Bell's theorem refuted, and 't Hooft's superdeterminism rejected, as we factor quantum entanglements in full accord with commonsense local realism

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Abstract: Commonsense local realism (CLR) is the fusion of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively). Advancing our case for a wholly CLR-based quantum mechanics, we use undergraduate maths and logic to factor the quantum entanglements in EPRB and Aspect (2002). Such factors (one factor relating to beables in Alice's domain, the other to beables in Bob's), refute Bell's theorem and eliminate the need for 't Hooft's superdeterminism. An obvious unifying algorithm (based on spin-s particles in a single thought-experiment) is foreshadowed and left as an exercise. That is, to emphasize the physical significance of our component results, we here factor EPRB and Aspect (2002) separately and in detail.

On one supposition we absolutely hold fast; that of local/Einstein causality: "The real *factual* situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former," after Einstein (1949:85).

"It is a matter of indifference . . . whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous. However, [Bell writes] as if λ were a single continuous parameter," Bell (1964:195). λ may denote "any number of hypothetical additional complementary variables needed to complete quantum mechanics in the way envisaged by EPR," Bell (2004:242).

#**1.** Bound by commonsense local realism (CLR), the fusion of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively), let Alice and Bob be two experimenters, independent and (for now) free-willed.

#**2.** Re Bell (1964) (available online; see References): Let the unnumbered equations between Bell's (14)-(15) be (14a)-(14c); let unit-vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ replace his $\vec{a}, \vec{b}, \vec{c}$. Let Z be shorthand for EPRB, the experiment that Bell (1964) considers. Let expectation $\langle A(\mathbf{a})B(\mathbf{b}) \mid Z\rangle$ replace Bell's equivalent term $P(\vec{a}, \vec{b})$; etc. Let $P(. \mid Z)$ denote a probability conditioned on Z; etc.

#**3.** Given Bell's broad λ specifications (above), let primes ($'$) identify any λ in Bob's domain. Then, under the conservation of total angular momentum in EPRB and (for later) in Aspect (2002), let $\lambda + \lambda' = 0$; ie, $\lambda' = -\lambda$. Thus, combining Bell's (1)-(3) and (12)-(13) in our terms:

$$A(\mathbf{a}, \lambda) = \pm 1 \equiv A^{\pm};\ B(\mathbf{b}, \lambda') = B(-\lambda, \mathbf{b}) = \pm 1 \equiv B^{\pm};\ \int d\lambda \rho(\lambda) = 1; \tag{1}$$

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$$\langle A(\mathbf{a})B(\mathbf{b})\,|\,Z\rangle = \int_Z d\lambda\rho(\lambda)A(\mathbf{a},\lambda)_Z B(\mathbf{b},\lambda')_Z = \int_Z d\lambda\rho(\lambda)A(\mathbf{a},\lambda)_Z B(-\lambda,\mathbf{b})_Z \neq -\mathbf{a}{\cdot}\mathbf{b}. \tag{2}$$

#**4.** (1) captures a vital CLR assumption, in full accord with Bell (1964:196) and Einstein (1949:85): the result B does not depend on the setting $\mathbf{a}$; nor A on $\mathbf{b}$.

#**5.** (2) is the 'impossibility theorem' – Bell 1964:(2) $\neq$ Bell 1964:(3) – that we reject. For (2) captures Bell's allegation that (1) and LHS (2) cannot equal RHS (2); RHS (2) being the quantum mechanical expectation that we endorse.

#**6.** So we now refute (2), in full accord with CLR. That is, bound by (1) and LHS (2), we show that the probability of any result is determined by local factors alone; one factor relating to beables in Alice's domain, the other to beables in Bob's. To that end, let a trigonometric argument ($\mathbf{u}$,$\mathbf{v}$) denote the angle between vectors $\mathbf{u}$ and $\mathbf{v}$, and let s denote the relevant spin: ie, $s = 1/2$ for the spin-half particles in EPRB, experiment Z; $s = 1$ for the photons in Aspect (2002), experiment X. Then, under a unifying thought-experiment Q, a generalized base-factor derivation of the related quantum mechanical expectations follows:

$$\text{If } A(\mathbf{a},\lambda)_Q = \sqrt{2}\cos 2s(\lambda,\mathbf{a});\; B(-\lambda,\mathbf{b})_Q = \sqrt{2}\cos 2s(-\lambda,\mathbf{b});\; \int_Q d\lambda\rho(\lambda) = \int_0^{4\pi} \frac{d\lambda}{4\pi} = 1: \tag{3}$$

$$\text{Then } \langle A(\mathbf{a})B(\mathbf{b})\,|\,Q\rangle = \int_Q d\lambda\rho(\lambda)A(\mathbf{a},\lambda)_Q B(\mathbf{b},-\lambda)_Q \tag{4}$$

$$= \int_0^{4\pi} \frac{d\lambda}{4\pi}\sqrt{2}\cos 2s(\lambda,\mathbf{a})\sqrt{2}\cos 2s(-\lambda,\mathbf{b}) = (-1)^{2s}\cos 2s(\mathbf{a},\mathbf{b}). \tag{5}$$

$$\text{So } \left\langle A(\mathbf{a})B(\mathbf{b})\,|\,Q, s=\frac{1}{2}\right\rangle = \langle A(\mathbf{a})B(\mathbf{b})\,|\,Z\rangle = -\cos(\mathbf{a},\mathbf{b}) = -\mathbf{a}{\cdot}\mathbf{b}; \tag{6}$$

$$\langle A(\mathbf{a})B(\mathbf{b})\,|\,Q, s=1\rangle = \langle A(\mathbf{a})B(\mathbf{b})\,|\,X\rangle = \cos 2(\mathbf{a},\mathbf{b}). \tag{7}$$

#**7.** Thus Bell's 'impossibility theorem' is refuted: at (6) in the context of Z, EPRB; at (7) in the context of X, Aspect (2002). However, to be clear about the physical significance of our base-factors (which together deliver the correct overall expectations), we now expand each base-factor in (3). In this way, the physically-significant components (the measured components) of each expectation are exposed. Further, for clarity, we derive the components of $\langle A(\mathbf{a})B(\mathbf{b})\,|\,Z\rangle$ and $\langle A(\mathbf{a})B(\mathbf{b})\,|\,X\rangle$ separately. So, for EPRB, using (3) with $s = \frac{1}{2}$:

$$A(\mathbf{a},\lambda)_Z = \sqrt{2}\cos(\lambda,\mathbf{a}) = (\sqrt{2}\cos^2\frac{(\lambda,\mathbf{a})}{2} \pm \frac{1}{2}) - (\sqrt{2}\sin^2\frac{(\lambda,\mathbf{a})}{2} \pm \frac{1}{2}); \tag{8}$$

$$B(\mathbf{b},\lambda')_Z = B(-\lambda,\mathbf{b})_Z = \sqrt{2}\cos(-\lambda,\mathbf{b}) = (\sqrt{2}\cos^2\frac{(-\lambda,\mathbf{b})}{2} \mp \frac{1}{2}) - (\sqrt{2}\sin^2\frac{(-\lambda,\mathbf{b})}{2} \mp \frac{1}{2}). \tag{9}$$

#**8.** Then from LHS (2) and using (8)-(9):

$$\langle A(\mathbf{a})B(\mathbf{b})\,|\,Z\rangle = \int_Z d\lambda\rho(\lambda)A(\mathbf{a},\lambda)_Z B(\mathbf{b},-\lambda)_Z \tag{10}$$

$$= \int_0^{4\pi} \frac{d\lambda}{4\pi}[(\sqrt{2}\cos^2\frac{(\lambda,\mathbf{a})}{2} \pm \frac{1}{2})(\sqrt{2}\cos^2\frac{(-\lambda,\mathbf{b})}{2} \mp \frac{1}{2}) - (\sqrt{2}\cos^2\frac{(\lambda,\mathbf{a})}{2} \pm \frac{1}{2})(\sqrt{2}\sin^2\frac{(-\lambda,\mathbf{b})}{2} \mp \frac{1}{2})$$

$$-(\sqrt{2}\sin^2\frac{(\lambda,\mathbf{a})}{2} \pm \frac{1}{2})(\sqrt{2}\cos^2\frac{(-\lambda,\mathbf{b})}{2} \mp \frac{1}{2}) + (\sqrt{2}\sin^2\frac{(\lambda,\mathbf{a})}{2} \pm \frac{1}{2})(\sqrt{2}\sin^2\frac{(-\lambda,\mathbf{b})}{2} \mp \frac{1}{2})] \tag{11}$$

$$= \frac{1}{2}\sin^2\frac{(\mathbf{a},\mathbf{b})}{2} - \frac{1}{2}\cos^2\frac{(\mathbf{a},\mathbf{b})}{2} - \frac{1}{2}\cos^2\frac{(\mathbf{a},\mathbf{b})}{2} + \frac{1}{2}\sin^2\frac{(\mathbf{a},\mathbf{b})}{2} \tag{12}$$

$$= P(A^+B^+ \,|\, Z) - P(A^+B^- \,|\, Z) - P(A^-B^+ \,|\, Z) + P(A^-B^- \,|\, Z) = -\mathbf{a}\cdot\mathbf{b}. \text{ QED.} \tag{13}$$

#**9.** Comparing (13) with RHS (2), Bell's impossibility-claim re EPRB is refuted; the physical significance of our component-by-component factor-analysis being evident in (13). Further, any probability $P(. \mid Z)$ can be derived from the above factors. The expanded derivation for Aspect 2002:(6) follows, using (3) with $s = 1$. Let

$$A(\mathbf{a}, \lambda)_X = \sqrt{2}\cos 2(\lambda, \mathbf{a}) = (\sqrt{2}\cos^2(\lambda, \mathbf{a}) \pm \frac{1}{2}) - (\sqrt{2}\sin^2(\lambda, \mathbf{a}) \pm \frac{1}{2}); \tag{14}$$

$$B(\mathbf{b}, \lambda')_X = B(-\lambda, \mathbf{b})_X = \sqrt{2}\cos 2(-\lambda, \mathbf{b}) = (\sqrt{2}\cos^2(-\lambda, \mathbf{b}) \mp \frac{1}{2}) - (\sqrt{2}\sin^2(-\lambda, \mathbf{b}) \mp \frac{1}{2}). \tag{15}$$

#**10.** Then, adapting LHS (2) and using (14)-(15):

$$\langle A(\mathbf{a})B(\mathbf{b}) \,|\, X\rangle = \int_X d\lambda \rho(\lambda) A(\mathbf{a}, \lambda)_X B(\mathbf{b}, -\lambda)_X \tag{16}$$

$$= \int_0^{4\pi} \frac{d\lambda}{4\pi}[(\sqrt{2}\cos^2(\lambda, \mathbf{a}) \pm \frac{1}{2})(\sqrt{2}\cos^2(-\lambda, \mathbf{b}) \mp \frac{1}{2}) - (\sqrt{2}\cos^2(\lambda, \mathbf{a}) \pm \frac{1}{2})(\sqrt{2}\sin^2(-\lambda, \mathbf{b}) \mp \frac{1}{2})$$

$$-(\sqrt{2}\sin^2(\lambda, \mathbf{a}) \pm \frac{1}{2})(\sqrt{2}\cos^2(-\lambda, \mathbf{b}) \mp \frac{1}{2}) + (\sqrt{2}\sin^2(\lambda, \mathbf{a}) \pm \frac{1}{2})(\sqrt{2}\sin^2(-\lambda, \mathbf{b}) \mp \frac{1}{2})] \tag{17}$$

$$= \frac{1}{2}\cos^2(\mathbf{a}, \mathbf{b}) - \frac{1}{2}\sin^2(\mathbf{a}, \mathbf{b}) - \frac{1}{2}\sin^2(\mathbf{a}, \mathbf{b}) + \frac{1}{2}\cos^2(\mathbf{a}, \mathbf{b}) \tag{18}$$

$$= P(A^+B^+ \,|\, X) - P(A^+B^- \,|\, X) - P(A^-B^+ \,|\, X) + P(A^-B^- \,|\, X) = \cos 2(\mathbf{a}, \mathbf{b}). \text{ QED.} \tag{19}$$

#**11.** Bell's 'impossibility theorem' is thus doubly refuted. For, via (8)-(13) and (14)-(19), CLR delivers the correct quantum mechanical expectation values for two important experiments.

#**12.** We now move to refute 'Bell's theorem', a term coined by CHSH (1969) for Bell 1964:(15) when they generalized it to realizable experiments. As expected, the results of such experiments continue to contradict Bell's theorem to our total satisfaction: for all loopholes are closed under CLR. So, in our terms, using (6) or (13) and variants, here's Bell 1964:(15):

$$1 + \langle A(\mathbf{b})B(\mathbf{c}) \,|\, Z\rangle \geq |\langle A(\mathbf{a})B(\mathbf{b}) \,|\, Z\rangle - \langle A(\mathbf{a})B(\mathbf{c}) \,|\, Z\rangle|; \tag{20}$$

$$\text{ie, } 1 - \cos(\mathbf{b}, \mathbf{c}) = 1 - \mathbf{b}\cdot\mathbf{c} \geq |\,\mathbf{a}\cdot\mathbf{c} - \mathbf{a}\cdot\mathbf{b}\,| = |\cos(\mathbf{a}, \mathbf{c}) - \cos(\mathbf{a}, \mathbf{b})\,|: \tag{21}$$

which is false for $-\pi/2 < \phi < \pi/2$ if $(\mathbf{a}, \mathbf{b}) = (\mathbf{b}, \mathbf{c}) = \phi$ and $(\mathbf{a}, \mathbf{c}) = 2\phi$. So Bell's 1964:(15), "Bell's theorem" per CHSH (1969), is refuted.

#**13.** We conclude that Bell's theorem is irrelevant to any serious physical theory. In particular, it should no longer be a constraint on 't Hooft's (2014) program, especially not at 't Hooft 2014:(8.22)-(8.23). Finally, reviewing paragraph #1 in the light of all our results, we conclude that Alice and Bob have sufficient free-will to complete any experiment to our CLR satisfaction. For, in refuting Bell's theorem, we eliminate the need for 't Hooft's superdeterminism in physics.

#**14.** So, thanks to the team acknowledged below, the story that began with Mermin (1988) continues. And thanks to viXra.org, there's http://vixra.org/abs/1405.0020: a rough draft that also meets Bell's (1990:10) expectation that relativity and quantum mechanics would be reconciled; ie, it too delivers Bell's hope (2004: 167) for a simple constructive locally-causal (CLR) model of reality.

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