## 2.1_A SYSTEMIC APPROACH TO COMPLEX ADAPTIVE SYSTEMS SUSTAINABILITY

### *2.1.1_COMPLEX ADAPTIVE SYSTEMS CHARACTERIZATION*

### 2.1.1.1_CONCEPT AND MODELLING OF SYSTEMS

The **concept of ‘system’** *emerges when two or more elements interact differently from how they interact with other elements, forming a distinguishable entity in relation to their environment.*

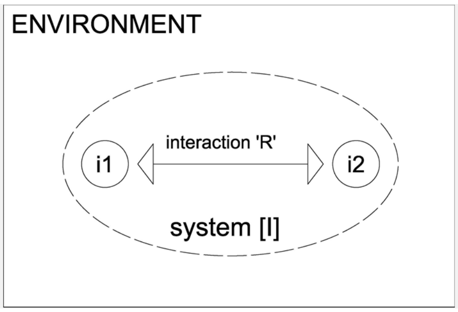

**Figure 10: A ‘system’** *may be defined as 'a set of interacting elements forming a distinguishable entity from its environment'.*

*Alternative definitions may be "a 'complex' of elements interrelated and with their surrounding medium" [von Bertalanffy 1968] or "an interplay of elements that constitute a global entity or whole" [Morin 1977, p.123 ]*

The former description requires us to explain the 'meaning' of four particularly important terms:

An **element or part** of a system I is any physical or conceptual component 'i' that interacts with other components of I, and does not imply 'elementary particle' in an absolute but relative sense; it may be "elementary" from certain perspectives yet being decomposable into smaller parts from other perspectives[1].

An **interaction or relationship** is a relation R between elements 'i' of a system I that modifies their behavior compared to when they were not part of the system, "if their behaviors do not differ [...] there is no interaction between the elements" [von Bertalanffy, 1968:56][2]

The **environment or surrounding medium** E of a system I is ‘everything that is not part of the system'.

And the term **entity** refers to the quality of having a global recognizable form, and implies the ideas of differentiation and autonomy; of environment and identity:

- *A system is only possible within an environment from which it can be differentiated;* the environment is the background that allows the system to be recognized as such.
- *A system requires an identity* capable of maintaining its meaning with some independence or autonomy of the environment in which it is located

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[1] For example, an atom is a complex system in nuclear physics, while a planet may be an element in astronomy [Simon 1962]

[2] And this idea of behaviors that differ anticipates the idea of 'emerging properties' that we review next.

The **concept of complex system**[3] is going to be linked to two key issues; the ideas of 'organization' and 'emergency':

- **Organization** means that not all the elements belonging to the system interact in the same way; *some interactions are more frequent or intense than others;* not all relationships are possible and not all that are possible are equally likely.
- **Emergence** implies that when elements interact, *new properties appear [‘emerge’] that were not in the elements before they joined as a system.*

Organization and Emergence appear inextricably linked: we use the concept of system when two or more elements interact in a stable manner generating a 'whole' which is "greater than the sum of its parts"[4], **organization makes possible and implies the emergence of a global identity.**

Image 01: St. Peter's Piazza**.** *Bernini’s colonnade [1656-1667] acquires as much importance as the Basilica facade [1506-1626] or the longitudinal axis of Via della Conciliazione that leads the view towards Castell Sant'Angello [135-139] or even the empty space that ‘emerges’ as a ‘piazza’.*

*Certain 'organization' of the parts allows the 'emergence' of a global identity that is more than the sum of those parts; 'organization' and 'emergence' are implicit in the idea of system.* Photograph: es.wikimedia.org.

Consequently, these are the two issues that we identify with the "complexity" of the systems, and which we require to be in a group of elements to consider them a 'complex' system. If the elements interact without sustaining a stable 'organization', we do not consider them to form a complex system. And if the elements come together and a global identity does not 'emerge', neither we consider them to be a complex system.

Additionally from both perspectives we can appreciate another feature of complex systems; they structure hierarchically:

**The organization of complex systems has hierarchy; the system is a set of subsystems that in turn include subsystems of lower dimension, and so on down to the basic elements.**

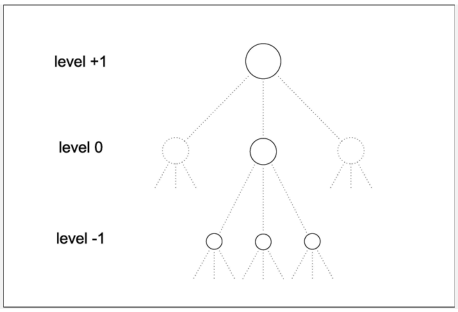

**Figure 11: The concept of hierarchy** *relates to a series of subsystems containing other subsystems, and so on until we reach the elementary parts of the system.*

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[3] In a strict sense, 'complex' and 'system' are synonym and therefore 'all systems are complex'. However, in this text we will use the term 'complex system' in its usual sense [the one that was originally proposed]; referring to systems with organization, hierarchy and some stability.

[4] "A complex system is one made up of a large number of parts that interact in a nonsimple way [so that] the whole is more than the sum of its parts" [Simon 1962, p.468]

Also, **the emergence in systems is an 'architecture' of levels,** the interaction between the fundamental elements makes 'emerge' properties with interacting capability, and their interaction makes other properties 'emerge' on higher levels, and so on, *in a succession of levels that enables the global identity to 'emerge'*[5].

Complex systems are 'structured' as hierarchies located far from thermal equilibrium, but **Second Law of Thermodynamics states that 'reality' tends towards thermal equilibrium and thus systems structures tend to degrade over time; any system inevitably progresses towards its dissolution.**

$$\frac{dH}{dt} > 0 \qquad (1)$$

Where H is Entropy

Entropy [H] necessarily increases over time but Complex Systems seem to oppose the Second Principle by importing negative entropy [negentropy] from its environment and 'dissipating' Entropy towards it; Complex Systems are 'dissipative structures'.

**Complex systems can only exist as open systems to their environment exchanging negentropy and entropy** [can be in the form of matter, energy and/or information], and their sustainability requires the sustainability of their entropy exchanges with the environment.

## 'SUSTAINABILITY DEGREE' AS DISTANCE OF THE SYSTEM TO ITS DISSOLUTION

Change is inherent to complex systems; if a group of elements cannot 'change' we do not consider them a system. But systems 'sustainability' requires those changes to maintain its identity[6] and system stability becomes a central issue of its 'sustainability' which requires something 'recognizable' and 'stable' to 'sustain'.

We have said that Second Law of Thermodynamics leads systems towards their dissolution, and this allows us to state the first characterization of the sustainability of complex systems sustainability from the idea of 'stability' as their capacity to maintain themselves far from their dissolution state.

**The Sustainability Degree of a system is its 'relative distance to total unsustainability or system dissolution'** and complementarily its Unsustainability Degree is its 'relative distance to total sustainability or most stable state of the system', and the two extreme values mean the following:

- $S_T[I]=1$ the system is in its most stable status
- $S_T[I]=0$, the system has reached its unsustainability threshold or dissolution point; its global identity will no longer be 'perceptible'.

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[5] The parts are more in the system; emergence is a result of the organization that appears not only at a global level but eventually at the component level: the part is more than the part [Morin 1977, p. 131]. The whole transforms the parts in relation to how they were before constituting the system.

[6] If various elements change continuously without actually taking any particular 'identity' they cannot be called a system, it simply would be an indefinite transformation that we could not 'name'.

We can represent it graphically as:

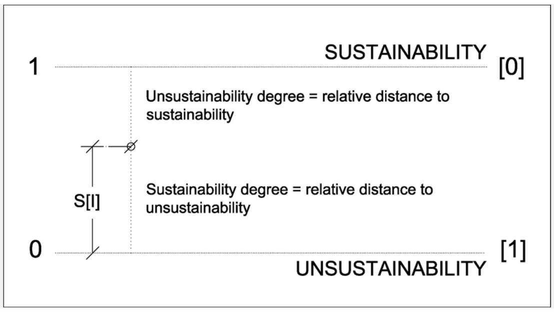

**Figure 12: The Sustainability Degree as relative distance of a system to its unsustainability threshold** *or dissolution, and the Unsustainability Degree as 'distance from sustainability status' are complementary values.*

$$S_T[I] = 1 - \neg S_T[I]$$

Sustainability Degree of a system is therefore a measure of its relative position to its 'dissolution as a system' or unsustainability threshold, which not necessarily implies the disappearance of its constituent members.

**Image 02: A democratic system** *is an example that allows us to understand that the total unsustainability of a system does not necessarily imply the disappearance of all its elements; a coup that a dictatorship imposes can be considered a situation of complete unsustainability for a democratic system, in which democracy is 'dissolved' yet many of its parts still remain; inhabitants, security forces, educational system, health system, etc. ... are usually present in dictatorial systems, although they assume different 'functions'.*
Photograph: commons.wikimedia.org

## HIERARCHICAL DESCRIPTION OF SYSTEMS AS 'NESTED HIERARCHIES'

### *SYSTEM DESCRIPTION*

Systems are described by variables[7] that inform of their status, allowing us to model and represent them in order to:

- Understand their status at a given time; the relation between the parts and the whole.
- Understand their behavior, being able to make short and long term predictions of their evolution or statues in future time points.

And both issues can be described through a series of differential equations, which expresses both the relationship between the 'possible states' of the whole and each of its elements, as between its possible states and modifications over time[8]:

$$\forall i \in I \longrightarrow \begin{array}{c} di_1/dt = f[i_1, i_2 \dots i_n] \\ di_2/dt = f[i_1, i_2 \dots i_n] \\ \dots \\ di_n/dt = f[i_1, i_2 \dots i_n] \end{array} \qquad (2)$$

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[7] We call 'variables' to those parameters that can "vary" over time.

[8] Dynamical systems are described through a set of n measurements, called state 'variables'. Its change in time is expressed by a set of 'n' simultaneous differential equations [von Bertalanffy 1968, p. 264].

However, **most systems contain so much information that** a complete list of their variables and relationships is not usually feasible; **its complete description is impossible or lead to inoperative descriptions,** both to perform and to understand.

**Describing a system requires summarizing its information** in relation to a purpose that determines the criteria that makes 'relevant' certain information while another becomes 'expendable'[9].

But **describing systems also requires sorting their information,** what can be accomplished by searching the underlying ‘organization’ of the relevant variables.

The variables that describe complex systems constitute parallel information systems, which also have hierarchical organization, and this organization allows us to describe systems as ‘nested hierarchies’ [which are similar to logical decompositions] that we can interpret in two ways:

- *From a bottom up perspective,* they can be understood as a series of successive aggregations of system components, obtaining a value that describes its overall state.
- *From a top down perspective,* they can be understood as a decomposition of the description of the global status of the system into the description of different partial features, reaching a desired level of detail.

We first review the process of ‘composing’ a nested hierarchy from a bottom-up perspective, i.e., by aggregating its information.

### *BOTTOM-UP PROCESS: THE COMPOSITION OF A NESTED HIERARCHY*

Systems descriptions are always ‘intentional’; and such intention requires that we select, interpret, structure and add their information according to certain analytical perspective, which can be undertaken in three steps:

- Selecting the level of detail and information [variables] relevant to the description[10]
- Transforming the information according to the perspective of analysis; turning variables into indicators.
- Structuring the information aggregation; establishing the organization of the nested hierarchy.

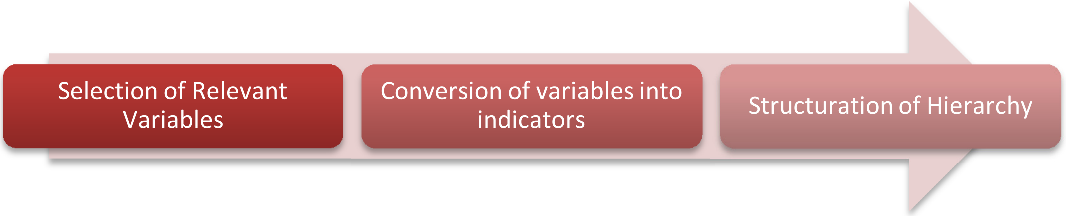

***Diagram 03: Composition of a nested hierarchy***

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[9] To 'describe' is "defining something imperfectly […]" [Drae 2014]; 'incompleteness' will be inherent to any description as well as the ‘map is not the territory’.

[10] To describe a city we do not start describing each of the atoms in each of its molecules; we add the information to the level that we consider relevant to the description; people, cars, buildings, neighborhoods, ...

The purpose of composing a nested hierarchy is to establish an aggregation structure which global aggregated value can be considered as a measure of relative distance to the unsustainability threshold, and this provides the criteria to undertake the first two steps:

**Selection of relevant variables:** we consider relevant for the description any variable for which there is a range of values that can modify the relative position of the system between its sustainability and unsustainability states, i.e. its 'Sustainability Degree'

**Conversion of variables into indicators:** we formulate mathematical formulations that transform the relevant variables into measures of relative distance of the system between the closest possible to sustainability and unsustainability for the possible values of each variable.

These two steps allow us to 'prepare' the system information; determine the relative position in which each variable tends to place the system between its most stable state and its dissolution threshold. But determining the 'global position' of the system requires aggregating its indicators and first we revise the rules for aggregating information in Complex Systems.

*Complex systems have characteristics that allow us to greatly synthesize the information that describes them;* they have 'structure' or ‘organization’ which determines the optimal way to add their information.

We can determine the underlying organization of system’s indicators reviewing the intensity of their interactions and grouping them into aggregation subsystems that meet the following conditions [adapted from Simon 1955]:

- *they ‘move together’ [covariate]; i.e.: are closer to each other than to the rest of the system*
- *they relate to other system’s indicators in the same way*

And if we add the indicators in each subsystem, we obtain a series of indicators related to more general aspects of the system, which in turn can be also grouped into aggregation subsystems and added again, obtaining more "global" indicators, and so on ... composing a ‘hierarchical structure’ in which each higher level is an aggregation of lower levels information.

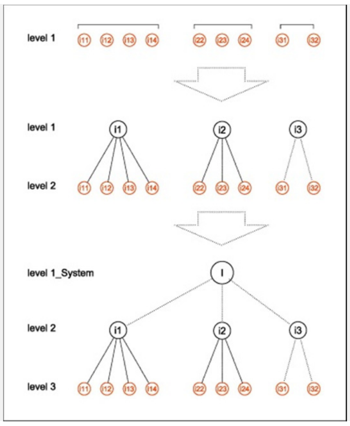

**Figure 13: Composition of a nested hierarchy.**

*We identify 'sets of indicators' which are connected to the rest of the system in a similar way, and we add them so that each aggregate indicator 'contains' a set of indicators in a lower level, generating a 'nested hierarchy'.*

*Indicators at lower levels report of smaller parts of the system; while at higher levels indicators report of bigger parts of the system; up to the top indicator which describes the overall behavior of the system with a single aggregate value.*

**Nested hierarchies are 'structures' that** show us how the parts of the system modify as we ascend levels in the hierarchy, getting to define its global 'identity', i.e., **allow us to understand how the 'whole' relates to its 'parts':**

- Its *'organization'*; possible and non-possible interactions, more probable and more improbable ones.
- Its *‘emergent properties’*; each indicator aggregation must not only summarize the information of the lower level indicators, but include the effect of the properties that emerge from their interaction.

Interactions among indicators produce a 'behavior' in the upper level which is different to the 'sum' of individual behavior of indicators at a lower level, **the meaning of the indicators is understood in their level while the meaning of their interactions needs to be understood on the upper level.**

We have reviewed the conditions that allow us to hierarchically describe a system based on a bottom up approach; now let us review the reverse process:

### *TOP-DOWN PROCESS: HIERARCHICAL DECOMPOSITION OF A SYSTEM*

It is the reverse process of the previous [and similar to the logical decomposition of concepts], in which we start with a 'global characterization of the system', and decompose it into a small set of indicators, which in turn we decompose in other sets indicators, and so on up to the level of detail of the information sought, considering that each decomposition must satisfy the following:

- Must be based on *an inclusion relationship;* elementary indicators must be ‘implied’ in the aggregated/decomposed indicator.
- Each decomposition must generate indicators with *similar level of significance.*
- *The joint consideration of all lower-level indicators must provide all the information that is required to fully determine the value of the indicator on the higher level.*

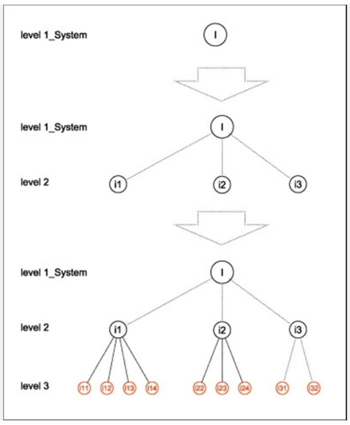

**Figure 14: Hierarchical decomposition of a system**. *Starting from the global characterization of the system, decomposing it into indicators [usually 3 to 5 each decomposition] which in turn we decompose again into other indicators and so on up to the required level of detail.*

*In each decomposition [aggregation subsystem] indicators should have a similar significance level, the variation of any of them for any value in the range 0-1 must have a similar impact on the added value, regardless of the chosen indicator; i.e. 'the maximum proximity to sustainability or unsustainability that each indicator in a subsystem can produce must be almost equal'*

A **Nested Hierarchy** is a structure composed of 'levels' and 'holons' [subsystems] that maintain a different kind of interaction among them [Wu and David 2002]:

- *Levels* present different interaction frequencies, and they hold nonsymmetrical relations.
- *Holons* develop similar interactions in both directions at their level, stronger and more frequent among indicators that belong to the same subsystem than between holons.

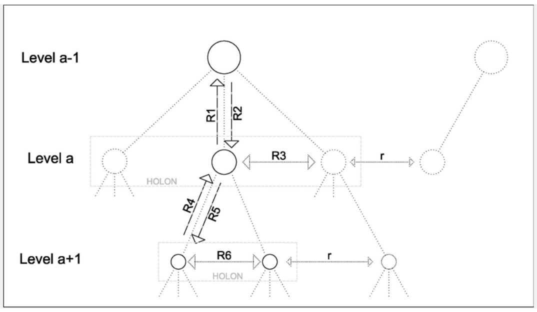

**Figure 15: Interactions in a nested hierarchical description of a system**. *The interactions between adjacent levels are not symmetrical. Interactions at the same level are symmetrical, but are much stronger among elements belonging to the same holon [R] than those between elements belonging to different holons [r]*

Disaggregating the information of a system is equivalent to decomposing its information into mutually exclusive categories or classes. It implies considering it to be a ‘tree’ that can be broken down into parts and relationships, and assumes that there are no interactions between the parts that are 'broken', what is called 'decomposability'[11].

*However, Complex Systems are not ‘decomposable’, there are always interactions between parts*[12] *and breaking them down implies considering some interactions to be 'irrelevant' from the analysis perspective; it requires considering the systems as ‘nearly decomposable’ [Simon 1962].*

Aggregation and Disaggregation/decomposition are two methods that used in combination allow us to 'structure' systems’ information in a way that its 'global state' can be determined as an aggregated measure of its 'partial states'. Their importance is therefore fundamental for sustainability measuring.

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[11] From an information perspective, 'decomposability' refers to data 'separability' discussed before [see 2.0.0.4_FUZZY SETS DECOMPOSITION].

[12] Or information of lower level indicators not entirely contained in their upper level indicators that contain them, but in other upper level indicators; this relates to 'semi lattice' structure of reality [Alexander 1965].

### 2.1.1.2_THE COMPLEXITY OF COMPLEX SYSTEMS

Etymologically the term 'complex' is derived from 'complexus' or ‘that which is woven together’ leading us to an **equivalence between the terms 'complex' and 'system'; they both refer to a ‘fabric’ or structure of relations that allows us a dual interpretation of a set of elements as 'parts' and as a 'whole' or global identity[13].**

And this approach to the Complexity of the systems can be related to two qualities of systems already anticipated:

The first is from **Complexity as Organization or arrangement of relations** that 'knits' a set of elements together enabling an 'entity' or system to emerge with its own 'identity'[14].

The organization of a system ‘is’ and *this perspective leads to ‘deterministic’ approaches,* which may be revised from the membership functions of Fuzzy Logic as restrictions to the possible states of a system.

And the second is from the **Complexity as Emergence or qualities [properties]** of the system that were not implicit in the elements, which allows us to perceive the system as an 'entity' [or identity] making it distinguishable [identifiable] from its surrounding environment and other systems.

Emergence ‘happens’ and *this perspective leads to ‘probabilistic’ approaches[15],* which may be revised from the probability functions of Probability Theory as stable frequency or expectation on its occurrence.

The complexity of systems is therefore a combination of ‘determinism’ and ‘probabilism’; of what necessarily must ‘be’ [or organization] and what may 'happen' [or emerge] as a result, and when we extend this idea over time [something necessary since we talk about sustainability], this dual nature forces us to understand Logic and Probability as two non-separable approaches to the same issue.

The extent to which a system 'is' in time is equivalent to the degree that it 'happens' at every moment[16], and the ‘determinism’ of Organization merges with the 'probabilism' of Emergence. However, this question may be somehow ‘difficult’, so we are going to revise it progressively, and for more clarity we continue the analysis from both approaches separately, starting with the first:

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[13] “The term 'complex' implies the paradox of the one and the many" [Rueda 2006]

[14] "Organization is the arrangement of relationships between components that produces a complex unit or system, endowed with qualities unknown to the component or individual level” [Morin 1977, p. 127].

[15] "Emergence is an event, that ‘appears’ discontinuously once the system has been constituted" [Morin 1977, p. 132] and this starts to show us that emergence requires to be understood in relation to Probability Theory that deals with 'events'.

[16] The 'dual' character of probability [as both stable frequency and degree of belief] can be related to this issue, since in essence the first refers to the probability as ‘event’ while the second refers to probability as ‘being'. See 2.2_A PROBABILISTIC APPROACH TO SUSTAINABILITY

## COMPLEXITY AS ORGANIZATION OR 'ORGANIZED COMPLEXITY

Organization refers to the structure of relationships between different elements that enables a recognizable system to emerge as a global entity, and it implies three key 'features':

- **A stable Structure of relationships** that determines the characteristics of the relations between different elements [their intensity, frequency, etc…] and should have sufficient level of permanence in time.
- **A Differentiation of elements,** a system of relations necessarily implies ‘different elements to relate’, either because they were different before joining the system or because they differentiate when integrating into the system.
- **An Order,** which refers to the relationship between each of the parts and the whole, for the system to achieve its optimum status.

Image 03: Company as an example of organization [sketch: es.wikimedia.org]**:**

- *Stable Structure of Relations: its members, functions and way of intercommunicating may be modified, but they should not be changed every day; if a company changes its staff or duties every day, we cannot recognize it as such.*
- *Differentiation: organization differences the people that it integrates [Director, Commercial, etc...]; the ‘parts’ are changed in the system.*
- *Order: the organization involves rules of optimum relationship between the parts and the whole. A company can operate with 15 salesmen and a director, but it hardly could with 15 directors and one salesman.*

The idea of ‘Order’ leads us to a key issue; *the relationship between the different parts of a system can make it perform better or worse* and this allows us to propose an ‘optimal’[17] situation or organization of systems; that cannot be improved.

*The idea of system implies the possibility of 'change', i.e., the system can be in different states [if a set of elements cannot change we do not consider them a system] but it also implies the idea of 'identity' or maintenance of a "sufficiently" recognizable global form through its transformations. Therefore not all changes are possible.*

The Organization is the limitation of transformations and possible states of the system[18], and we can distinguish two extreme states of system’s elements: one that is its optimal organization [the organization a system has when it is at its optimum status], and one which is the first state[19] of its elements that prevents the emergence of the system’s identity.

The existence of these extreme configurations [optimally organized and non-possible states] allow us to conceptualize the sustainability degree of a system from the perspective of organized complexity as a measure of 'relative coincidence' between its state and its optimal organization [in relation to its first non-possible state].

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[17] The definition of Optimal is "extremely good, which cannot be better" [Drae 2014]

[18] "the presence of organization between variables is equivalent to the existence of a constraint in the product-space of the possibilities” [Ashby 1962, p.257], i.e., not all ‘theoretically’ possible ‘is’ actually possible.

[19] Though we designate it this way, usually this ‘first state’ is not ‘one state’, but admits several different configurations.

## SUSTAINABILITY DEGREE AS THE DEGREE THAT THE ORGANIZATION OF A SYSTEM IS OPTIMAL

The sustainability degree of a system can be defined from an Organized Complexity perspective as a measure of agreement between its structure and its optimal organization, related to its first non-possible state, and its representation as a nested hierarchy allows us measuring it:

- Sustainability Degree is a measure of the degree to which the structure of a system coincides with its optimal or best possible configuration.
- Unsustainability degree is a measure of the degree to which the structure of a system does not match its optimal state [or matches a non-possible state in which the elements cannot constitute the system]

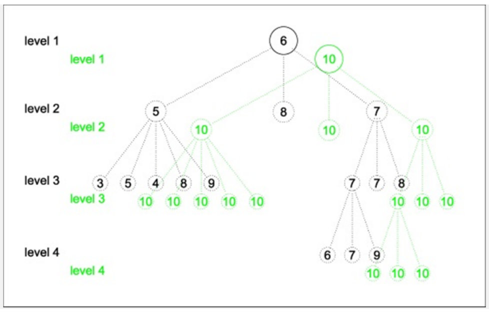

***Figure 16: Sustainability Degree as a measure of the degree to which a structure 'matches' its optimal.***

Both can be measured from the degree of overlap of information that characterizes the particular assessed system and its optimal [and not-possible] organizational state, and the two extreme values mean the following:

- $S_T[I]=1$ system's structure totally matches its optimal organization; it is in its 'optimal' state.
- $S_T[I]=0$ system's elements have disappeared or are in a state where the 'system' is not possible; they can only exist 'separated' [or as parts of other different systems]

From the perspective of ' Organized Complexity' a sustainability indicator of a system can be understood as a 'rule' that compares certain aspect of the system with the character it would have when the system is in its optimum organizational state; i.e.: a measure of the degree that some part of the system 'is' in relation to what it 'should be' in its optimum organization.

## HIERARCHICAL ORGANIZATION FROM COMPLEXITY

A hierarchical organization is a way of representing a structure of relationships among the parts of a system, i.e., a structured representation of its 'organization rules'. And to measure the sustainability degree we shall represent the structure that the system has in its optimal state; it's 'Optimal Organization'.

It incorporates the rules the system must meet to be in its optimal state', which specify both the conditions that place the system in its optimal state as well as those that take it to a non-possible state.

Image 04: Sports facility [photograph: commons.wikimedia.org]. *An indicator of 'adequacy of sports areas' compares the surface and type of existing sports facilities and its allocation in a given urban area, with the optimal values [i.e.: which has the 'optimal organization'] for that type of urban fabric.*

*The 0 value is taken usually as the unsustainability limit in urban facilities indicators, because it is the limit of possible states [no urban area can have less than 0 provisions], but it is not necessarily the unsustainability threshold for other types of indicators.*

And from a logical perspective it is possible to interpret it as a structure of 'rules' or 'statements':

- *From Fuzzy Sets Theory* membership to a certain class equates meeting a series of rules that determine its membership to some subclasses.
- *From Propositional Logic* the degree of truth of a global statement is determined by the degree to which certain partial statements are true.

We can therefore establish a direct relationship between complexity as ‘Organization’ and the logical inference rules specified before: **a system belongs to the class of sustainable systems if its organization meets certain rules, and the extent to which it complies with them determines the degree of truth of the proposition 'the system is sustainable'.**

## COMPLEXITY AS EMERGENCE

‘Emergence’ refers to the presence of properties in the systems that were not present in its components before constituting the system and usually involves some 'unpredictability'; we can expect certain properties to 'emerge' when the elements join together, but we cannot predict the time of their onset or their development with total accuracy.

Therefore **'Emergence' implies the existence of organizational structures**, as it occurs as a result of elements interaction. And it is important to say that the structures that allow emergence can only exist far from thermal equilibrium:

- Are *dissipative structures,* ‘sustaining’ them over time requires continuously importing 'negentropy' from the environment and constantly ‘dissipating’ entropy to the environment.
- Imply *different levels of Entropy* [or information]; at equilibrium there can be no difference; everything becomes equal and no system can exist[20].

'Emergence' of a system as an entity is going to require a difference of Entropy between the system and its environment, leading us to the possibility of measuring ‘Degree of Emergence’ in terms of relative negentropy. Additionally, we can measure the emergence of certain properties in a system, which also require a reduction in entropy from the situation prior to their appearance. And when we review the emergence of 'sustainability' as a system’s property, we are measuring its ‘Sustainability Degree'.

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[20] “At equilibrium, all is level and no more energy is available” [Lovelock 1979, p. 30]

### *SUSTAINABILITY DEGREE AS EMERGENCE DEGREE OR RELATIVE NEGENTROPY*

In terms of entropy, *we can define an organization as a distribution of elements different from its environment and which 'formation' from equilibrium requires a reduction of entropy, i.e.: a contribution of energy or negentropy*[21]*.*

A relationship appears between negentropy and 'emergence' that serves to measure 'Sustainability Degree' as 'Emergence Degree' or relative negentropy between two situations: **null and complete emergency of system’s identity/sustainability:**

- $S_T[I]=0$ is the maximum entropy 'threshold' that if exceeded makes system’s emergency non-possible; *its global 'identity' cannot emerge*[22]*.*
- $S_T[I]=1$ is the negentropy 'threshold' required for *system’s sustainability to fully emerge*, reducing entropy no longer increases the emergence of its ‘sustainability’[23].

### *LOGICAL DECOMPOSITION AS EMERGENCE LEVELS*

Emergent properties do not designate system’s elements but its 'qualities'[24] [which appear after the system constitutes] and its analysis shows a complete equivalence with Fuzzy Logic; *the degree to which certain properties/qualities emerge in a system is equivalent to the degree of truth when referring these 'properties/qualities' to the system.*

The logical decomposition of the sustainability of a system is logically interpretable as a ‘hierarchy of emergence levels’; a hierarchically structured set of measures of the degree of truth of different qualities referred to the system.

Each level involves system’s properties not present in the lower levels generating a succession of "emergence levels"[25]; the emergence of properties at one level requires the emergence of certain properties at a lower level. And the emergence of system’s identity [and its sustainability] at the global level is only possible if the emergence of properties at lower levels satisfies certain conditions.

Image 05: Paris [photograph: commons.wikimedia.org]

*When we value the 'Urban Quality' of a city, almost all the issues we care about are emergent properties [Cityscape, Accessibility, Acoustic Comfort, Public Safety, Vitality, etc,...]*

*'Good urban quality' emerges as a result of the interaction of 'good urban landscape', 'good accessibility', etc ... The interaction of emergent properties or qualities at a level causes other to emerge at a higher level.*

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[21] Adapted from Lovelock [1979, p. 31]

[22] As commented before, complete unsustainability of a system does not imply the disappearance of its elements, and this implies that unsustainability of real systems is usually far away from thermal equilibrium, as the assembly of the elements also requires energy

[23] It is noteworthy that for many systems their ‘identity’ implies the idea of their ‘sustainability’, and therefore it is possible to measure either the emergence of sustainability as a system property or the emergence of system’s identity.

[24] “Emergence is a new quality in relation to the constituents of the system” [Morin 1977, p.132].

[25]Picking up Morin [1977, p. 134] assertions that any ‘whole’ implies emergence and that nature is poly-systemic, i.e. builds systems on other systems; an architecture of ‘emergences’ that can be understood as ‘emergences of emergences’

Each emergent property [each concept of the logical decomposition] can be reviewed in terms of 'relative negentropy' from the non-emergence situation or unsustainability; as the relative entropy reduction [or energy supply] required for the property to emerge.

And an indicator can be considered as a rule that allows translating information concerning various components of a system into a measure of the degree that their interaction causes a certain property to emerge.

Simplifying the above, Complexity as Organization can be understood as a top-bottom approach to systems [the existence of the system implies a specific arrangement of its constituent elements] while Complexity as Emergence can be understood as a bottom-up approach [the emergence of some properties at a level enables higher levels of emergence][26].

However, reality usually implies a circularity that almost always requires a bi-directional approach.

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[26] The meaning of the term 'emerge' leads to this bottom-up understanding

## MEASURING THE COMPLEXITY DEGREE OF A SYSTEM

We have proposed two approaches to 'Sustainability Degree' of systems from the perspective of Complexity:

- One in which we equate it to the *Degree that the system structure matches its optimal organization*, which can be calculated in terms of 'mutual information'
- Other in which we equate it to the *Degree of Emergence of Sustainability as a systems property*, which can be calculated in terms of relative negentropy.

And both approaches are compatible in the context of Communication Theory [Shannon, 1949], which conceptualization of 'information' as 'uncertainty reduction' leads to a mathematical isomorphism with the entropy of thermodynamics, and using its formulations we can measure both Entropy and Information at the same time.

The formula of the 'Mutual Information' allows us to compare the state of a system I at a given T moment with its status at sustainability 'S'

- As agreement degree with its optimal organization [complexity as organization]
- As relative negentropy regarding the non-emergency situation [complexity as emergence]

$$I[I;s]_{\%} = H[s] - H_s[I] \tag{3}$$

Where the first term H[s]:

- The information contained in a complete description of the optimal organization or sustainable state 's'[27]
- The entropy difference between unsustainability '¬s' and sustainability 's' states

And the second term $H_S$[I] is:

- the amount of information of the system I that does not match its optimal organization or state 's' [or ignorance regarding I when 's' is known]
- The Entropy increase of the system I in relation to state 's' [conditional entropy of I].

And if we express the above formula in 'relative' terms, it is a measure of 'Complexity Degree', which can be considered a measure of 'Sustainability Degree'[28].

$$I[I;X]_{\%} = 1 - \frac{H_X[I]}{H[X]} \tag{4}$$

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[27] Actually it refers not to information but to 'rules' content, but we review it later.

[28] Further development of this formulation is undertaken in A-VI.1.2_FORMULATION AS DEGREE OF CERTAINTY / NEGENTROPY. For now, we just want to state that it is possible to measure this quantity.

### 2.1.1.3_COMPLEX ADAPTIVE SYSTEMS: SYSTEMS THAT EVOLVE

Complex Adaptive Systems [CAS] are a class of complex systems which **permanence over time implies 'evolution'[29] or shift towards 'more developed' states.**

CAS are ‘evolving’ by nature and therefore their sustainability implies the sustainability of their ‘development’ or ‘evolution’. CAS 'sustainability' equates their 'sustainable development', and this makes the review of 'evolution' concept necessary, that we undertake from three interrelated approaches to evolution:

- *As an increase in the amount of systems’ organization.*
- *As a comparative measure of adaptation to the environment or coevolution*
- *As progress toward states of greater desirability,* what relates to learning, decision-making and teleology inherent to CAS

Let us review in this section the first two.

### EVOLUTION AS AN INCREASE OF COMPLEXITY OR AMOUNT OF ORGANIZATION

The majority of authors consider that a system ‘evolves’ when it increases its 'amount of organization', equating it with an increase in the number of different elements [or quantity of information] and relationships between them [or quantity of rules]; **CAS evolution materializes in their ever-increasing amount of ‘Organized complexity’,** and some alternative CAS designations are proposed:

- Self-Organizing Systems [Von Foerster 1960]
- Self-Differentiating Systems [Von Bertalanffy 1968]
- Systems of Increasing Complexity [Margulis and Sagan 1998 cited in Maldonado, 2010][30]

The above designations refer to CAS as systems that 'evolve' or which organized complexity increases over time; **the increase in organized complexity is set as the condition for the existence or not of 'evolution' between two states of a system,** and CAS may 'evolve' in two ways:

- *Increasing their 'structure of relations',* incorporating new rules in its structure; modifying and increasing the number of their possible and not possible states
- *Increasing their 'differentiation',* incorporating new different elements, and consequently increasing the number of possible interactions.

**CAS evolution can be measured in terms of increased organized complexity or organization,** and their description as nested hierarchies becomes a tool for measuring ‘organization’; the amount of 'rules' governing the interactions between system elements[31].

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[29] Although some authors differentiate between CAS [as systems that ‘adapt’] and Complex Evolutionary Systems, CES [as systems that ‘evolve’], we consider them to be equal for two reasons:

- Many systems adapt in the short term but evolve in the long run; the difference is not in the system but in the time interval.
- If a system adapts to an 'evolving environment' with high probability it will develop an 'evolutionary' behavior.

[30] It relates to von Bertalanffy [1968, p. 101] definition of 'self-differentiating systems' as "systems that evolve towards increasing complexity"

And therefore, the complexity of a system is approximately proportional to the amount of indicators in its description; **as CAS evolve the number of parameters needed to completely describe their optimal organization increases.**

## INDEPENDENCE OF SUSTAINABILITY AND EVOLUTION

We have related the 'Sustainability Degree' of a system with its 'Organization Degree', and thus sustainability and evolution link each other through CAS 'organization'; however both properties are essentially independent in the sense that variation of one for a specific time interval does not imply variation of the other in the same direction.

**An evolutionary process is a process that increases the amount of 'organization' of a CAS, but an organization can be 'good or bad' and therefore CAS sustainability is not related to its amount of organization[32].**

Previously, we have differentiated three aspects in systems' organization [structure of relations, differentiation and order], and CAS ever-increasing complexity only affects [or implies] the first two; their 'structures' an ever increasing amount of different 'rules and elements', but the extent to which the optimal 'order' is maintained, is an independent 'event'.

CAS Organized Complexity provides a measure of their 'Development' or 'Evolution', but it is independent of their 'Sustainability Degree' which is mainly related to the aforementioned concept of 'order'; i.e.: with the extent to which the relationship between parts and whole approach the system to its optimal state.

And the evolving nature of adaptive systems introduces a fundamental issue; **CAS optimal organization continuously modifies over time, not only incorporating new rules but also modifying existing ones**, and this implies a constant adaptation and evolution effort. **A CAS that does not 'evolve' cannot maintain an optimal situation.**

Therefore, **'sustainable development'** *can be understood as a process in which a CAS increases its 'amount of organization' maintaining the highest degree of coincidence with its optimal organization*; increases its 'differentiation' maintaining its 'optimum order'.

The environments are also systems and the characteristics of the 'optimal organization and non-possible states for a system are determined by comparison with the characteristics of the environment and with other CAS of the same class; leading us straight to the concept of coevolution, that we review next.

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[31] The number of "rules" in a system can be considered as a measure of its amount of organization, since each different rule implies a different relationship between the same or different elements.

[32] The amount of 'organization' is not relevant for CAS sustainability. As sustainable may be a 'beehive' as a 'city', yet the amount of 'organization' is much larger in the second case.

## SYSTEMS COEVOLUTION

CAS evolution materializes in a particular organization, and a relationship emerges between evolution and sustainability embodied in **the concept of 'co-evolution' which refers to the extent to which the evolution of a system is consistent with that of its environment[33].**

'Co-evolution' is an emergent phenomenon[34] that implies that systems' optimal organization and non-possible states [equivalent to fittest and un-fit states] are in part determined by the evolution of their environments as a whole[35], and the permanence of a system requires its evolution towards optimal organizational states.

### *SUSTAINABILITY DEGREE AS FITNESS OR CO-EVOLUTION DEGREE*

The concept of co-evolution introduces a new relevant issue for CAS sustainability; systems need to evolve consistently with their environments, and this consistency is measurable as both 'fitness and co-development degree'; making a revision of the concept of "optimal organization" necessary.

Evolutionary environments force us to incorporate the fitness and coevolution requirements into the optimal organization of systems so it implies consistency with the environment and from this perspective, S extreme values take the following meaning:

- $S_T[I]=1$ means that at temporal moment T the system is a state of the highest possible development/fitness to its environment [optimal organization]; at time moment T no higher adapted states are possible [however, CAS optimal organization modifies over time, and at a later point in time, there may be better states possible].
- $S_T[I]=0$ means that at temporal moment T the system has reached a totally un-developed or un-fit [non-possible] state; the system disappears due to lack of adaptation to the environment.

The review of CAS from the evolutive perspective shows many important implications that shall be integrated as constraints for determining a system's optimal organization:

The first is that **it requires introducing the concept of class of systems;** coevolution implies that *the optimal organization of a system is partly determined by [it has to be consistent with that of] its class of systems;* and this takes us from the apparent determinism of evolution towards the probabilism of emergence.

---

[33] In biological systems it can be related to 'natural selection' [Darwin 1859] or 'survival of the fit'; the survival of an unfit CAS is not possible and $S_T[I]=0$ coincides with the 'unfitness' threshold imposed by 'natural selection'.

[34] Evolution is an 'event' and coevolution inherits this character of 'event' or 'emergence'. Coevolution can also be considered as an emergent phenomenon from the perspective that it arises from the interaction between different systems.

[35] "change needs to be seen in terms of co-evolution with all other related systems, rather than as adaptation to a separate and distinct environment" [Mitleton-Kelly 2002 p. 8]

"An organism's DNA thus is not only a "book" about the organism, but is also a book about the environment it lives in, including the species it co-evolves with" [Adami et Al 2000, p. 4464]

CAS usually exist as classes with numerous systems and the characteristics of the optimal/unfit organization emerge as a result of the interaction of numerous individuals; *its evolution can be understood as an emergent property of chaotic evolving systems,* and thus the evolutionary approach departs from the determinism of logic, *becoming essentially non-determinable*[36]**.**

The second is that environments impose fitness as a constraint for system's sustainability; and **evolutionary environments [and it is questionable whether there is any environment which is not evolutionary] lead us to an understanding of complete sustainability as a steady state that shall be maintained through continuous adaptation/evolution.**

The environment may also impose some restrictions to the inequality of development between CAS: *excessive disparity may reduce environment stability* [and therefore its sustainability degree], *while excessive equality would prevent differentiation and evolution*[37]*;* the optimal state is located in that "middle" almost always difficult to establish[38].

The third is that development implies a degree of desirability **introducing directionality in the processes of CAS evolution** which in part happens 'unconsciously' but in part happens 'consciously'; *CAS evolution not only seeks optimizing processes but also reaching 'desired' states* [Holland 1996].

And evolution is also important at a conceptual level. **CAS evolving nature transforms 'Fuzzy Logic' into a 'Temporal Logic'; the degree of truth of propositions modifies in time, what today constitutes an 'optimal or fit' state for a system may not be so tomorrow.**

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[36] The review of systems unpredictability is in 2.1.2_THE UNPREDICTABILITY OF SYSTEMS

[37] It also could imply a Sustainability reduction if there are other similar environments with greater differentiation [i.e.; highly developed] which they compete to.

[38] Referred to SES, this issue relates and may provide solution to equality issues.

### 2.1.1.4_THE ENVIRONMENT OF SYSTEMS

Systems sustainability requires that of their environment; *any system requires the existence of an environment in which to settle and be able to sustain its Entropy/Negentropy flows, so let us review this issue from the* **system-environment model,** usual approach of Ecology.

SYSTEM ENVIRONMENT MODEL

We have defined the environment E of a system I as its 'surrounding environment' which includes 'everything that is not part of the system', however, this definition of 'environment' pose some disadvantages:

- The first is **conceptual,** as the 'Environment actually contains the System', and this forces us to consider that the 'Environment' containing the system is the *System-Environment.*
- The second is **operational**; the 'Environment' as the 'non-system' embraces an area which makes it inoperative, *we cannot work with that much information.*

However, this large amount of information is much bigger than that actually required to determine the sustainability of a system that in the time intervals we do care is expected to have little to do with what happens on the other end of the universe [which may be decisive on longer time scales].

And though the greater the extent of the environment reviewed apparently the better the assessment is, working with more information than required is actually inefficient and increases the likelihood of errors, making necessary to propose a criterion to limit the extent of the environment to be evaluated.

This criterion is to assess the minimum environment E which total unsustainability necessarily implies unsustainability of the system, and we determine it by an 'accessibility' condition. The environment that implies complete system's unsustainability is its **accessible environment**; the environment such system can access and settle to.

However, some systems can move and settle in different environments, and this requires reformulating the condition; *the sustainability of a system I has an upper bound in that of its Global Accessible Environment* $E_A$ *that includes all of its accessible environments* $E_{Ai}$[39].

$$E_A = \bigcup_{i \in R} E_{A_i} \rightarrow \forall I \subset E_A \tag{5}$$

Where $E_A$ is the environment which Sustainability Degree limits system's Sustainability Degree

This accessibility condition allows us to understand that even systems' from the same class in a very similar state can have different sustainability degree if they have different accessible environments. **The sustainability degree of a system is 'conditioned' to that of its Global Accessible Environment that stands as a limit to its sustainability or capacity to endure.**

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[39] Otherwise, there could always be an environment $E_i$ different from E to which such a system can 'move', in which case its sustainability is not limited by the environment E. The fact that $E_A$ is the set of all environments where the system 'I' can settle implies the containment condition is always satisfied, because 'I' always is 'contained' in $E_A$

### *SUSTAINABILITY DEGREE OF THE GLOBAL ACCESSIBLE ENVIRONMENT AS UPPER BOUND TO THE SUSTAINABILITY DEGREE OF A SYSTEM*

The former review allows us to state that the sustainability of a system is only possible if it is consistent with the sustainability of its environment. And the limits to the Sustainability Degree of a system acquire the following meaning:

- $S_T[I]=1$ means that the state of the system is fully consistent with the sustainability of its environment; *the optimal organization of systems must be ‘consistent’ with environmental limits or needs.*
- $S_T[I]=0$ means that the system has achieved its dissolution threshold as a result of complete unsustainability of its environment; *if a system no longer has environments in which to settle, its endurance becomes not-possible.*

The condition that the optimal organization of the systems is consistent with the sustainability of its Global Accessible Environment brings us to the interpretation of systems’ evolution in terms of 'efficiency'. There is a limit to the amount of entropy that the Global Accessible Environment can assimilate and so its efficient use by systems seems a condition for its ability to 'endure'.

However, the efficiency of a system is not directly related to its sustainability degree, so we have to adapt the classical ‘efficiency’ formula transforming it into a measure of 'efficiency degree'[40].

### THE HIERARCHICAL REPRESENTATION OF THE SYSTEM-ENVIRONMENT MODEL

The sustainability of the Global Accessible Environment $E_A$ becomes a necessary condition for the sustainability of CAS, and thus **any variable relevant for $E_A$ sustainability becomes also relevant for I sustainability,** which is incorporated in the hierarchical representation.

The system-environment conceptualization is inherent to the 'description' of any system; it is implicitly adopted when considering that a part of a whole is a CAS[41]. But the hierarchical representation that divides the issues relevant to the environment separate from those relevant to system sustainability usually ignores two issues:

- In high complexity systems, this separation at the top of the hierarchy is usually incorrect; there always are issues relevant to both system and environment at different levels.
- These representations usually calculate the sustainability degree of the System as an average of system’s and environment’s sustainability degree, not complying with the 'containment' condition that states:

$$I \subset E_A \;\rightarrow S_T[I] \leq S_T[E_A] \rightarrow S_T[I] \equiv S_T[I] \cap S_T[E_A] \tag{6}$$

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[40] This issue is undertaken as a separate annex [see ANNEX IV_SYSTEMS EFFICIENCY: EFFICIENCY VS 'DEGREE OF EFFICIENCY']

[41] The concept of CAS is itself a System-Environment Model [see Foerster 1960]. In real systems, any analysis that does not check the entire 'Universe' will be implicitly assuming a 'system-environment' model.

Therefore, if for any system it were possible to separate the issues that determine the sustainability of the system from those that determine the sustainability of its environment, then their aggregation operation should not be an averaging but an intersection.

**'System-Environment' model shall not be understood as the aggregation of two values to determine an overall Sustainability Degree of the whole, but as the modelling of their intersection;** setting environmental sustainability as an upper bound to the sustainability of the system.

Usually, for Socio Ecological Systems it is not possible to completely separate system's sustainability from environment's sustainability, and the 'system-environment' model should be understood as a transversal criterion to all the subclasses in the 'nested hierarchy':

- when determining the mathematical formulation of membership functions to the classes in the hierarchy, $E_A$ sustainability needs to be imposed as 'restriction' to the maximum 'Grade of membership' of the system to any class:

$$\forall i \in I: S_T[I_i] \leq S_T[E_{Ai}] \rightarrow S_T[I_i] \equiv S_T[I_i] \cap S_T[E_{Ai}] \qquad (7)$$

- When checking the completeness of the description, any relevant variable for the sustainability of $E_A$ becomes relevant for the system, and its effects need to be assessed.

### *2.1.2_THE UNPREDICTABILITY OF SYSTEMS*

General Systems Theory is founded on a ‘deterministic’ approach that considers possible to predict the future state of systems, but in reality the behavior of most systems is not fully predictable [for some of them is not predictable at all], mainly due to non-linear feedback processes.

*Systems non-linearity implies that they can be located in more than one state in the future*[42] not being possible to exactly determine in which one the system will be. And it can relate to CAS from two perspectives:

- They can develop chaotic behaviors.
- They can ‘learn’[43] and ‘decide’

Let us start with the first one.

#### 2.1.2.1_CHAOS THEORY: SENSITIVE DEPENDENCE AND NONLINEAR FEEDBACK

Chaos theory arises in order to explain why a system behaving in a deterministic way can incorporate ‘seemingly random’ [unpredictable] events; ‘chaos’ considers that reality is *‘deterministic and obeys the fundamental laws, yet is unpredictable; its unstable aperiodic behavior makes accurate predictions impossible’ [Gleick, 1988]*[44]*.*

As a consequence, chaos theory analyzes systems’ behaviors from a combination of two previous theoretical models [Sabino 1996, p.79]:

- The **deterministic model** which holds that knowing the initial conditions of a system allows us to predict its final state.
- The **probabilistic model** which holds that its final state cannot be accurately predicted, because it depends partly on chance and it may not be possible to accurately determine their initial conditions.

According to chaos theory is **not possible to know precisely the state of a system in the future, not due to chance but to the fact that we cannot accurately model its initial state and the processes that modify them**[45]**.**

Chaotic systems [or processes] present **sensitive dependence on initial conditions**; two states with minimal differences can evolve into very different states even following the same processes[46].

---

[42] “Non-linearity means that problems have more than one possible solution” [Maldonado 2005, p. 32]

[43] Any learning system has nonlinear feedback [Wiener 1949]

[44] “An aperiodic system is a system that never reaches stability and 'almost' repeats its status but it never does” [Gleick 1988, p. 30]

[45] “Chaos retains from classical determinism the idea that [...] it is possible to draw a model that explains the behavior of a system, but it claims that systems can reach an -in occasions- infinite variety of possible outcomes. The difference to the probabilistic model is obvious: there are laws and a way to calculate the result of a process. But - the apparent similarity is that, the result cannot be predicted; as in a random pattern” [Sabino 1996, p. 79]

In addition, their **non-linear feedback makes it impossible to accurately model the processes that modify the systems.** The result of a process can be part of its next entry modifying its future states in an unpredictable manner.

These two qualities announce that **the prediction of the future state of any system incorporating ‘chaotic behaviors’ has a degree of uncertainty that increases with the prediction time.**

However, *chaotic systems present 'regularities' o 'patterns' that make them timely unpredictable but globally stable* [Gleick 1988, p.56]; the interaction among great number of elements generates an underlying pattern or shape in their behavior as a system, ‘the overall results show precise and well defined regularities’ [Sabino 1996, p. 79].

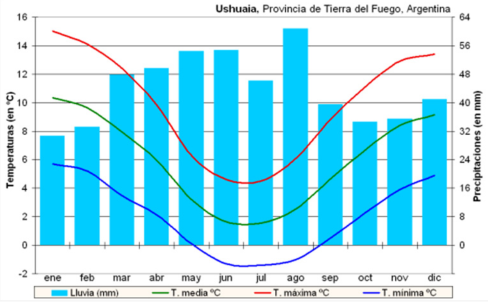

Image 06: Precipitation and temperature charts [image: commons.wikimedia.org]. *Climate emerges as a result of the interaction of a chaotic system [the Earth's atmosphere] with other elements or systems -such as land, seas, etc…- which generates globally stable patterns.*

*It is impossible to accurately predict the temperature or precipitation of a particular day a month in advance, but we can predict a 'probable' range of precipitation and temperature for a whole year or at certain times.*

Statistics and fractals are useful tools to detect 'patterns' in chaotic systems, allowing us to understand their behavior and to perform 'approximate predictions'[47]:

**Statistics** allows us to detect 'patterns' that can be considered properties of 'disorganized complexity', which may help us to explain and predict systems that in their 'elementary' level cannot be predicted.

- Any *analysis of a ‘sample’ and subsequent extrapolation of its results to a population* assumes that the population has an organizational pattern[48].
- Any *'trend' analysis based on 'time series'* seeks to detect 'patterns' or 'attractors' in the revised phenomena that limit its ranges of possible values, allowing us to 'estimate the probability of a specific result to be in a range of values' [adapted from Feigenbaum 1980].

**Fractal geometry** allows us to use graphic representation of chaotic systems in order to detect their organizational patterns:

- *Self-similarity phenomena;* properties that repeat at different scales within the systems.
- *Strange attractors* that represent the behavior of chaotic systems in the 'phase space'.

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[46] Something Pointcaré [1903 cited in Crutchfield et Al 1986] suggests: “it happens that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible”.

[47] See also ANNEX III_ STATISTICS AND FRACTALS; PATTERNS IN CHAOS

[48] An example is the behavior of a large group of people that can often be predicted with sufficient accuracy, though not their individual behavior [vote surveys, etc…].

If we represent the 'state' of a system in a phase space at a time moment T by a single point, that point will move through the space showing the variations of the system status.

We say that *a system has an attractor if there is a point, line or region in the phase space, to which the state of the system settles down to in the long term*[49], and its analysis allows us to distinguish three types of systems or behaviors:

- If the attractor is a point the system tends to 'stop'; to settle in a certain position.
- If the attractor is a linear figure [circle, polygon, etc...] the system has a 'stable' behavior; i.e.: is 'regular and predictable. "
- If the attractor is formed by curves that 'approximately repeat' without being cut, the system is 'chaotic'; i.e.: has an unstable aperiodic behavior[50].

This type of 'fractal' attractors are called **'strange attractors'**, and can be considered 'patterns' in apparently 'random' behavior.

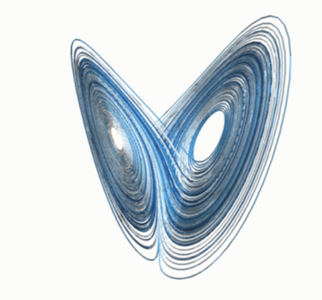

***Image 07: Lorenz Attractor***[image: commons.wikimedia.org]

The shape of the attractor gives name to the 'Butterfly Effect' which states that a minimum difference in initial conditions can have an enormous impact on the future state of the system, 'the flapping of a butterfly in Brazil could cause a tornado in the United States' [adapted from Lorenz 1972].

"There is order in chaos: underlying chaotic behavior there are elegant geometric forms that create randomness" [Crutchfield et Al 1986]

In summary, the former issues provide us with three perspectives to work with and understand chaotic systems:

- Seeking properties that repeat regardless of the scale [self-similar and fractal properties].
- Understanding and making predictions of chaotic behaviors or systems in terms of 'probabilities'; based on 'trend' analysis and statistical records.
- Limiting the time interval of the predictions

And the unpredictability of Chaos forces CAS to maximize their resilience or ability to withstand unforeseen impacts, which becomes a relevant variable for their 'sustainability':

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[49] "An attractor is what the behavior of a system settles down to, or its attracted to" [Crutchfield et Al 1986:5]

[50] 'The attractor consists of 'similar repetitive curves that do not cut themselves', and therefore its length will be infinite, i.e., it must be a 'fractal'' [Ruelle and Takens cited in Gleick 1988:146]. The system becomes 'regular' and 'stable' if the curves get to cut themselves.

## SUSTAINABILITY DEGREE OF A SYSTEM AS ITS DEGREE OF RESILIENCE

The unpredictability of Chaos introduces a new variable relevant for CAS sustainability; chaotic environments imply the possibility of unforeseen impacts and for CAS located in chaotic environments [and it is questionable whether there is any environment that does not incorporate chaotic behavior], a revision of the "stability" concept becomes necessary.

*Chaos forces to* **understand the 'stability' of CAS in terms of their 'Resilience'** *or ability to assimilate unforeseen disturbances maintaining their 'structure'. The higher the 'resilience' of a system, the lower its unsustainability degree is, reducing the potential negative effects of unpredictable external shocks*[51]*.*

And from this perspective, the extreme values of S mean the following:

- $S_T[I]=1$ means that at T moment the system is in the maximum Resilience possible state.
- $S_T[I]=0$ means that at T moment the system has reached its null Resilience, and therefore its dissolution[52].

High resilience states are more sustainable, and referred to CAS *the concepts of 'stability' and 'resilience' become virtually 'synonyms' and a necessary and sufficient condition for 'sustainability'.*

**When stability and resilience 'extend' indefinitely over time they become sustainability[53].**

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[51] Resilience is a variable present in many common facets of SSE, especially in economy. An example are public debt limitations that EU imposes their members, in order to be able to withstand [be more resilient to] economic crisis.

[52] The null Resilience of a system necessarily involves its dissolution; since there are always some impacts from the environment [systems own tendency to increase entropy following second law of thermodynamics would lead to its dissolution].

[53] CAS sustainability implies their maximum resilience, which in turn implies their maximum stability.

### 2.1.2.2_CAS UNPREDICTABILITY: SYSTEMS THAT LEARN, DECIDE AND HAVE TELEOLOGY

CAS have several features that make their future states highly unpredictable:

The first is that **their survival requires them to be in continuous adaptation to the environment;** CAS are constantly gathering information about the state of their environment and adapting [changing their status] according to the information received.

And CAS are located and interact with environments that are -at least partly- chaotic and thereby their behaviors internalize the unpredictability of such environments; *any system that 'adapts' to a chaotic environment necessarily develops chaotic and unpredictable behavior.*

The second is that **they can learn and develop behaviors;** CAS are constantly exchanging information with their environment and converting part of this information into knowledge and 'rules of conduct' that serve to explain their behavior[54].

These rules are continuously evaluated and improved; whenever a CAS applies a rule of conduct, it checks if it has been a satisfactory answer; maintains the satisfactory aspects and improves the aspects that can be improved.

And when a CAS modifies a 'rule of conduct', its next response to the same situation will be different to previous responses –a lot or a little depending on the modification- and this *'non-linear feedback' makes impossible to predict the future state of the CAS[55].*

The modus operandi of CAS is related to their 'experience' and if that 'experience' is not known then it may not be possible to predict its behavior to certain stimuli or in certain situations.

In addition, the multiplicity of possible inputs from the environment makes it impossible for a CAS to develop a rule for each different input, and therefore *they design their rules so that they can interact with each other* [Holland 1996]:

- The rules can operate in parallel, and when they involve different 'choices' CAS give priority to those containing more information.
- The interaction between rules allows confronting a multiplicity of very different situations with a small number of rules.
- The combination of these rules enables 'creativity', allowing CAS to respond to situations not previously experienced[56]

*CAS creativity increases the unpredictability of their future states, allowing them to design unpredictable responses.*

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[54] Information becomes a key variable to explain the very nature [and individuality] of CAS. CAS adapt and evolve based on 'information received'; a system that receives no information neither adapts nor 'evolves'; two systems that receive different information may evolve in a different way.

[55] It would require modeling all 'rules of conduct' developed by a CAS; which are different for each different CAS [not all CAS of a class have the same experience, and therefore not all develop the same 'rules' or behavior]. And yet, CAS future responses would be predictable only in the very short term.

[56] 'If we had well-defined parts of the situation that we handle, and we could combine those parts, then we could handle situations that we had never seen before' [Holland 1996].

The third is that **CAS are decision-making systems; when facing a range of possible 'courses of action', they can choose which one to develop,** basing their decisions on criteria that often cannot be fully modeled.

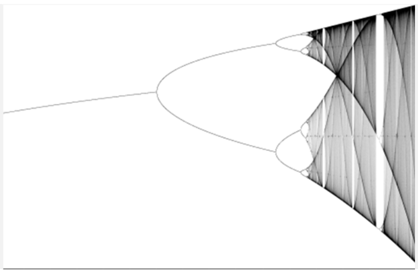

***Image 08: Feigenbaum Fractal*** [image: en.wikimedia.org]

*If we consider the attractor as a representation of a system capable of continuously deciding between two states; we see that from a known situation, the system can locate in a few steps in a large number of different situations. Prediction becomes impossible.*

CAS generally make 'rational' decisions[57], and this implies some predictability if the decision criteria are known, but becomes unpredictability if they are not or the decision is made as a response to a chaotic environment. *Interaction with chaotic environments involves unpredictability even in rational decision making:*

- Limiting the options[58]
- Forcing choices relating to unforeseen impacts or not completely predictable future situations[59].

**CAS decision making ability is another source of unpredictability.**

However, **CAS have teleology; a quality that reduces the unpredictability of their future states;** they can establish goals [equivalent to 'desired states' or possible 'futures'] and develop strategies to achieve them; and this 'teleology' reduces unpredictability if desired states are 'known'[60].

CAS teleology directs their changes towards the state they consider will be their 'optimal state'; but it partly depends on the status of the environment and therefore CAS have to 'anticipate' environment's future state in order to 'imagine' how their optimal state will be.

And this 'need for anticipation' faces the unpredictability of chaotic environments; Chaos hinders CAS "teleology" by preventing them to accurately anticipate the future; the real situations are always different than expected, and to counteract it CAS, develop two strategies:

- Increase their **resilience** [already mentioned]

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[57] A rational decision-maker may be defined as one which always chooses from the set of possible options the one with the highest 'utility', measured based on 'rational criteria'.

[58] For example, a "non-predictable" financial crisis caused by a chaotic behavior limits the availability of economic resources of essentially organized systems [cities, countries...], limiting their possible 'rational choices'.

[59] It is equivalent to making decisions with 'incomplete information', what can turn 'rational choices' into erroneous decisions

[60] In SES the 'desirability degree' of different system states may be relatively 'known' using statistical analysis.

- **Monitor their environments,** reviewing the actual course of events in relation to the forecasts and correct prediction errors.

**Unpredictability does not prevent CAS from making predictions, but forces them to set time limits for each type of prediction; to monitor the actual course of events -making the necessary corrections as soon as they are required-, and to increase their resilience in order to stand unforeseeable negative impacts.**

## SUSTAINABILITY DEGREE AS THE DEGREE OF RATIONAL DESIRABILITY OF A STATE

CAS have teleology, their actions require constant decision-making that incorporates directionality; systems obtain different utility from different states and their decision making processes intend to 'direct' them towards the most preferred or desired states[61].

And for any ‘rational’ CAS sustainability must be its ‘preferred’ state because it is its optimal state; provides the higher utility for the system and therefore systems must 'direct' their processes in that direction. *The existence of optimal states introduces ‘directionality’ in decision making processes.*

'Rational' desirability is incorporated as a relevant variable for the sustainability of any possible system status and 'rational undesirability' as a relevant variable for 'unsustainability'. **No CAS tries to perpetuate an undesired situation or status; it always tries to change it towards more 'preferred' situations, and the ‘undesirability degree’ becomes an ‘unsustainable degree’.**

And from this perspective, the extreme values of S mean the following:

- $S_T[I]=1$ means that at T moment, the system is in the state of highest possible rational desirability[62].
- $S_T[I]=0$ means that at T moment, the system is in a totally non-desired state [no rational desirability at all] in which the system disappears as a decision making entity.

It is important to emphasize that the 'preference' of some states over others is sustainable as long as it is based on rational criteria, i.e. **the preference degree needs to be based on the degree to which a state is optimal for the system and involves 'consistency' with the environment.**

Sustainability constitutes a 'desired state' and this allows us to consider the Sustainability Degree as an utility function for decision making, with many interesting implications especially referred to SES, where collective decisions need to be addressed constantly on a rational basis[63].

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[61] We are going to consider that "preferred" equates "desired" and that both involve "rationality" or ‘rational’ utility maximization. It is therefore a "desirability" that seeks to direct the system towards states that are preferred because they are optimal; no better states are possible.

[62] This fact leads us to alternatively designate the sustainability limit of relevant variables as ‘sustainability objective or goal’

[63] See ANNEX IX_ DECISION MAKING