

Redshift and the Curvature of Space

Valery Zhukov¹⁾

If space is curved, then the nature of the cosmological redshift and time dilation is the same effect as the gravitational redshift and time dilation in a gravitational field. This is confirmed by the fact that the ratio between redshift and time dilation is the same for both gravitational and cosmological redshifts.

Let us assume that the space is infinite and:

> the space is uniformly filled with matter with density ρ

> let's introduce two points in space : we are the point A . We are observing photons from the point B .

Let us consider a curved space (Ricci curvature tensor). In these coordinates the volume form expands in a Taylor series around the vicinity of any point p :

$$d\mu_q = [1 - \frac{1}{6}R_{jk}x^j x^k + O(|x|^3)]d\mu_{Euclidean} \quad (1)$$

Thus, if the Ricci curvature $Ric(\xi, \xi)$ is negative in the direction of a vector ξ , a narrow conical region of the geodesic, emanating from p in the direction , will have larger volume than it would in Euclidean space .

Let's assume that the Ricci curvature is negative in all the directions. If the Ricci curvature is negative, the density integral of the current volume per volume unit, from the point of view A , will also be greater in B , just as the gravitational potential

$$\phi = -G \int_V \frac{\rho dV}{r} \quad (2)$$

In other words, the gravitational potential (ϕ) for the photon travelling from B (ϕ_1) to $A(\phi_0)$ from the point of view A will decrease. As a result, we get a gravitational redshift (in the linear approximation)

$$Z = \Delta\phi/c^2 \quad (3)$$

and the difference of potentials fo B and A is proportional to the difference in volumes

$$\phi_1/\phi_0 = V_1/V_0 \quad (4)$$

moreover we also observe time dilation

$$dt_1 = dt_0(1 + (d\phi)/c^2), \quad (5)$$

observed at the point A for the photon from point B , which fully corresponds to the existing relation between the redshift Z and time dilation

$$dt_1 = (1 + Z)dt_0 \quad (6)$$

for distant galaxies.

¹⁾valeryzhukov@mail.ru / Valery Zhukov, Netanya, Israel