A commentary about the solution in 2nd order Schwarzschild's Equation

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In section 3 of my recent paper [1] is obtained the 2nd order approximation the solution

$$u = \frac{mc^2}{l^2}\,(1 + ecos\varphi + e(\delta\omega)sen\varphi), \tag{1}$$

for the Schwarzschild's equation

$$\frac{d^2u}{d\varphi^2} + u = \frac{mc^2}{l^2} + 3mu^2, \tag{2}$$

where

$$\delta\omega = \frac{3m^2c^2}{l^2}\,\varphi. \tag{3}$$

How $\delta\omega << 1$ the equation (1) is approximated to

$$u = \frac{mc^2}{l^2}\,(1 + ecos(\varphi - \delta\omega)), \tag{4}$$

which explains the precession of the perihelion of Mercury on General Relativity.

We criticize this reasoning, which is broadly adopted by several authors [2], by the fact that in addition to not solve the differential equation (2) that originated, leads to a range of different variation that (1). While in (4) the maximum value of u is given by

$$u\ = \frac{mc^2}{l^2}\,(1 + e), \tag{5}$$

and the minimum is

$$u\ = \frac{mc^2}{l^2}\,(1 - e), \tag{6}$$

with the primary equation (1) u may vary between $-\infty$ to $+\infty$, since the angle φ may also vary between these two infinite extremes and not only as a function argument $cos\varphi$, but appears as the product $\varphi sen\varphi$, as can be seen easily by replacing (3) in (1).

For a full turn counterclockwise is made $\varphi = 2\pi$ in (3) to obtain the angular displacement of perihelion

$$\delta\omega = \frac{6\pi GM}{ac^2(1-e^2)}, \qquad (7)$$

being $a = mc^2/l^2$ the semi-major axis of the ellipse, $m = GM/c^2$ and $l^2 = aGM(1-e^2)$, where l is the angular momentum that is conserved.

For two complete revolutions must make $\varphi = 4\pi$, for k complete revolutions must make $\varphi = 2k\pi$ in (3), which would (1) become arbitrarily large in modulus with increasing k. The assumption $\delta\omega \ll 1$ mentioned in the passage (4) can only apply to a limited number of turns, but far from the reality of our planetary system, as said earlier, where billions of revolutions have already been distributed around the Sun and probably many others will still be given by a long time, perhaps infinite.

What we want to comment here, and we did not realize before, is that, regardless of previous criticism, the solution (1) can bring a greater truth about the physical reality that the solution (4): more important than a displacement of perihelion and even more dramatic, would be a spiral movement.

Since $u = 1 / r$, r is the distance of the planet to the origin of the system, even the infinite and zero initial speed motion converge to the origin of the system, which in this model would be located the Sun (or other power source), in a spiral motion modulated with sine and cosine functions.

Perhaps this conclusion may not be physically feasible, but it is mathematically possible.

REFERENCES

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