:

- $z = w \int_0^D n_e\, ds$ provides a link between n_e and the redshift of spectral lines in the optical range or near the hydrogen line[25], which come from the vicinity of the pulsar.
- Due to the free electron plasma resonance[26], the arrival time of high frequency pulses (about 1 GHz) of a pulsar depends on the frequency[27]. The relationship between n_e and the *dispersion measure* DM, whose value is calculated from the time difference[28], is
  $DM = \int_0^D n_e\, ds$.

The comparison of the two formulas shows that *dispersion measure* and *redshift* depend on each other in spite of different causes: $z = w \cdot DM$. In all previous *DM* mesasurements on pulsars in our galaxis, results to about 1000 pc/cm³ were measured[29]. If one were to measure the *DM* of a pulsar in the LMC (eg, SNR 0538-69.1.), this should have a much lower value, because for LMC the values of H_0 and redshift $z_{LMC} = 1.24 \cdot 10^{-5}$ are quite accurately known[22].

By applying the competing methods, the spiral structure and mass distribution of our galaxy can be measured more exactly.

It would be very instructive if someone can derive a similar formula as $z = w\, n_e D$, assuming light energy is transfered by tiny bullets, called photons. The corpuscular theory can not explain the double-slit experiment[30] nor the Poisson spot[5], not even the Fresnel zones. Also, the *HBT* effect[31] was predicted by classical wave theory and can be explained far easier and better than with quantum mechanics, which originally failed. Only years later an explanation was delivered[32].

To quote Hanbury Brown (1991, p. 121)[33]: "To me the most interesting thing about all this fuss was that so many physicists had failed to grasp how profoundly mysterious light really is, and were reluctant to accept the practical consequences of the fact that modern physics doesn't claim to tell us what things are like 'in themselves' but only how they 'behave'.[…] If our system was really going to work, one would have to imagine photons hanging about waiting for each other in space!"

[1] http://en.wikipedia.org/wiki/Thomson_scattering
[2] http://en.wikipedia.org/wiki/Fresnel_zone_plate
[3] http://en.wikipedia.org/wiki/Airy_Disk
[4] http://en.wikipedia.org/wiki/Fresnel_number
[5] http://en.wikipedia.org/wiki/Zone_plate
[6] http://en.wikipedia.org/wiki/Arago_spot
[7] http://wwwth.mpp.mpg.de/members/raffelt/mypapers/200604.pdf
[8] http://astronomy.swin.edu.au/cms/astro/cosmos/p/Pulsar+Dispersion+Measure
[9] http://en.wikipedia.org/wiki/Larmor_formula
[10] http://en.wikipedia.org/wiki/Compton_scattering
[11] http://en.wikipedia.org/wiki/Bremsstrahlung
[12] C.S.Chen et al. "Investigation of the mechanism of spectral emission and redshifts of atomic line in laser-induced plasmas". Optik 120 (2009) 473-478
[13] http://en.wikipedia.org/wiki/Planck%27s_law
[14] H. Weidner, Inherent Energy Loss of the Thomson Scattering, http://vixra.org/abs/1312.0079
[15] http://en.wikipedia.org/wiki/Redshift
[16] http://en.wikipedia.org/wiki/Large-scale_structure_of_the_cosmos#Large-scale_structure
[17]http://en.wikipedia.org/wiki/Redshift_quantization
[18] Dispersion and Techniques for its Correction, http://gmrt.ncra.tifr.res.in/gmrt_hpage/Users/doc/WEBLF/LFRA/node155.html
[19] http://en.wikipedia.org/wiki/Cosmic_distance_ladder
[20] http://en.wikipedia.org/wiki/Parallax
[21] http://en.wikipedia.org/wiki/Hipparcos
[22] An eclipsing binary distance to the Large Magellanic Cloud accurate to 2 per cent, http://arxiv.org/abs/1303.2063
[23] Carnegie Hubble Program: A Mid-Infrared Calibration of the Hubble Constant, http://arxiv.org/abs/1208.3281
[24] http://en.wikipedia.org/wiki/Lambda-CDM_model
[25] http://en.wikipedia.org/wiki/Hydrogen_line
[26] http://en.wikipedia.org/wiki/Plasma_oscillation
[27] J. H. Taylor, Pulsar distances and the galactic distribution of free electrons, 1993ApJ, 411:674-684
[28] http://en.wikipedia.org/wiki/Dispersion_measure#Dispersion_in_pulsar_timing
[29] Petroff et. al., Dispersion measure variations in a sample of 168 pulsars, http://arxiv.org/pdf/1307.7221.pdf
[30] http://en.wikipedia.org/wiki/Double-slit_experiment
[31] http://en.wikipedia.org/wiki/Hanbury_Brown_and_Twiss_effect
[32] Brannen, Ferguson: The Question of Correlation between Photons in Coherent Light Rays, Nature 178, 1956
[33] http://quantum-history.mpiwg-berlin.mpg.de/news/workshops/hq3/hq3_talks/23_silva.pdf